



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

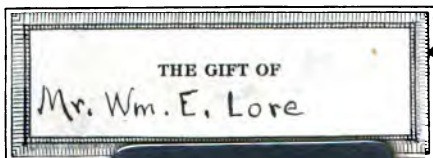
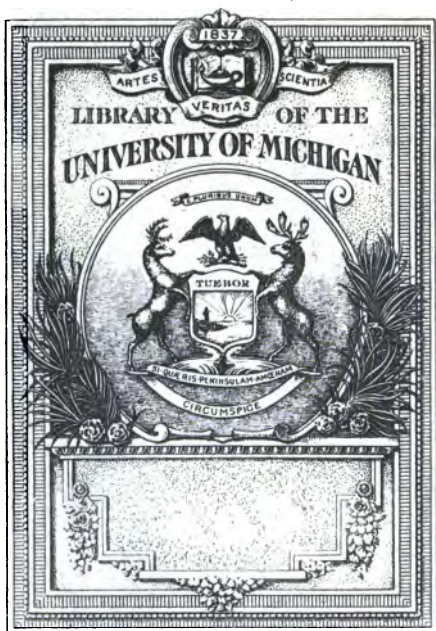
Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

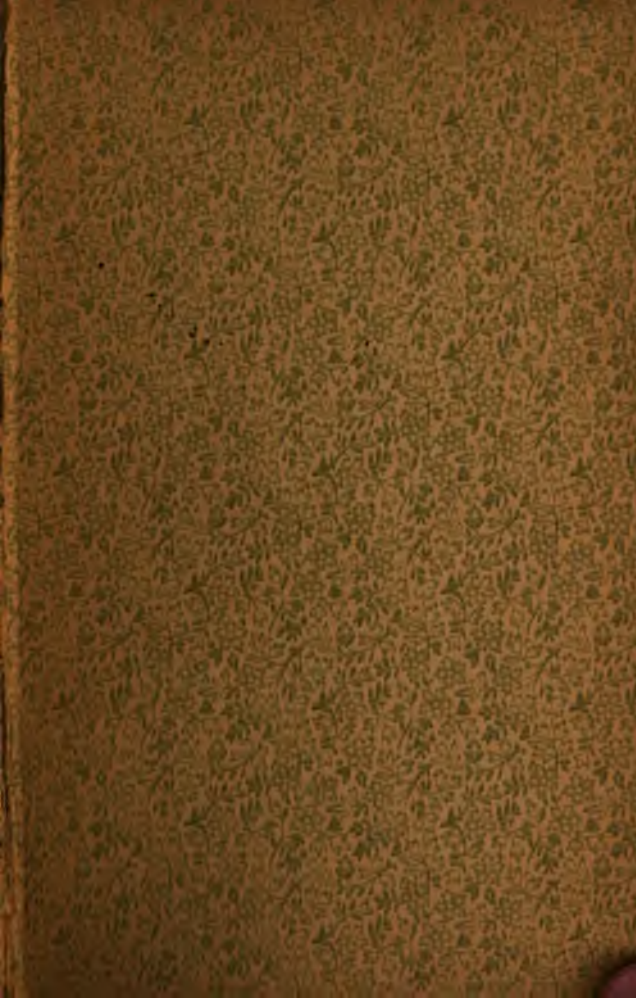
We also ask that you:

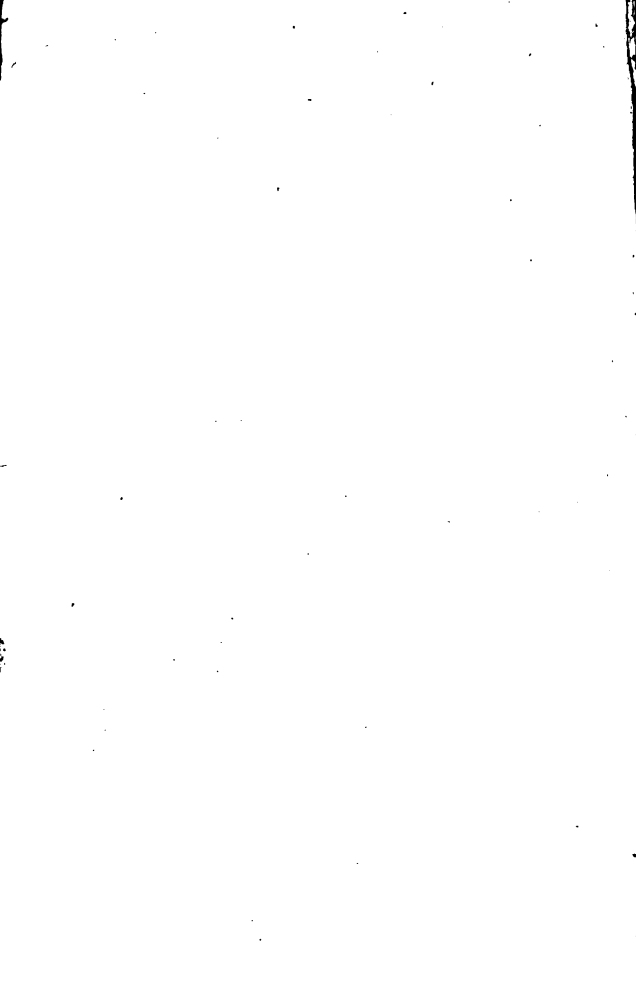
- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>





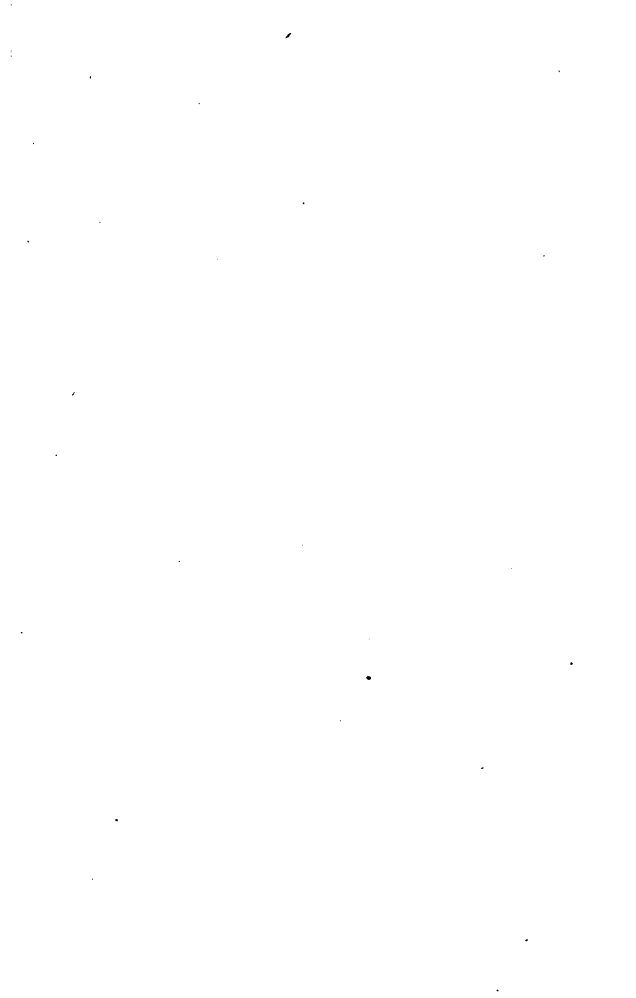


TA

151

,I6

1900



MECHANICS' POCKET MEMORANDA

A CONVENIENT POCKETBOOK

FOR ALL PERSONS INTERESTED IN

Mechanical Engineering, Steam Engineering, Electrical
Engineering, Railroad Engineering, Hydraulic
Engineering, Bridge Engineering, Etc.

BY

INTERNATIONAL CORRESPONDENCE SCHOOLS

SCRANTON, PA.

6th Edition, 202d Thousand, 13th Impression

SCRANTON, PA.

INTERNATIONAL TEXTBOOK COMPANY

COPYRIGHT, 1893, 1894, 1897, 1898, 1900.
BY
THE COLLIERY ENGINEER COMPANY.

All rights reserved.



PRINTED BY
INTERNATIONAL TEXTBOOK Co.,
SCRANTON, PA.

*Gift
Mr. William E. Linn
6-6-1928*

PREFACE.

The first edition (2,000 copies) of the pocket-book of which this is the outcome was issued in October, 1893, in the form of a notebook containing 74 printed pages, with about the same number of blank pages for memoranda, whence the title *Mechanics' Pocket Memoranda*. The little book proved so popular that a new edition (10,000 copies) enlarged to 110 pages was issued 8 months later. In June, 1897, the blank pages were discarded, the work was entirely recast and enlarged to 318 pages, and the edition (third) consisted of 25,000 copies. Before printing the fifth edition (March, 1898), a large amount of matter relating especially to Plumbing, Heating, and Ventilation and the Building Trades was taken out, replaced by tables of logarithms, trigonometric functions, etc., together with directions for using them, and other new matter, the result being to confine the work more particularly to different branches of engineering.

It has been the aim of the publishers, from the first, to present to the public a handbook of a size convenient to carry in the coat or hip pocket—a pocketbook in reality—which would contain rules, formulas, tables, etc. in most common use by

engineers, together with explanations concerning them and practical examples illustrating their use. We have not endeavored to produce a condensed cyclopedia of engineering or of any branch of it, but we have striven to anticipate the daily wants of the user and to give him the information sought in the manner best suited to his needs. Our aim has been to meet the necessities not only of the engineer but of all in any manner interested in engineering, and in accomplishing this we have selected that rule, formula, or process which was, in our opinion, best adapted to the circumstances of the case, describing it fully, giving full directions how and when to use it, and not mentioning other methods (when such were available); in other words, *we* have made the selection instead of leaving the choice to the judgment of the user, which is frequently at fault. The exceedingly large sale, 147,000 copies in 10 years, proves that the idea was popular and has vindicated our judgment. We hope that succeeding editions will meet and merit the same approval that has been accorded those preceding.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

December 1, 1903.

INDEX.

A.		PAGE
Absolute pressures		25
Alloys		19
Alternating system, Size of wires for.....		239
Aluminum and copper, Properties of		250
Ampere		230
Angles or arcs, Measures of.....		2
Annunciator system		243
Anode of an electric battery		263
Arc-light circuits, Size of wire for		253
Arcs or angles, Measures of.....		2
Areas and circumferences of circles		82-90
" Irregular		119
" of circles, Table of.....		82
Avoirdupois weight		3

B.		
Batteries, Storage.....		267
" " Regulation of		267
" Various chemical		263-266
Beams, Bending moments in.....		152
" Cantilever		152
" Deflection of.....		150
" fixed at both ends		150
" Simple		150
Bearing of a line		276
" " deduced		283
Bearings for line shafting, Distance apart of.....		193
Bell wiring		241
Belting		140
" Rope		211
Belt pulleys		204
Bending moments in beams		152
Birmingham wire gauge		248
Blow, Force of a		140

	PAGE
Blueprint paper, To make	175
Blueprints	175
Bollers	158
" (steam)	158
" " Foaming and priming of	168
" " Horsepower of	168
" " Inspection and care of	162
" " Prevention of scale in	164
" " To develop dome of	158
" " To develop slope sheet of	159, 160
Bolts for cylinder heads	217
" " steam chests	217
" Standard proportions of	22
Booster	267
Brake, Prony	260
Bridge, Wheatstone	270
Briggs, or common, logarithms	82

C.

Cables, Carrying capacity of	239
" Chain	14
" Testing of (electrical)	271
Calendar, Perpetual	327
Candlepower	236
Capacity, Measures of	6
" of cables	239
Cathode of an electric battery	263
Center of gravity	121
Centrifugal force	121
Chain cables, Wrought-iron	14
Chains and ropes	157
Change gears	178
Characteristic of a logarithm	33
Chemical treatment of feedwater	165
Chimneys	170
" Formulas for	171
" Table of sizes of	172
Chord of circle	116
Circle, Area of	113
" Chord of	116
" Circumference of	113
" Segment of	115
Circles, Tables of circumferences and areas of	82-90
Circuits, Derived, or shunt	234
" Motor	253
" Size of wire for arc-light	253
Circular pitch, Formula for	228
" " Table of	229
" rings, Area of	116
" " Volume of	117
Circumferences and areas of circles	82-90

INDEX.

vii

PAGE

Clearance, Piston	216
Coefficient of elasticity.....	152
Coefficients of expansion.....	19
Columns, Formulas for strength of.....	156
Commutator, Sparking at	259
Compass surveying.....	276-279
Compound-gear ed lathes, Screw-cutting with	182
Compound pulley, Formula for	138
Compression, Table of ultimate strength for	151
Conductivity, Electrical	232
Conductor, Direction of motion of.....	233
" Size of	235-240
Cone, Formulas for.....	117
Conical frustum, Formulas for	117
Connecting-rods	224
Connections for dynamo-electric machines	252
Copper and aluminum, Properties of	250
Corliss engine crank-shaft	223
" " cylinder.....	221
Corrosion of boilers	163
Cotters for connecting-rods.....	226
Couplings, Flange	194
" Flexible	195
" Proportions of	195
" Shaft	194
Course of a line in surveying.....	276
Crank-shafts for Corliss engines	223
" " " high-speed engines	223
Cross-over tracks	325
Cube root.....	107
Cubes, squares, and fifth powers.....	112
Cubic expansion, Coefficient of	19
" measure	2
Current, Rules for direction of electrical.....	232
" Strength of	231
Curves, Deflection angles of	286
" Degree of.....	286
" Elevation of railroad.....	311
" of Saturation.....	256
" Tangent distance of	287
" To lay out with transit.....	288
" " " without transit	290
Curving of rails.....	309
Cylinder heads	217
" " Bolts for	217
Cylinders for Corliss engines.....	221
" Formulas for strength of	157
" Proportions of	216-219
" Stuffingbox for	228
" Surface of	116
" Volume of	116

D.

PAGE

Decimal equivalents of parts of one inch	91
Decimals of a foot, Equivalent in inches of	91
Declination of needle	277
Deflected line.....	281
Deflection of beams	152
Deflections, Tangent and chord	297
" " " " Formulas for	297
Density.....	25
Derived circuits	234
Designs of machine details.....	192
Development of boiler dome	158
" " " slope sheet	159
Diagram, Slide-valve.....	188
Diametral pitch, Formula for.....	228
" " Table of.....	229
Differential pulley	138
Division by logarithms.....	42
Dome of boiler, To develop.....	158
Double movable pulley.....	137
Draft of chimneys, Formulas for	171
Drills, Speed of twist	177
Dry measure	3
Duty of pumps	144
Dynamo design.....	254
" electric machines, Connections for.....	252
" machines	246
" wiring, Underwriters' rules for.....	245
Dynamos and motors	253
" Faults of	258

E.

Earthwork, Calculation of	306
Eccentric	227
Efficiency, Lamp	240
" Motor	253-260
Elastic limit, Table of	152
Electric gas lighting	269
" motors, Application of	261
Electricity	230-275
Electrodeposition	269
Electrolyte of an electric battery.....	263
Electromagnet, Polarity of	233
Electromotive force	230
" " Formula for.....	254
Elements, Table of chemical.....	16
Elevation of railroad curves	311
Ellipse, Formulas for.....	115
Emery wheels, Speed of	176
Engine horsepower, Formula for.....	185
English and metric measures, Conversion tables of	7

INDEX.

ix

PAGE

Equivalent decimal parts of one foot.....	91
“ “ “ “ inch	91
Evolution by logarithms	46
Exhaust heating	173-174
“ ports, Dimensions of	217
Expansion, Coefficients of	19
Exponents	32

F.

Factors of safety, Table of	151
“ Prime	80
Failure of dynamos	258
Falling bodies	120
Feedwater heaters	166
“ Methods of purifying	165
“ Testing of	164
Field magnet.....	255
“ Reversal of	258
Fifth powers, squares, and cubes.....	112
“ root.....	110
Filtration of feedwater.....	165
Flange coupling	194
Flanges, Pipe.....	215
Flexible coupling	195
Flexure, Ultimate strength of	151
Flow of water in pipes.....	146
Fluxes for soldering and welding	24
Foaming of boilers	168
Foot, Decimals of a	91
Force, Formula for electromotive	254
“ Magnetizing	259
“ of a blow	140
Forces, Resultant of	137
Formulas.....	93-102
“ How to use.....	93
Frog (railroad work).....	312
“ Angle	313
“ Crotch or middle	324
“ distance	314
Frustum of cone, Formulas for	117
“ “ pyramid, Formulas for	118
Fusion, Latent heat of	18
“ Temperature of	18

G.

G ² , Values of	153
Galvanometer	271
Gases, Weights of.....	14
Gas lighting, Electric.....	269
Gauge, Birmingham wire	248
“ B. & S. wire.....	248

Gauge, sizes of wire, with equivalent sectional areas	243
Gearing, Formulas for	228
Gears, Change, for screw-cutting	178
" To calculate speed of	142
Gibs for connecting-rods	224
Gland	228
Grade lines	296
" Rate of	296
Gravity, To find center of	121
Grindstone, Speed of	176
Gyratation, Square of least radius of	153
" To find radius of	131
" " " " experimentally	133

H.

Hangers, Shaft	202
Heat	19
" Latent, of fusion	18
" of liquid	25
Heating by exhaust steam	173
" of dynamos	259
" surface, Square feet of, per horsepower	163, 169
" " Ratio of, to grate area	168
Helix, Formula for	116
" To construct a	116
High-speed engines, Crank-shaft for	223
Horsepower of belts	140
" " boilers	163
" " electrical currents	231
" " engines	185
" " pumps	184
" " rope belting	210
" Theoretical	184
Hydrokinetics	145
Hydromechanics	144
Hydrostatics	144
Hyperbolic logarithms	32

I.

I, Values of	153
Incandescent lamp data	240
" wires, Underwriters' regulations for	245
Inch, Equivalent decimal parts of	91
Inches and parts thereof in decimals of one foot	91, 92
Inclined planes, Formula for	138
Incrustation in boilers	164
Indicated horsepower of engines, Formula for	185
Inertia, To find moment of	125
" " " " " experimentally	133
" " " " " for various sections	153
Inspection of boilers	162

INDEX.

xi

PAGE

Insulation, Test of	272
Interior wiring	235
Involution by logarithms	44
Iron bars, Weight of round and square	21
Irregular areas	119

J.

Joint coupling, Universal	195
Journal box, Design of	195

K.

Kerosene in boilers	167
Keys for shafting, Proportions of	194
Kilowatt	232

L

Lamps, Efficiency of	240
" Incandescent, data	240
" in series (electric light)	253
Lap, Inside and outside	187
Latent heat of fusion	18
" " vaporization	18
Lathe, Change gears of	178
" Compound-gearred	182
" Simple-gearred	178
Law, Ohm's	231
Lead of valve	187
Lead, Weight of sheet	21
Leakage, Magnetic	258
Leclanché cell	244, 253
Legal ohm	230
Length, Measures of	5
Leveling, Direct	292
" Grade lines in	296
" notes, How to check and keep	294
" Profiles in	296
Levers	136
Linear expansion, Coefficient of	18
" measure	1
Line shafting	193
Lines of force, Leakage of	257
" " " Number of	254
Lining for seats	216
Liquid, Heat of	25
" measures	4
Liquids, Weights of	13
Locknuts	192
Logarithmic table	50-67
" " Use of	34
Logarithms	32
Long ton table	3

M.

PAGE

Machine design	175
" tools, Cutting speeds for	176
" " Motors for	263
Magnetic meridian	277
" permeability	257
Manila rope belting	209
" " Weight of	209
Mantissa of a logarithm	33
Materials, Strength of	150
Mean effective pressure	185
Measure, Cubic	2
" Dry	3
" Linear	1
" Liquid	4
" Surveyor's	1
" Surveyor's square	2
Measures and weights	1-4
" " Metric	5
" of angles or arcs	2
" " capacity	6
" " length	5
" " surface (not land)	5
" " volume	5
" " weight	6
Mechanical powers	136
Mechanics	120, 149
Mensuration	113
Meridian, Magnetic	277
" True	277
Metals, Weights of	10
Metric and English measures, Conversion table for ..	7
" system	5
Mil	235
Mil, Circular	235
Miscellaneous table	4
Moment of inertia defined	125
" " " of various sections	153
" " " To find experimentally	133
" " resistance defined	134
" " " of various sections	133
Moments, Bending	152
Motor circuits	253
" efficiency, Approximate	253
Motors, Application of electric	261
" for machine tools	263
" Output and efficiency of	260
" Polarity of	234
" Underwriters' rules for	247
Multiple arc, Lamps in	253
Multiplication by logarithms	41

N.

PAGE

Needle, Declination of	277
Neutral axis	135
Numbers, Prime	79
Nuts, Proportions of	22

O.

Oblique fixed pulley, Formula for	138
Ohm, Legal	230
Ohm's law	231
Oil cup	198
Oscillation, To find center of	127
" To find experimentally radius of	133
Output of motors	260

P.

Packing rings	224
Paper, To make blueprint	175
Parallel, Lamps in	253
Parallelogram	114
" of forces, Explanation of	137
Passage, Steam	217
Pedestals, Design and proportions of	195-202
Percussion, To find center of	130
Permeability, Magnetic	257
Perpetual calendar	327
Pipe flanges	215
" Weight of cast-iron	23
Pipes and cylinders, Strength of	157
" Flow of water in	146
" Sizes of wrought-iron	24
Piston clearance	216
Pistons	223
Pitch, Formula for circular	228
" Table of circular	230
" Formula for diametral	228
" Table of diametral	230
" of bolts in cylinder heads	217
" " " steam-chest covers	217
Polarity of a dynamo, To determine	233
" " an electromagnet, To determine	233
Polishing wheels, Speed of	176
Polygon of forces, Explanation of	137
Polygons, Regular	119
Port, Exhaust	217
" Steam	217
Power transmitted by leather belting	141
" " rope	209
Powers, Mechanical	136
Powers, Table of squares, cubes, and fifth	112
Pressure, Absolute	26

	PAGE
Pressure, Mean effective	185
Prime factors, Table of	80
" numbers	79
Priming of boilers	168
Prismoidal formula	306
Profiles in leveling	296
Projectiles	120
Prony brake	260
Properties of aluminum and copper	250
" " saturated steam	29
Proportions of belt pulleys	204-209
" " flange couplings	194
" " journal-boxes	195-202
" " keys	193
" " rope-pulley rims	211
" " shaft hangers	202
Pulleys, Belt	204
" Differential	138
" Double movable	137
" Formula for compound	138
" Proportions of	204-209
" Quadruple movable	138
" Rope	211
" Single fixed	137
" Single movable	137
" Speed of	142
Pumps, Discharge of	143
" Duty of	143
" Horsepower of	143
Pyramid, Formulas for	118
" Formulas for frustum of	118

Q.

Quadruple movable pulley, Formula for	138
---	-----

R.

R, Values of	153
Radiating surface in exhaust-steam heating	174
Radii and deflections, Table of	298-300
Radius of gyration, To find	181
" " " " " experimentally	183
" " " " " for various sections	153
" " oscillation, To find experimentally	183
Rate of transmission of electricity	231
Reciprocal of a number	234
Rectangle, Formula for	114
Regular polygons	119
Resistance, Electrical	231
" Moment of	184
" " " for various sections	153
" of copper wire	251-258

INDEX.

XV

PAGE

Resistance of derived circuit.....	234
Resultant of forces.....	137
Retaining walls.....	300
" " Resistance of, to overturning.....	303
Reversal of field.....	258
Ribs for piston.....	224
" " steam-chest cover.....	220
Ring, Formula for circular.....	117
" Formula for.....	116
Root, Cube.....	107
" Fifth.....	109
" Square.....	104
Roots, Method of extracting.....	103
" Table of.....	112
Rope belting.....	209
" " Pulleys for.....	211
" Weight of manila.....	209
Ropes and chains, Strength of.....	157
" Wire.....	212
" " Pulleys for.....	213

S.

Safety, Table of factors of.....	151
" valves.....	173
Saturated steam, Properties of.....	29
Saturation curves (electrical).....	256
Scale in boilers, Prevention of.....	164
Screw cutting, Change gears for.....	178
Screw, Formulas for.....	139
Screw threads, Proportion of.....	22
Seats, Lining for.....	216
Sector, Formula for.....	115
Segment, Formula for.....	115
Series, Lamps in.....	253
Shaft couplings.....	194
" hangers.....	202
Shafting, Formulas for.....	159
" Line.....	193
Shafts, Crank.....	223
Shearing strengths, Table of.....	151
Sheaves for rope gearing.....	213
Sheet lead, Weight of.....	21
Shunt circuit.....	234
Simple-gear lathe, Screw cutting with.....	178
Single fixed pulley, Formula for.....	137
Size of copper wire for circuits.....	235-252
Slide valve.....	187
" " diagram.....	188
Slope sheet of boiler, To develop.....	160
Soldering, Fluxes for.....	24
Solders.....	20

	PAGE
Sparking at commutator.....	259
Specific gravity, Table of.....	10
" heat, Table of	18
" volumes.....	25
Speed, Cutting.....	176
" of emery wheels	176
" " gears, To calculate.....	142
" " grindstones	176
" " polishing wheels.....	176
" " pulleys, To calculate.....	142
" " twist drills	177
Sphere, Formula for	117
Spiral, Length of.....	118
Square measure	2
" root	104
Squares, cubes, and fifth powers.....	112
Standard pipe flanges	217
Steam chest	219
" " bolts.....	220
" " covers.....	221
" Heating by exhaust.....	173, 174
" port area	219
" Properties of saturated	29
" tables	25
" Velocity of, through ports	219
Steel, Tempering of.....	6
Stone, Weight of	10
Storage batteries	263
Strands in wire rope.....	212
Strap, Eccentric	226
" end of connecting-rod.....	224
Strength of materials	150
Stroke of engine	216
Stuffingbox.....	228
Surcharged walls, Pressure on	305
Surface expansion, Coefficients of	19
" Measures of (not land)	5
Surveying	276-326
" with compass.....	276
" " transit	280
Surveyor's measure	1
" square measure	2
Switch	315
" Point or split.....	315
" Stub	315
" " To lay out	320
Systems, Annunciator	243
T.	
Table, Long ton	3
" Miscellaneous.....	4
" of chemical elements.....	16

INDEX.

xvii

	Page
Table of squares, cubes, and fifth powers.....	112
Tables, Steam	25
" Wire	247-251
Teeth of wheels	228
Temperature of fusion.....	18
" " vaporization	18
Tempering steel	6
Tensile strength of materials.....	151
Tension of rope belting, Formula for.....	210
Testing of cables (electrical).....	271
Threads, Cutting screw	178
" Proportions of screw	22
Three-wire system, Edison	253
Ton, Long	8
Tools, Cutting speeds for machine	176
" " Motors for.....	263
Torque	261
Tracks, Cross-over	825
Trackwork	809
Transit notes, How to keep	291
" surveying	279
Trapezium and trapezoid, Formula for	115
Triangles, Formulas for.....	114
Triangulation	283
Trigonometric functions, Directions for use of table of	68
" " Table of	74-78
Troy weight	8
Tunnel sections	806
Turnouts	812
Type metals.....	15

U.

Ultimate strength of materials	151
Underwriters' line wire	247
" rules for incandescent wire	245
Units, Electrical	230
Universal joint coupling.....	195
Useful tables	1-92

V.

Valve diagram	188
Valves, Safety	178
" Slide	187
Vaporization, Latent heat of.....	18, 25
" Temperature of	18
Vapors, Weights of.....	14
Velocity of steam through ports.....	219
Vernier	279
Volt	230
Volume, Measures of.....	5
Volumes, Specific.....	25

W.

PAGE

Water, Testing of feed	163
“ Flow of, in pipes	146
Watt, The unit	231
Wedge, Formula for	117
Weight, Avoirdupois	3
“ Measures of	6
“ of bar iron, round and square	21
“ “ copper wire	238
“ “ manila rope	209
“ “ sheet lead	21
“ “ various substances	10
“ Troy	3
Weights and measures	1-4
“ “ Metric system of	5-6
Welding fluxes	24
Wheatstone bridge	270
Wheel and axle	136
Wheels, Speed of emery	176
“ “ “ polishing	176
Wheelwork, Formulas for	136
Width of belts, Formulas for	140
Wire, copper, Sizes for circuit	235-240
“ “ Weight of	238
“ gauges, Sizes of B. & S. and Birmingham	248
“ rope, Steel	212
“ “ Strands in	212
“ tables	247-251
“ Underwriters' line	247
Wires, Equivalent areas of, B. & S. gauge	248
Wiring, Bell	241
“ Interior	235
Work, Definition of	139
Wristpin brasses	224
Wrought-iron pipe, Sizes of	24

MECHANICS'

POCKET MEMORANDA.

USEFUL TABLES.

WEIGHTS AND MEASURES.

LINEAR MEASURE.

12 inches (in.)	= 1 foot	ft.
3 feet	= 1 yard	yd.
5.5 yards	= 1 rod	rd.
40 rods	= 1 furlong	fur.
8 furlongs	= 1 mile	mi.

in.	ft.	yd.	rd.	fur.	mi.
36	= 3	= 1			
198	= 16.5	= 5.5	= 1		
7,920	= 660	= 220	= 40	= 1	
63,360	= 5,280	= 1,760	= 320	= 8	= 1

SURVEYOR'S MEASURE.

7.92 inches	= 1 link	li.
25 links	= 1 rod	rd.
4 rods	} = 1 chain	ch.
100 links		
66 feet		
80 chains	= 1 mile	mi.
1 mi. = 80 ch. = 320 rd. = 8,000 li. = 63,360 in.		

USEFUL TABLES.

SQUARE MEASURE.

144 square inches (sq. in.).....	=	1 square foot	sq. ft.		
9 square feet.....	=	1 square yard	sq. yd.		
30½ square yards.....	=	1 square rod	sq. rd.		
160 square rods	=	1 acre	A.		
640 acres.....	=	1 square mile	sq. mi.		
sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
1	= 640	= 102,400	= 3,097,600	= 27,878,400	= 4,014,489,600

SURVEYOR'S SQUARE MEASURE.

625 square links (sq. li.)	= 1 square rod	sq. rd.
16 square rods	= 1 square chain	sq. ch.
10 square chains	= 1 acre	A.
640 acres	= 1 square mile	sq. mi.
36 square miles (6 mi. square) = 1 township		Tp.
1 sq. mi. = 640 A. = 6,400 sq. ch. = 102,400 sq. rd.		
	= 64,000,000 sq. li.	

The acre contains 4,840 sq. yd., or 43,560 sq. ft., and is equal to the area of a square measuring 208.71 ft. on a side.

CUBIC MEASURE.

1,728 cubic inches (cu. in.).....	= 1 cubic foot	cu. ft.
27 cubic feet	= 1 cubic yard.....	cu. yd.
128 cubic feet	= 1 cord.....	cd.
24½ cubic feet	= 1 perch	P.
1 cu. yd. = 27 cu. ft. = 46,656 cu. in.		

MEASURE OF ANGLES OR ARCS.

60 seconds (")	= 1 minute	'
60 minutes	= 1 degree	°
90 degrees	= 1 rt. angle or quadrant	□
360 degrees	= 1 circle	cir.
1 cir. = 360° = 21,600' = 1,296,000"		

AVOIRDUPOIS WEIGHT.

437.5 grains (gr.)	= 1 ounce	oz.
16 ounces	= 1 pound	lb.
100 pounds	= 1 hundredweight	cwt.
20 cwt., or 2,000 lb.	= 1 ton	T.
1 T.	= 20 cwt. = 2,000 lb. = 32,000 oz. = 14,000,000 gr.	
The avoirdupois pound contains 7,000 grains.		

LONG TON TABLE.

16 ounces	= 1 pound	lb.
112 pounds	= 1 hundredweight	cwt.
20 cwt., or 2,240 lb.	= 1 ton	T.

TROY WEIGHT.

24 grains (gr.)	= 1 pennyweight	pwt.
20 pennyweights	= 1 ounce	oz.
12 ounces	= 1 pound	lb.
1 lb.	= 12 oz. = 240 pwt. = 5,760 gr.	

DRY MEASURE.

2 pints (pt.)	= 1 quart	qt.
8 quarts	= 1 peck	pk.
4 pecks	= 1 bushel	bu.
1 bu.	= 4 pk. = 32 qt. = 64 pt.	

The U. S. struck bushel contains 2,150.42 cu. in. = 1.2444 cu. ft. By law, its dimensions are those of a cylinder 18½ in. in diameter and 8 in. deep. The heaped bushel is equal to 1½ struck bushels, the cone being 6 in. high. The dry gallon contains 268.8 cu. in., being ⅔ of a struck bushel.

For approximations, the bushel may be taken at 1½ cu. ft.; or a cubic foot may be considered ⅔ of a bushel.

The British bushel contains 2,218.19 cu. in. = 1.2837 cu. ft. = 1.032 U. S. bushels.

USEFUL TABLES.

LIQUID MEASURE.

4 gills (gi.)	= 1 pint	pt
2 pints	= 1 quart	qt
4 quarts	= 1 gallon	gal
81½ gallons	= 1 barrel	bbl
2 barrels, or 63 gallons	= 1 hogshead	hhd
1 hhd.	= 2 bbl. = 63 gal. = 252 qt. = 504 pt. = 2,016 gi.	

The U. S. gallon contains 231 cu. in. = .134 cu. ft., nearly, or 1 cu. ft. contains 7.481 gal. The following cylinders contain the given measures very closely:

	<i>Diam.</i>	<i>Height.</i>		<i>Diam.</i>	<i>Height.</i>
Gill.....	1½ in.	3 in.	Gallon	7 in.	6 in.
Pint	3½ in.	3 in.	8 gallons	14 in.	12 in.
Quart	3½ in.	6 in.	10 gallons	14 in.	15 in.

When water is at its maximum density, 1 cu. ft. weighs 62.425 lb. and 1 gallon weighs 8.345 lb.

For approximations, 1 cu. ft. of water is considered equal to 7½ gal., and 1 gal. as weighing 8½ lb.

The British imperial gallon, both liquid and dry, contains 277.274 cu. in. = .16046 cu. ft., and is equivalent to the volume of 10 lb. of pure water at 62° F. To reduce British to U. S. liquid gallons, multiply by 1.2. Conversely, to convert U. S. into British liquid gallons, divide by 1.2: or, ~~increase~~ the number of gallons ⅓.

MISCELLANEOUS TABLE

12 articles = 1 dozen.	20 quires = 1 ream.
12 dozen = 1 gross.	1 league = 3 miles.
12 gross = 1 great gross.	1 fathom = 6 feet.
2 articles = 1 pair.	1 hand = 4 inches.
20 articles = 1 score.	1 palm = 3 inches.
24 sheets = 1 quire.	1 span = 9 inches.
1 sea mile (U. S.) = 6,080 ft. = 1½ statute miles (roughly).	
1 meter = 3 feet 3¼ inches (nearly).	

THE METRIC SYSTEM.

The metric system is based on the meter, which, according to the U. S. Coast and Geodetic Survey Report of 1884, is equal to 39.370432 inches. The value commonly used is 39.37 inches, and is authorized by the U. S. government. The meter is defined as one ten-millionth the distance from the pole to the equator, measured on a meridian passing near Paris.

There are three principal units—the meter, the liter (pronounced lee-ter), and the gram, the units of length, capacity, and weight, respectively. Multiples of these units are obtained by prefixing to the names of the principal units the Greek words deca (10), hecto (100), and kilo (1,000); the submultiples, or divisions, are obtained by prefixing the Latin words deci ($\frac{1}{10}$), centi ($\frac{1}{100}$), and milli ($\frac{1}{1000}$). These prefixes form the key to the entire system. In the following tables, the abbreviations of the principal units of these submultiples begin with a small letter, while those of the multiples begin with a capital letter; they should always be written as here printed.

MEASURES OF LENGTH.

10 millimeters (mm.)	= 1 centimeter	cm.
10 centimeters	= 1 decimeter	dm.
10 decimeters	= 1 meter	m.
10 meters	= 1 decameter	Dm.
10 decameters	= 1 hectometer	Hm.
10 hectometers	= 1 kilometer	Km.

MEASURES OF SURFACE (NOT LAND).

100 square millimeters (mm ² .)	= 1 square centimeter	cm ² .
100 square centimeters	= 1 square decimeter	dm ² .
100 square decimeters	= 1 square meter	m ² .

MEASURES OF VOLUME.

1,000 cubic millimeters (mm ³ .)	= 1 cubic centimeter	cm ³ .
1,000 cubic centimeters	= 1 cubic decimeter	dm ³ .
1,000 cubic decimeters	= 1 cubic meter	m ³ .

MEASURES OF CAPACITY.

10 milliliters (ml.)	= 1 centiliter	cl
10 centiliters	= 1 deciliter	dl
10 deciliters	= 1 liter	l
10 liters	= 1 decaliter	Dl.
10 decaleters	= 1 hectoliters	Hl.
10 hectoliters	= 1 kiloliters	Kl.

NOTE.—The liter is equal to the volume that is occupied by 1 cubic decimeter.

MEASURES OF WEIGHT.

10 milligrams (mg.)	= 1 centigram	cg.
10 centigrams	= 1 decigram	dg.
10 decigrams	= 1 gram	g.
10 grams	= 1 decagram	Dg.
10 decagrams	= 1 hectogram	Hg.
10 hectograms	= 1 kilogram	Kg.
1,000 kilograms	= 1 ton	T.

NOTE.—The gram is the weight of 1 cubic centimeter of pure distilled water at a temperature of 39.2° F.; the kilogram is the weight of 1 liter of water; the ton is the weight of 1 cubic meter of water.

TEMPERING OF STEEL

The following colors may be made use of in tempering steel-cutting tools:

	<i>Corresponding Temperature F.</i>
Lancets	Pale yellow 430°
Razors	Straw yellow 450°
All kinds of wood-cutting tools	Darker straw yellow 470°
Screw taps	Yellow 490°
Chipping chisels, hatchets, and saws	Brown yellow 500°
All kinds of percussive tools	Brown (slightly tinged purple) 520°
	Light purple 530°
	Clear black 570°
Springs	Dark blue 600°

CONVERSION TABLES.

By means of the tables on pages 8 and 9, metric measures can be converted into English, and *vice versa*, by simple addition. All the figures of the values given are not required, four or five digits being all that are commonly used; it is only in very exact calculations that all the digits are necessary. Using table, proceed as follows:

Change 6,471.8 feet into meters. Any number, as 6,471.8, may be regarded as $6,000 + 400 + 70 + 1 + .8$; also, $6,000 = 1,000 \times 6$; $400 = 100 \times 4$, etc. Hence, looking in the left-hand column of the upper table, page 8, for figure 6 (the first figure of the given number), we find opposite it in the third column, which is headed "Feet to Meters," the number 1.8287838. Now, using but five digits and increasing the fifth digit by 1 (since the next is greater than 5), we get 1.8288. In other words, 6 feet = 1.8288 meters; hence, 6,000 feet = $1,000 \times 1.8288 = 1,828.8$, simply moving the decimal point three places to the right. Likewise, 400 feet = 121.92 meters; 70 feet = 21.336 meters; 1 foot = .3048 meter, and .8 foot = .2438 meter. Adding as shown above, we get 1,972.6046 meters.

1,828.8
121.92
21.336
.3048
.2438
1,972.6046

Again, convert 19.635 kilos into pounds. The work should be perfectly clear from the explanation given above. The result is 43.2875 pounds.

22.046
19.8416
1.3228
.0661
.0110
43.2875

The only difficulty in applying these tables lies in locating the decimal point; it may always be found thus: If the figure considered lies to the left of the decimal point, count each figure in order, beginning with units (but calling unit's place zero), until the desired figure is reached, then move the decimal point to the *right* as many places as the figure being considered is to the left of the unit figure. Thus, in the first case above, 6 lies three places to the left of 1, which is in unit's place; hence, the decimal point is moved three places to the *right*. By exchanging the words "right" and "left," the statement will also apply to decimals. Thus, in the second case above, the 5 lies three places to the *right* of unit's place; hence, the decimal point in the number taken from the table is moved three places to the *left*.

USEFUL TABLES.

CONVERSION TABLE—ENGLISH MEASURES INTO METRIC.

English.	Metric.	Metric.	Metric.	Metric.
	Inches to Meters.	Feet to Meters.	Pounds to Kilos.	Gallons to Liters.
1	.0253998	.3047973	.4535925	3.7853122
2	.0507996	.6095946	.9071850	7.5706244
3	.0761993	.9143919	1.3607775	11.3559366
4	.1015991	1.2191892	1.8143700	15.1412488
5	.1269989	1.5239865	2.2679625	18.9265610
6	.1523987	1.8287838	2.7215550	22.7118782
7	.1777984	2.1335811	3.1751475	26.4971854
8	.2031982	2.4383784	3.6287400	30.2824976
9	.2285980	2.7431757	4.0823325	34.0678098
10	.2539978	3.0479730	4.5359250	37.8531220

CONVERSION TABLE—ENGLISH MEASURES INTO METRIC.

English.	Metric.	Metric.	Metric.	Metric.
	Square Inches to Square Meters.	Square Feet to Square Meters.	Cubic Feet to Cubic Meters.	Pounds per Square Inch to Kilo per Square Meter.
1	.000645150	.092901394	.028316094	703.08241
2	.001290300	.185802788	.056632188	1,406.16482
3	.001935450	.278704182	.084948282	2,109.24723
4	.002580600	.371605576	.113264376	2,812.32964
5	.003225750	.464506970	.141580470	3,515.41205
6	.003870900	.557408364	.169896564	4,218.49446
7	.004516050	.650309758	.198212658	4,921.57687
8	.005161200	.743211152	.226528752	5,624.65928
9	.005406350	.836112546	.254844846	6,327.74169
10	.006451500	.929013940	.283160940	7,030.82410

CONVERSION TABLE—METRIC MEASURES INTO ENGLISH.

Metric.	English.	English.	English.	English.
	Meters to Inches.	Meters to Feet.	Kilos to Pounds.	Liters to Gallons.
1	39.370432	3.2808693	2.2046223	.2641790
2	78.740864	6.5617386	4.4092447	.5283580
3	118.111296	9.8426079	6.6138670	.7925371
4	157.481728	13.1234772	8.8184894	1.0567161
5	196.852160	16.4043465	11.0231117	1.3208951
6	236.222592	19.6852158	13.2277340	1.5850741
7	275.593024	22.9660851	15.4323564	1.8492531
8	314.963456	26.2469544	17.6369787	2.1134322
9	354.333888	29.5278237	19.8416011	2.3776112
10	393.704320	32.8086930	22.0462234	2.6417902

CONVERSION TABLE—METRIC MEASURES INTO ENGLISH.

Metric.	English.	English.	English.	English.
	Square Meters to Square Inches.	Square Meters to Square Feet.	Cubic Meters to Cubic Feet.	Kilos per Square Meter to Pounds per Square Inch.
1	1,550.03092	10.7641034	35.3156163	.001422310
2	3,100.06184	21.5282068	70.6312326	.002844620
3	4,650.09276	32.2923102	105.9468489	.004266930
4	6,200.12368	43.0564136	141.2624652	.005689240
5	7,750.15460	53.8205170	176.5780815	.007111550
6	9,300.18552	64.5846204	211.8936978	.008533860
7	10,850.21644	75.3487238	247.2093141	.009956170
8	12,400.24736	86.1128272	282.5249304	.011378480
9	13,950.27828	96.8769306	317.8405467	.012800790
10	15,500.30920	107.6410340	353.1561630	.014223100

SPECIFIC GRAVITY.

The specific gravity of a body is the ratio between its weight and the weight of a like volume of distilled water at a temperature of 39.2° F. For gases, air is taken as the unit. One cubic foot of water at 39.2° F. weighs 62.425 pounds.

Name of Substance.	Specific Gravity.	Weight per Cu. In Pounds.
METALS.		
Platinum, rolled.....	22.009	.819
Platinum, wire	21.042	.760
Platinum, hammered	20.337	.735
Gold, hammered.....	19.361	.699
Gold, pure cast	19.258	.696
Gold, 22 carats fine.....	17.486	.632
Mercury, solid at - 40° F.....	15.632	.565
Mercury, at + 32° F.....	13.619	.492
Mercury, at 60° F.....	13.580	.491
Mercury, at 212° F.	13.375	.483
Lead, pure.....	11.330	.409
Lead, hammered	11.388	.411
Silver, hammered	10.511	.380
Silver, pure	10.474	.378
Bismuth.....	9.746	.352
Copper, wire and rolled	8.878	.321
Copper, pure	8.788	.317
Bronze, gun metal.....	8.500	.307
Brass, common	8.500	.307
Steel, cast steel	7.919	.286
Steel, common soft	7.833	.283
Steel, hardened and tempered.....	7.818	.282
Iron, pure.....	7.768	.281
Iron, wrought and rolled.....	7.780	.281
Iron, hammered.....	7.789	.281
Iron, cast	7.207	.260
Tin, from Böhmen	7.312	.264
Tin, English.....	7.201	.263
Zinc, rolled	7.101	.260
Antimony.....	6.712	.242
Aluminum	2.660	.096
STONES AND EARLHS.		
Emery.....	4.000	.145
Limestone.....	2.700	.098
Asbestos, starry	3.073	.111

TABLE—(Continued).

Name of Substance.	Specific Gravity.	Weight per Cu. In. Pounds.
Glass, flint	3.500	.1260
Glass, white	2.900	.1050
Glass, bottle	2.732	.0987
Glass, green	2.642	.0954
Marble, Parian	2.838	.1025
Marble, African	2.708	.0978
Marble, Egyptian	2.668	.0964
Mica	2.800	.1012
Chalk	2.784	.1006
Coral, red	2.700	.0975
Granite, Susquehanna	2.704	.0977
Granite, Quincy	2.652	.0958
Granite, Patapsco	2.640	.0954
Granite, Scotch	2.625	.0948
Marble, white Italian	2.708	.0978
Marble, common	2.686	.0970
Talc, black	2.900	.1005
Quartz	2.660	.0961
Slate	2.800	.1012
Pearl, oriental	2.650	.0957
Shale	2.600	.0939
Flint, white	2.594	.0937
Flint, black	2.582	.0933
Stone, common	2.520	.0910
Stone, Bristol	2.510	.0907
Stone, mill	2.484	.0897
Stone, paving	2.416	.0873
Gypsum, opaque	2.168	.0788
Grindstone	2.143	.0774
Salt, common	2.130	.0769
Saltpeter	2.090	.0755
Sulphur, native	2.033	.0734
Common soil	1.984	.0717
Rotten stone	1.981	.0716
Clay	1.900	.0686
Brick	2.000	.0723
Niter	1.900	.0686
Plaster Paris	1.872	.0676
Ivory	2.473	.0893
Sand	1.822	.0659
Phosphorus	2.650	.0957
Borax	1.770	.0639
Coal, anthracite	1.714	.0619
	1.640	.0592
	1.436	.0519

TABLE—(Continued).

Name of Substance.	Specific Gravity.	Weight per Cu. In. Pounds.
Coal, Maryland	1.355	.0490
Coal, Scotch	1.300	.0470
Coal, Newcastle	1.270	.0459
Coal, bituminous	1.350	.0488
Earth, loose	1.360	.0491
Lime, quick	1.500	.0542
Charcoal	.441	.0159
WOODS (DRY).		
Alder	.800	.0289
Apple tree	.793	.0287
Ash, the trunk	.845	.0305
Bay tree	.822	.0297
Beech	.852	.0308
Box, French	.960	.0347
Box, Dutch	1.328	.0480
Box, Brazilian red	1.031	.0372
Cedar, wild	.596	.0215
Cedar, Palestine	.613	.0221
Cedar, American	.561	.0203
Cherry tree	.672	.0243
Cork	.250	.0090
Ebony, American	1.220	.0441
Elder tree	.695	.0251
Elm	.560	.0202
Filbert tree	.600	.0217
Fir, male	.550	.0199
Fir, female	.498	.0180
Hazel	.600	.0217
Lemon tree	.703	.0254
Lignum-vitæ	1.330	.0481
Linden tree	.604	.0218
Logwood	.913	.0330
Mahogany, Honduras	.560	.0202
Maple	.790	.0285
Mulberry	.897	.0324
Oak	.950	.0343
Orange tree	.705	.0255
Pear tree	.661	.0239
Poplar	.383	.0138
Poplar, white Spanish	.529	.0191
Sassafras	.482	.0174
Spruce	.500	.0181
Spruce, old	.460	.0166

TABLE—(Continued).

Name of Substance.	Specific Gravity.	Weight per Cu. In. Pounds.
Pine, southern	.720	.0260
Pine, white	.400	.0144
Walnut	.610	.0220
LIQUIDS.		
Acid, acetic	1.062	.0384
Acid, nitric	1.217	.0440
Acid, sulphuric	1.841	.0665
Acid, muriatic	1.200	.0434
Acid, phosphoric	1.558	.0563
Alcohol, commercial	.833	.0301
Alcohol, pure	.792	.0286
Beer, lager	1.034	.0374
Champagne	.997	.0360
Cider	1.018	.0368
Ether, sulphuric	.739	.0267
Egg	1.090	.0394
Honey	1.450	.0524
Human blood	1.054	.0381
Milk	1.032	.0373
Oil, linseed	.940	.0340
Oil, olive	.915	.0331
Oil, turpentine	.870	.0314
Oil, whale	.932	.0337
Proof spirit	.925	.0334
Vinegar	1.080	.0390
Water, distilled (62.425 lb. per cu. ft.)	1.000	.0361
Water, sea	1.030	.0372
Wine	.992	.0358
MISCELLANEOUS.		
Beeswax	.965	.0349
Butter	.942	.0340
India rubber	.933	.0337
Fat	.923	.0333
Gunpowder, loose	.900	.0325
Gunpowder, shaken	1.000	.0361
Gum arabic	1.452	.0525
Lard	.947	.0342
Spermaceti	.943	.0341
Sugar	1.605	.0580
Tallow, sheep	.924	.0334
Tallow, calf	.934	.0337
Tallow, ox	.923	.0333
Atmospheric air	.0012	

TABLE—(Continued).

Name of Substance.	Specific Gravity.	Weight per Cu. Ft. Grains.
GASES AND VAPORS.		
At 32° and a tension of 1 atmosphere.		
Atmospheric air	1.0000	565.11
Ammonia gas	.5894	333.1
Carbonic acid	1.5201	859.0
Carbonic oxide	.9673	546.6
Light carbureted hydrogen	.5527	312.3
Chlorine	2.4502	1,384.6
Olefiant gas	.9672	546.6
Hydrogen	.0692	39.1
Oxygen	1.1056	624.8
Sulphureted hydrogen	1.1747	663.8
Nitrogen	.9713	548.9
Vapor of alcohol	1.5890	898.0
Vapor of turpentine spirits	4.6978	2,654.8
Vapor of water	.6219	351.4
Smoke of bituminous coal	.1020	57.6
Smoke of wood	.9000	508.6
Steam at 212° F.	.4880	275.8

The weight of a cubic foot of any solid or liquid is found by multiplying its specific gravity by 62.425 lb. avoirdupois. The weight of a cubic foot of any gas at atmospheric pressure and at 32° F. is found by multiplying its specific gravity by .08073 lb. avoirdupois.

WROUGHT-IRON CHAIN CABLES.

The strength of a chain link is less than twice that of a straight bar of a sectional area equal to that of one side of the link. A weld exists at one end and a bend at the other, each requiring at least one heat, which produces a decrease in the strength. The report of the committee of the U. S. Testing Board, on tests of wrought-iron and chain cables, contains the following conclusions:

“That beyond doubt, when made of American bar iron with cast-iron studs, the studded link is inferior in strength to the unstudded one.

"That, when proper care is exercised in the selection of material, a variation of 5% to 17% of the strongest may be expected in the resistance of cables. Without this care the variation may rise to 25%.

"That with proper material and construction the ultimate resistance of the chain may be expected to vary from 155% to 170% of that of the bar used in making the links, and show an average of about 163%.

"That the proof test of a chain cable should be about 50% of the ultimate resistance of the weakest link."

From a great number of tests of bars and unfinished cables, the committee considered that the average ultimate resistance and proof tests of chain cables made of the bars, whose diameters are given, should be such as are shown in the accompanying table.

ULTIMATE RESISTANCE AND PROOF TESTS OF CHAIN CABLES.

Diam. of Bar. Inches.	Average Resist. = 163% of Bar. Pounds.	Proof Test. Pounds.	Diam. of Bar. Inches.	Average Resist. = 163% of Bar. Pounds.	Proof Test. Pounds.
1	71,172	33,840	1 $\frac{3}{8}$	162,283	77,159
1 $\frac{1}{8}$	79,544	37,820	1 $\frac{5}{8}$	174,475	82,956
1 $\frac{1}{4}$	88,445	42,053	1 $\frac{3}{4}$	187,075	88,947
1 $\frac{3}{8}$	97,731	46,468	1 $\frac{7}{8}$	200,074	95,128
1 $\frac{1}{2}$	107,440	51,084	1 $\frac{1}{2}$	213,475	101,499
1 $\frac{5}{8}$	117,577	55,903	1 $\frac{3}{4}$	227,271	108,058
1 $\frac{3}{4}$	128,129	60,920	1 $\frac{1}{2}$	241,463	114,806
1 $\frac{7}{8}$	139,103	66,138	2	256,040	121,737
1 $\frac{1}{2}$	150,485	71,550			

TYPE METALS.

Name.	Proportions.
Smallest type	3 L, 1 A
Small type	4 L, 1 A
Medium type	5 L, 1 A
Large type	6 L, 1 A
Largest type	7 L, 1 A

In the above table, L represents the lead, and A the antimony in the alloy

TABLE OF ELEMENTS.

	Symbol.	Atomic Weight.*
Aluminum	<i>Al</i>	27.04
Antimony (stibium)	<i>Sb</i>	119.96
Arsenic	<i>As</i>	74.9
Barium	<i>Ba</i>	136.9
Beryllium	<i>Be</i>	9.08
Bismuth	<i>Bi</i>	207.5
Boron	<i>B</i>	10.9
Bromine	<i>Br</i>	79.76
Cadmium	<i>Cd</i>	111.7
Cæsium	<i>Cs</i>	133.0
Calcium	<i>Ca</i>	39.91
Carbon	<i>C</i>	11.97
Cerium	<i>Ce</i>	141.2
Chlorine	<i>Cl</i>	35.37
Chromium	<i>Cr</i>	52.45
Cobalt	<i>Co</i>	58.6
Columbium	<i>Cb</i>	93.7
Copper (cuprum)	<i>Cu</i>	63.18
Didymium	<i>D</i>	147.0
Erbium	<i>E</i>	169.0
Fluorine	<i>F</i>	19.06
Gallium	<i>G</i>	69.8
Germanium	<i>Ge</i>	72.32
Gold (aurum)	<i>Au</i>	196.2
Hydrogen	<i>H</i>	1.0
Indium	<i>In</i>	113.4
Iodine	<i>I</i>	126.54
Iridium	<i>Ir</i>	196.7
Iron (ferrum)	<i>Fe</i>	55.88
Lanthanum	<i>La</i>	139.0
Lead (plumbum)	<i>Pb</i>	206.39
Lithium	<i>Li</i>	7.01
Magnesium	<i>Mg</i>	23.94
Mercury (hydrargyrum)	<i>Hg</i>	199.8
Manganese	<i>Mn</i>	54.8
Molybdenum	<i>Mo</i>	95.6
Nickel	<i>Ni</i>	58.6
Niobium	<i>Nb</i>	94.0
Nitrogen	<i>N</i>	14.01
Osmium	<i>Os</i>	198.6
Oxygen	<i>O</i>	15.96

* Principally from the 16th edition *Des Ingenieurs Taschen-
uch*. The names of the non-metals are printed in heavy type.

TABLE—(Continued).

	Symbol.	Atomic Weight.
Palladium	<i>Pd</i>	106.2
Phosphorus	<i>P</i>	30.96
Platinum	<i>Pt</i>	194.43
Potassium (kalium)	<i>K</i>	39.04
Rhodium	<i>Rh</i>	104.1
Rubidium	<i>Rb</i>	86.2
Ruthenium	<i>Ru</i>	103.5
Scandium	<i>Sc</i>	44.04
Selenium	<i>Se</i>	78.00
Silicon	<i>Si</i>	28.00
Silver (argentum)	<i>Ag</i>	107.66
Sodium (natrium)	<i>Na</i>	23.0
Strontium	<i>Sr</i>	87.3
Sulphur	<i>S</i>	31.98
Tantalum	<i>Ta</i>	182.0
Tellurium	<i>Te</i>	128.0
Thallium	<i>Tl</i>	203.6
Thorium	<i>Th</i>	231.5
Tin (stannum)	<i>Sn</i>	117.35
Titanium	<i>Ti</i>	48.0
Tungsten (wolfram)	<i>W</i>	183.6
Uranium	<i>U</i>	240.0
Vanadium	<i>V</i>	51.2
Ytterbium	<i>Yb</i>	93.0
Yttrium	<i>Y</i>	172.6
Zinc	<i>Zn</i>	64.88
Zirconium	<i>Zr</i>	90.0

TABLE OF SPECIFIC HEATS.

SOLIDS.

Copper	.0951	Cast iron	.1298
Gold	.0324	Lead	.0314
Wrought iron	.1138	Platinum	.0824
Steel (soft)	.1165	Silver	.0570
Steel (hard)	.1175	Tin	.0562
Zinc	.0956	Ice	.5040
Brass	.0939	Sulphur	.2026
Glass	.1937	Charcoal	.2410

LIQUIDS.

Water	1.0000	Lead (melted)	.0402
Alcohol	.7000	Sulphur (melted)	.2340
Mercury	.0333	Tin (melted)	.0637
Benzine	.4500	Sulphuric acid	.3350
Glycerine	.5550	Oil of turpentine	.4260

GASES.

Air	.23751	Superheated steam	.4805
Oxygen	.21751	Carbonic oxide (CO)	.2479
Nitrogen	.24380	Carbonic acid (CO_2)	.2170
Hydrogen	3.40900	Olefiant gas	.4040

TEMPERATURES AND LATENT HEATS OF FUSION
AND OF VAPORIZATION.

Substance.	Temperature of Fusion.	Temperature of Vaporization.	Latent Heat of Fusion.	Latent Heat of Vaporization.
Water	32°	212°	142.65	966.6
Mercury	-37.8°	662°	5.09	157
Sulphur	228.3°	824°	13.26	
Tin	446°		25.65	
Lead	626°		9.67	
Zinc	680°	1,900°	50.63	493
Alcohol	Unknown	173°		372
Oil of turpentine ...	14°	313°		124
Linseed oil		600°		
Aluminum	1,400°			
Copper	2,100°			
Cast iron	2,192°	3,300°		
Wrought iron	2,912°	5,000°		
Steel	2,520°			
Platinum	3,632°			
Iridium	4,892°			

EXAMPLE.—How many units of heat are required to melt 10 lb. of zinc from a temperature of 60° F.?

SOLUTION.—The specific heat of zinc is found from the table to be .0956. Hence, the number of heat units necessary to raise it to the melting point is $10 \times (680 - 60) \times .0956 = 592.72$. Latent heat of fusion = 50.63 heat units. Hence, the total number of heat units required is $592.72 + 10 \times 50.63 = 1,099.02$.

HEAT.

COEFFICIENT OF EXPANSION FOR A NUMBER OF SUBSTANCES.

Name of Substance.	Linear Expansion.	Surface Expansion.	Cubic Expansion.
Cast iron	.00000617	.00001234	.00001850
Copper	.00000955	.00001910	.00002864
Brass	.00001037	.00002074	.00003112
Silver	.00000690	.00001390	.00002070
Bar iron	.00000686	.00001372	.00002068
Steel (untempered)	.00000599	.00001198	.00001798
Steel (tempered)	.00000702	.00001404	.00002106
Zinc	.00001634	.00003268	.00004908
Tin	.00001410	.00002820	.00003229
Mercury	.00003334	.00006668	.00010010
Alcohol	.00019259	.00038518	.00057778
Gases			.00203252

EXAMPLE.—A wrought-iron bar 22 ft. long is heated from 70° to 300° . How much will it lengthen?

SOLUTION.— $22 \times (300 - 70) \times .00000686 = .0347116$ ft. = .41654 in.

ALLOYS.

NOTE.—*A* = Antimony, *B* = Bismuth, *C* = Copper, *G* = Gold, *I* = Iron, *L* = Lead, *N* = Nickel, *S* = Silver, *T* = Tin, *Z* = Zinc.

Name.	Proportions.
Brass, common yellow	2 <i>C</i> , 1 <i>Z</i>
Brass, to be rolled	32 <i>C</i> , 10 <i>Z</i> , 1.5 <i>T</i>
Brass castings, common ...	20 <i>C</i> , 1.25 <i>Z</i> , 2.5 <i>T</i>
Brass castings, hard	25 <i>C</i> , 2 <i>Z</i> , 4.5 <i>T</i>
Brass propellers	8 <i>C</i> , .5 <i>Z</i> , 1 <i>T</i>
Gun metal	8 <i>C</i> , 1 <i>T</i>

ALLOYS—(Continued).

<i>Name.</i>	<i>Proportions.</i>
Copper flanges	9 C, 1 Z, .26 T
Muntz's metal	6 C, 4 Z
Statuary	91.4 C, 5.53 Z, 1.7 T, 1.37 L
German silver	2 C, 7.9 N, 6.3 Z, 6.5 I
Britannia metal	50 A, 25 T, 25 B
Chinese silver	65.1 C, 19.3 Z, 13 N, 2.58 S, 12 I
Chinese white copper	20.2 C, 12.7 Z, 1.3 T, 15.8 N
Medals	100 C, 8 Z
Pinchbeck	5 C, 1 Z
Babbitt's metal.....	25 T, 2 A, .5 C
Bell metal, large	3 C, 1 T
Bell metal, small	4 C, 1 T
Chinese gongs	40.5 C, 9.2 T
Telescope mirrors	33.3 C, 16.7 T
White metal, ordinary	3.7 C, 3.7 Z, 14.2 T, 28.4 A
White metal, hard	35 C, 13 Z, 2.2 T
Sheeting metal	56 C, 45 Z, 12 arsenic
Metal, expands in cooling	75 L, 16.7 A, 8.3 B

ALLOYS FOR SOLDERS.

<i>Name.</i>	<i>Proportions.</i>	<i>Melting Point.</i>
Newton's fusible	8 B, 5 L, 3 T,	212°
Rose's fusible.....	2 B, 1 L, 1 T,	201°
A more fusible	5 B, 3 L, 2 T,	199°
Still more fusible.....	12 T, 25 L, 50 B, 13 cadmium,	155°
For tin solder, coarse,.....	1 T, 3 L,	500°
For tin solder, ordinary.....	2 T, 1 L,	360°
For brass, soft spelter.....	1 C, 1 Z,	550°
Hard, for iron	2 C, 1 Z,	700°
For steel	19 S, 3 C, 1 Z	
For fine brasswork	1 S, 8 C, 8 Z	
Pewterer's soft solder.....	2 B, 4 L, 3 T	
Pewterer's soft solder.....	1 B, 1 L, 2 T	
Gold solder	24 G, 2 S, 1 C	
Silver solder, hard	4 S, 1 C	
Silver solder, soft.....	2 S, 1 brass wire	
For lead	16 T, 33 L	

WEIGHT OF ROUND AND SQUARE ROLLED IRON.

From $\frac{1}{8}$ in. to $9\frac{1}{2}$ in. in Diameter, and 1 ft. in Length.

Side or Diam. Inches.	Weight. Lb. per ft.		Side or Diam. Inches.	Weight. Lb. per ft.	
	Round.	Square.		Round.	Square.
$\frac{1}{8}$	.010	.013	$3\frac{7}{8}$	39.864	50.756
$\frac{1}{4}$	.041	.053	4	42.464	54.084
$\frac{3}{8}$	.093	.118	$4\frac{1}{8}$	45.174	57.517
$\frac{1}{2}$	.165	.211	$4\frac{1}{4}$	47.952	61.055
$\frac{5}{8}$	.273	.347	$4\frac{3}{8}$	50.815	64.700
$\frac{3}{4}$	.403	.515	$4\frac{1}{2}$	53.760	68.448
$\frac{7}{8}$	.553	.703	$4\frac{5}{8}$	56.788	72.305
1	.823	1.053	$4\frac{3}{4}$	59.900	76.264
$1\frac{1}{8}$	1.113	1.403	$4\frac{7}{8}$	63.094	80.333
$1\frac{1}{4}$	1.423	1.803	5	66.350	84.480
$1\frac{3}{8}$	1.753	2.213	$5\frac{1}{8}$	69.731	88.784
$1\frac{1}{2}$	2.103	2.643	$5\frac{1}{4}$	73.172	93.168
$1\frac{5}{8}$	2.473	3.093	$5\frac{3}{8}$	76.700	97.657
$1\frac{3}{4}$	2.863	3.563	$5\frac{1}{2}$	80.304	102.240
$1\frac{7}{8}$	3.273	4.053	$5\frac{5}{8}$	84.001	106.953
2	3.703	4.563	$5\frac{3}{4}$	87.776	111.756
$2\frac{1}{8}$	4.153	5.093	$5\frac{7}{8}$	91.634	116.671
$2\frac{1}{4}$	4.623	5.643	6	95.552	121.664
$2\frac{3}{8}$	5.113	6.213	$6\frac{1}{8}$	103.704	132.040
$2\frac{1}{2}$	5.623	6.803	$6\frac{1}{4}$	112.160	142.816
$2\frac{5}{8}$	6.153	7.413	$6\frac{3}{8}$	120.960	154.012
$2\frac{3}{4}$	6.703	8.043	7	130.048	165.632
$2\frac{7}{8}$	7.273	8.693	$7\frac{1}{4}$	139.544	177.672
3	7.863	9.363	$7\frac{1}{2}$	149.328	190.136
$3\frac{1}{8}$	8.473	10.053	$7\frac{3}{8}$	159.456	203.024
$3\frac{1}{4}$	9.103	10.763	8	169.856	216.336
$3\frac{3}{8}$	9.753	11.493	$8\frac{1}{8}$	180.696	230.068
$3\frac{1}{2}$	10.423	12.243	$8\frac{1}{4}$	191.808	244.220
$3\frac{5}{8}$	11.113	12.993	$8\frac{3}{8}$	203.260	258.800
$3\frac{3}{4}$	11.823	13.763	9	215.040	273.792
$3\frac{7}{8}$	12.553	14.553	$9\frac{1}{4}$	227.152	289.220
4	13.303	15.363	$9\frac{1}{2}$	239.600	305.056

WEIGHT OF SHEET LEAD.

Thickness. Inches.	W'ght. Lb.	Thickness. Inches.	W'ght. Lb.	Thickness. Inches.	W'ght. Lb.
.017	1	.085	5	.152	9
.034	2	.101	6	.169	10
.051	3	.118	7	.186	11
.068	4	.135	8	.203	12

PROPORTIONS OF THE UNITED STATES STANDARD SREW THREADS, NUTS, AND BOLT HEADS.

Diam. of Screw.	Threads per in.	Diam. of Core.	Width of Flat.	Outside Diam.	Inside Diam.	Diagonal.	Height of Head.
							
1-4	20	.185	.0062	9-16	1-2	11-16	1-4
5-16	18	.240	.0070	11-16	19-32	13-16	19-64
3-8	16	.294	.0078	25-32	11-16	31-32	11-32
7-16	14	.344	.0089	43-48	25-32	1 1-16	25-64
1-2	13	.400	.0096	1	7-8	1 1-4	7-16
9-16	12	.454	.0104	1 7-64	31-32	1 5-16	31-64
5-8	11	.507	.0113	1 7-32	1 1-16	1 1-2	17-32
3-4	10	.620	.0125	1 7-16	1 1-4	1 3-4	5-8
7-8	9	.731	.0140	1 21-32	1 7-16	2 1-32	23-32
1	8	.837	.0156	1 7-8	1 5-8	2 5-16	13-16
1 1-8	7	.940	.0180	2 3-32	1 13-16	2 1-2	29-32
1 1-4	7	1.065	.0180	2 5-16	2	2 27-32	1
1 3-8	6	1.160	.0210	2 1-2	2 3-16	3 1-10	1 3-32
1 1-2	6	1.284	.0210	2 3-4	2 3-8	3 3-8	1 3-16
1 5-8	5 1-2	1.389	.0227	2 15-16	2 9-16	3 5-8	1 9-32
1 3-4	5	1.490	.0250	3 3-16	2 3-4	3 29-32	1 3-8
1 7-8	5	1.615	.0250	3 13-32	2 15-16	4 3-16	1 15-32
2	4 1-2	1.712	.0280	3 5-8	3 1-8	4 7-16	1 9-16
2 1-4	4 1-2	1.962	.0280	4 1-16	3 1-2	4 31-32	1 3-4
2 1-2	4	2.175	.0310	4 1-2	3 7-8	5 1-2	1 15-16
2 3-4	4	2.425	.0310	4 29-32	4 1-4	6	2 1-8
3	3 1-2	2.628	.0357	5 3-8	4 5-8	6 9-16	2 5-16
3 1-4	3 1-2	2.878	.0357	5 3-4	5	7 1-8	2 1-2
3 1-2	3 1-4	3.100	.0384	6 7-64	5 3-8	7 5-8	2 11-16
3 3-4	3	3.317	.0410	6 5-8	5 3-4	8 3-16	2 7-8
4	3	3.566	.0410	7 3-64	6 1-8	8 11-16	3 1-16
4 1-4	2 7-8	3.798	.0435	7 1-2	6 1-2	9 1-4	3 1-4
4 1-2	2 3-4	4.027	.0460	7 31-32	6 7-8	9 3-4	3 7-16
4 3-4	2 5-8	4.255	.0480	8 3-8	7 1-4	10 9-32	3 5-8
5	2 1-2	4.480	.0500	8 13-16	7 5-8	10 13-16	3 13-16
5 1-4	2 1-2	4.730	.0500	9 1-4	8	11 3-8	4
5 1-2	2 3-8	4.953	.0526	9 11-16	8 3-8	11 29-32	4 3-16
5 3-4	2 3-8	5.203	.0526	10 1-8	8 3-4	12 7-16	4 3-8
6	2 1-4	5.423	.0555	10 9-16	9 1-8	12 9-10	4 9-16

The threads have an angle of 60°, with flat tops and bottoms, and are of the following proportions:

Notation of letters. All dimensions in inches.

D = outside diameter of screw;
 d = diameter of root of thread, or of hole in the nut;
 p = pitch of screw;
 t = number of threads per inch;
 f = flat top and bottom;
 o = outside diameter of hexagon nut or bolt head;

i = inside diameter of hexagon, or side of square nut or bolt head;
 s = diagonal of square nut or bolt head;
 k = height of rough or unfinished bolt head.

The height of finished nut or bolt head is made equal to the diameter D of the screw.

$$p = \frac{\sqrt{16D + 10} - 2.909}{16.64}, \quad t = \frac{1}{p}, \quad s = 1.414 i.$$

$$d = D - \frac{1.299}{t}, \quad i = \frac{3D}{2} + \frac{1}{8}, \quad o = 1.155 i, \quad f = \frac{p}{8}.$$

WEIGHT OF CAST-IRON PIPE PER FOOT IN POUNDS.

These weights are for plain pipe. For hauboy pipe add 8 in. in length for each joint. For copper add $\frac{1}{2}$; for lead, $\frac{3}{4}$; for welded iron, add $\frac{1}{8}$, or multiply by 1.0667.

Diameter of Bore. Inches.	Thickness of Pipe in Inches.									
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$
1	3.07	5.07	7.38							
$1\frac{1}{4}$	3.69	6.00	8.61							
$1\frac{1}{2}$	4.30	6.92	9.84							
$1\frac{3}{4}$	4.92	7.84	11.10							
2	5.53	8.76	12.30	16.2						
$2\frac{1}{4}$	6.15	9.69	13.50	17.7						
$2\frac{1}{2}$	6.76	10.60	14.80	19.2	24.0					
$2\frac{3}{4}$	7.37	11.50	16.00	20.8	25.9					
3	7.98	12.50	17.20	22.3	27.7	33.4				
$3\frac{1}{8}$	9.21	14.30	19.70	25.4	31.4	37.7				
4	10.30	16.10	22.20	28.5	35.1	42.0				
$4\frac{1}{2}$	11.70	18.00	24.60	31.5	38.8	46.3				
5	12.90	19.80	27.10	34.6	42.5	50.6				
$5\frac{1}{2}$	14.20	21.70	29.50	37.7	46.1	54.9				
6	15.40	23.50	32.00	40.8	49.8	59.2	68.9			
$6\frac{1}{2}$	16.60	25.40	34.50	43.8	53.5	63.5	73.8	84.4		
7	17.80	27.20	36.90	46.9	57.2	67.8	78.7	89.4		
$7\frac{1}{2}$	19.10	29.10	39.40	50.0	60.9	72.1	83.7	95.5	108	
8	20.30	30.90	41.80	53.1	64.6	76.4	88.6	101.0	114	127
$8\frac{1}{2}$	21.50	32.80	44.30	56.1	68.3	80.7	93.5	107.0	120	134
9	22.80	34.60	46.80	59.2	72.0	85.1	98.4	112.0	126	140
$9\frac{1}{2}$	24.00	36.40	49.20	62.3	75.7	89.3	103.0	118.0	132	147
10	25.10	38.30	51.70	65.3	79.4	93.6	108.0	123.0	138	164
11	27.60	42.00	56.60	71.5	86.7	102.0	118.0	134.0	151	168
12	30.00	45.70	61.50	77.7	94.1	111.0	128.0	145.0	163	181
13	32.50	49.40	66.40	83.8	102.0	120.0	138.0	156.0	175	196
14	35.00	53.10	71.40	89.4	109.0	128.0	148.0	168.0	188	208
15	37.40	56.70	76.30	96.1	116.0	137.0	158.0	179.0	200	222
16	39.10	60.40	81.20	102.0	124.0	145.0	167.0	190.0	212	235
17	42.30	64.10	86.10	108.0	131.0	154.0	177.0	201.0	225	249
18	44.80	67.80	91.00	115.0	139.0	163.0	187.0	212.0	237	262
19	47.30	71.50	96.00	121.0	146.0	171.0	197.0	223.0	249	276
20	49.70	75.20	101.00	127.0	153.0	180.0	207.0	234.0	261	289
22	54.60	82.60	111.00	139.0	168.0	196.0	227.0	256.0	286	316
24	59.60	89.90	121.00	152.0	183.0	214.0	246.0	278.0	311	343
26	64.50	97.30	131.00	164.0	198.0	231.0	266.0	300.0	335	370
28	69.40	105.00	140.00	176.0	212.0	249.0	286.0	323.0	360	397
30	74.20	112.00	150.00	188.0	227.0	266.0	305.0	345.0	384	424

**TABLE OF STANDARD DIMENSIONS OF WROUGHT-
IRON WELDED PIPES.**

Nominal Diameter.	External Diameter.	Thickness.	Internal Diameter.	Internal Circumference.	External Circumference.	Length of Pipe per Sq. Ft. of Internal Surface.	Length of Pipe per Sq. Ft. of External Surface.	Internal Area.	Weight per Foot.	No. of Threads per Inch of Screw.
In.	In.	In.	In.	In.	In.	Ft.	Ft.	In.	Lb.	
$\frac{1}{8}$	.40	.068	.27	.85	1.27	14.15	9.440	.057	.24	27
$\frac{3}{8}$	.54	.088	.36	1.14	1.70	10.50	7.075	.104	.42	18
$\frac{1}{2}$	.67	.091	.49	1.55	2.12	7.67	5.657	.192	.56	18
$\frac{3}{4}$	.84	.109	.62	1.96	2.65	6.13	4.502	.305	.84	14
$1\frac{1}{4}$	1.05	.113	.82	2.59	3.30	4.64	3.637	.533	1.13	14
$1\frac{1}{2}$	1.31	.134	1.05	3.29	4.13	3.66	2.903	.863	1.67	$11\frac{1}{2}$
$2\frac{1}{2}$	1.66	.140	1.38	4.33	5.21	2.77	2.301	1.496	2.26	$11\frac{1}{2}$
$3\frac{1}{2}$	1.90	.145	1.61	5.06	5.97	2.37	2.010	2.038	2.69	$11\frac{1}{2}$
$4\frac{1}{2}$	2.37	.154	2.07	6.49	7.46	1.85	1.611	3.355	3.67	$11\frac{1}{2}$
$5\frac{1}{2}$	2.87	.204	2.47	7.75	9.03	1.55	1.328	4.783	5.77	8
$6\frac{1}{2}$	3.50	.217	3.07	9.64	11.00	1.24	1.091	7.388	7.55	8
$8\frac{1}{2}$	4.00	.226	3.55	11.15	12.57	1.08	0.955	9.887	9.05	8
$10\frac{1}{2}$	4.50	.237	4.03	12.65	14.14	.95	0.849	12.730	10.73	8
$12\frac{1}{2}$	5.00	.247	4.51	14.15	15.71	.85	0.765	15.939	12.49	8
$14\frac{1}{2}$	5.56	.259	5.04	15.85	17.47	.78	0.629	19.990	14.56	8
$16\frac{1}{2}$	6.62	.280	6.06	19.05	20.81	.63	0.577	28.889	18.77	8
$18\frac{1}{2}$	7.62	.301	7.02	22.06	23.95	.54	0.505	38.737	23.41	8
$20\frac{1}{2}$	8.62	.322	7.98	25.08	27.10	.48	0.444	50.039	28.35	8
$22\frac{1}{2}$	9.69	.344	9.00	28.28	30.43	.42	0.394	63.633	34.08	8
$24\frac{1}{2}$	10.75	.366	10.02	31.47	33.77	.38	0.355	78.838	40.64	8

FLUXES FOR SOLDERING & WELDING.

Iron	Borax	Zinc	Chloride of zinc
Tinned iron	Resin	Lead	Tallow or resin
Copper and brass		Lead and tin pipes	
Sal ammoniac		Resin and sweet oil	

Steel.—Pulverize together 1 part of sal ammoniac and 10 parts of borax and fuse until clear. When solidified, pulverize to powder.

STEAM TABLES.

Whenever the pressure of saturated steam is changed, there are other properties that change with it. These properties are the following:

1. The temperature of the steam, or, what is the same thing, the boiling point.

2. The number of B. T. U. required to raise a pound of water from 32° (freezing) to the boiling point corresponding to the given pressure. This is called the *heat of the liquid*.

3. The number of B. T. U. required to change the water at the boiling temperature into steam at the same temperature. This is called the *latent heat of vaporization*, or, simply, the *latent heat*.

4. The number of heat units required to change a pound of water at 32° to steam of the required temperature and pressure. This is called the *total heat of vaporization*, or, simply, the *total heat*.

It is plain that the *total heat* is the sum of the *heat of the liquid* and the *latent heat*. That is, total heat = heat of liquid + latent heat.

5. The *specific volume* of the steam at the given pressure; that is, the number of cubic feet occupied by a pound of steam of the given pressure.

6. The *density* of the steam; that is, the weight of 1 cubic foot of the steam at the given pressure.

All the above properties are different for different pressures. For example, if steam boils under atmospheric pressure, the temperature is 212°; the heat of the liquid is 180.531 B. T. U.; the latent heat, 966.069 B. T. U.; the total heat, 1,146.6 B. T. U. A pound of steam at this pressure occupies 26.37 cu. ft., and a cubic foot of the steam weighs about .037928 lb. When the pressure is 70 lb. per sq. in. above vacuum, the temperature is 302.774°; the heat of the liquid is 272.657 B. T. U.; the latent heat is 901.629 B. T. U.; the total heat is 1,174.286 B. T. U. A pound of the steam occupies 6.076 cu. ft., and a cubic foot of the steam weighs .164584 lb.

These properties have been determined by direct experiment for all ordinary steam pressures. They are given in the table of the properties of saturated steam, pages 29-31.

EXPLANATION OF THE TABLE.

Column 1 gives the pressures from 1 to 300 lb. These pressures are above vacuum. The steam gauges fitted on steam boilers register the pressure above the atmosphere. That is, if the steam is at atmospheric pressure, 14.7 lb. per sq. in., the gauge registers 0. Consequently, the atmospheric pressure must be added to the reading of the gauge to obtain the pressure above vacuum. In using the table, care must be taken *not* to use the gauge pressures without first adding 14.7 lb. per sq. in.

Pressures registered above vacuum are called *absolute pressures*. The pressures given in column 1 are *absolute*. Absolute pressure per square inch = gauge pressure per square inch + 14.7.

Column 2 gives the temperature of the steam when at the pressure shown in column 1.

Column 3 gives the *heat of the liquid*. It will be noticed that the values in column 3 may be obtained approximately by subtracting 32° from the temperature in column 2. If the specific heat of water were exactly 1.00, it would, of course, take exactly $212 - 32 = 180$ B. T. U. to raise a pound of water from 32° to 212° . But experiment shows that the specific heat of water is slightly greater than 1.00 when the temperature of the water is above 62° , and it therefore takes 180.531 B. T. U. to raise a pound of water from 32° to 212° .

Column 4 gives the *latent heat of vaporization*, which is seen to decrease slightly as the pressure increases.

Column 5 gives the *total heat of vaporization*. The values in column 5 may be obtained by adding together the corresponding values in columns 3 and 4.

Column 6 gives the weight of a cubic foot of steam in pounds. As would be expected, the steam becomes denser as the pressure rises, and weighs more per cubic foot.

Column 7 gives the number of cubic feet occupied by 1 pound of steam at the given pressure. It will be noticed that the corresponding values of columns 6 and 7 multiplied together always produce 1. Thus, for 31.3 pounds pressure, gauge, $.11088 \times 9.018 = 1.000$, nearly.

Column 8 gives the ratio of the volume of a pound of

steam at the given pressure, and the volume of a pound of water at 39.2°. The values in column 8 may be obtained by dividing 62.425, the weight of a cubic foot of water at 39.2°, by the numbers in column 6.

EXAMPLES ON THE USE OF THE STEAM TABLE.

EXAMPLE 1.—Calculate the heat required to change 5 lb. of water at 32° into steam at 92 lb. pressure above vacuum.

SOLUTION.—From column 5, the total heat of 1 lb. at 92 lb. pressure is 1,180.045 B. T. U.

$$1,180.045 \times 5 = 5,900.225 \text{ B. T. U.}$$

EXAMPLE 2.—How many heat units are required to raise 8½ lb. of water from 32° to 250° F.?

SOLUTION.—Looking in column 3, the heat of the liquid of 1 lb. at 250.293° is 219.261 B. T. U. $219.261 - .293 = 218.968$ B. T. U. = heat of liquid for 250°. Then, for 8½ lb. it is $218.968 \times 8\frac{1}{2} = 1,861.228$ B. T. U.

EXAMPLE 3.—How many foot-pounds of work will it require to change 60 lb. of boiling water at 80 lb. pressure, absolute, into steam of the same pressure?

SOLUTION.—Looking under column 4, the latent heat of vaporization is 895.108; that is, it takes 895.108 B. T. U. to change 1 lb. of water at 80 lb. pressure into steam of the same pressure. Therefore, it takes $895.108 \times 60 = 53,706.48$ B. T. U. to perform the same operation on 60 lb. of water.

$$53,706.48 \times 778 = 41,783,641.44 \text{ ft.-lb.}$$

EXAMPLE 4.—Find the volume occupied by 14 lb. of steam at 30 lb., gauge pressure.

SOLUTION.—30 lb., gauge pressure = $30 + 14.7 = 44.7$, absolute pressure. The nearest pressure in the table is 44 lb., and the volume of a pound of steam at that pressure is 9.403 cu. ft. The volume of a pound at 46 lb. pressure is 9.018 cu. ft. $9.403 - 9.018 = .385$ cu. ft., the difference in volume for a difference in pressure of 2 lb. $\frac{.385}{2} = .1925$ cu. ft., the difference in volume for a difference in pressure of 1 lb. $.1925 \times .7 = .135$ cu. ft., the difference in volume for a difference in pressure of 7 lb. Therefore, $9.403 - .135 = 9.268$ cu. ft. is the volume of 1 lb. of steam at 44.7 lb. pressure. The .135 cu. ft.

is subtracted from 9.403 cu. ft., since the volume is less for a pressure of 44.7 lb. than for a pressure of 44 lb.

$$9.268 \times 14 = 129.752 \text{ cu. ft.}$$

EXAMPLE 5.—Find the weight of 40 cu. ft. of steam at a temperature of 254° F.

SOLUTION.—The weight of 1 cu. ft. of steam at 254.002°, from the table, is .078839 lb. Neglecting the .002°, the weight of 40 cu. ft. is, therefore,

$$.078839 \times 40 = 3.15356 \text{ lb.}$$

EXAMPLE 6.—How many pounds of steam at 64 lb. pressure, absolute, are required to raise the temperature of 300 lb. of water from 40° to 130° F., the water and steam being mixed?

SOLUTION.—The number of heat units required to raise 1 lb. from 40° to 130° is $130 - 40 = 90$ B. T. U. (Actually a little more than 90 would be required, but the above is near enough for all practical purposes.) Then, to raise 300 lb. from 40° to 130° requires $90 \times 300 = 27,000$ B. T. U. This quantity of heat must necessarily come from the steam. Now, 1 lb. of steam at 64 lb. pressure gives up, in condensing, its latent heat of vaporization, or 905.9 B. T. U. But, in addition to its latent heat, each pound of steam on condensing must give up an additional amount of heat in falling to 130°. Since the original temperature of the steam was 296.805° F. (see table), each pound gives up by its fall of temperature $296.805 - 130 = 166.805$ B. T. U. Therefore, each pound of the steam gives up a total of

$$905.9 + 166.805 = 1,072.705 \text{ B. T. U.}$$

It will, therefore, take $\frac{27,000}{1,072.705} = 25.17$ lb. of steam to accomplish the desired result.

With the steam tables a reliable thermometer may be used for ascertaining the pressure of saturated steam or for testing the accuracy of a steam gauge. The temperature of the steam being measured by the thermometer, the corresponding absolute pressure is found from the steam tables; the gauge pressure is then found by subtracting 14.7 from the absolute pressure. Thus, the temperature of the steam in a condenser being 142°, we find from the steam tables that the corresponding absolute pressure is 3 lb. per sq. in., nearly.

THE PROPERTIES OF SATURATED STEAM.

Pressure Above Vacuum in Pounds per Square Inch.	Temperature, Fahrenheit Degrees.	Quantity of Heat in British Thermal Units.			Weight of a Cubic Foot of Steam in Pounds.	Volume of a Pound of Steam in Cubic Feet.	Ratio of Vol. of Steam to Vol. of Equal Weight of Dist. Water at Temp. of Maximum Density.
		Required to Raise Tem- perature of the Water From 32° to °.	Total Latent Heat at Pressure <i>p</i> .	Total Heat Above 32°.			
1	2	3	4	5	6	7	8
<i>p</i>	<i>t</i>	<i>q</i>	<i>L</i>	<i>H</i>	<i>W</i>	<i>V</i>	<i>R</i>
1	102.018	70.040	1,043.015	1,113.055	.003027	330.4	20,623
2	126.302	94.368	1,026.094	1,120.462	.005818	171.9	10,730
3	141.654	109.764	1,015.380	1,125.144	.008522	117.3	7,325
4	153.122	121.271	1,007.370	1,128.641	.011172	89.51	5,588
5	162.370	130.563	1,000.899	1,131.462	.013781	72.56	4,530
6	170.173	138.401	995.441	1,133.842	.016357	61.14	3,816
7	176.945	145.213	990.695	1,135.908	.018908	52.89	3,302
8	182.952	151.255	986.485	1,137.740	.021486	46.65	2,912
9	188.357	156.699	982.690	1,139.389	.023944	41.77	2,607
10	193.284	161.660	979.232	1,140.892	.026437	37.83	2,361
11	197.814	166.225	976.050	1,142.275	.028911	34.59	2,159
12	202.012	170.457	973.098	1,143.555	.031376	31.87	1,990
13	205.929	174.402	970.346	1,144.748	.033828	29.56	1,845
14	209.604	178.112	967.757	1,145.869	.036265	27.58	1,721
14.69	212.000	180.531	966.069	1,146.600	.037928	26.37	1,646
15	213.067	181.606	965.318	1,146.926	.038688	25.85	1,614
16	216.347	184.919	963.007	1,147.926	.041109	24.33	1,519
17	219.452	188.056	960.818	1,148.874	.043519	22.98	1,434
18	222.424	191.058	958.721	1,149.779	.045920	21.78	1,359
19	225.255	193.918	956.725	1,150.643	.048312	20.70	1,292

TABLE—(Continued).

1	2	3	4	5	6	7	8
<i>p</i>	<i>t</i>	<i>q</i>	<i>L</i>	<i>H</i>	<i>W</i>	<i>V</i>	<i>R</i>
20	227.964	196.655	954.814	1,151.469	.050696	19.730	1,231.0
22	233.069	201.817	951.209	1,153.026	.055446	18.040	1,126.0
24	237.803	206.610	947.861	1,154.471	.060171	16.620	1,038.0
26	242.225	211.089	944.730	1,155.819	.064870	15.420	962.3
28	246.376	215.293	941.791	1,157.084	.069545	14.380	897.5
30	250.293	219.261	939.019	1,158.280	.074201	13.480	841.3
32	254.002	223.021	936.389	1,159.410	.078839	12.680	791.8
34	257.523	226.594	933.891	1,160.485	.083461	11.980	948.0
36	260.883	230.001	931.508	1,161.509	.088067	11.360	708.8
38	264.093	233.261	929.227	1,162.488	.092657	10.790	673.7
40	267.168	236.386	927.040	1,163.426	.097231	10.280	642.0
42	270.122	239.389	924.940	1,164.329	.101794	9.826	613.3
44	272.965	242.275	922.919	1,165.194	.106345	9.403	587.0
46	275.704	245.061	920.968	1,166.029	.110884	9.018	563.0
48	278.348	247.752	919.084	1,166.836	.115411	8.665	540.9
50	280.904	250.355	917.260	1,167.615	.119927	8.338	520.5
52	283.381	252.875	915.494	1,168.369	.124433	8.037	501.7
54	285.781	255.321	913.781	1,169.102	.128928	7.756	484.2
56	288.111	257.695	912.118	1,169.813	.133414	7.496	467.9
58	290.374	260.002	910.501	1,170.503	.137892	7.252	452.7
60	292.575	262.248	908.928	1,171.176	.142362	7.024	438.5
62	294.717	264.433	907.396	1,171.829	.146824	6.811	425.2
64	296.805	266.566	905.900	1,172.466	.151277	6.610	412.6
66	298.842	268.644	904.443	1,173.087	.155721	6.422	400.8
68	300.831	270.674	903.020	1,173.694	.160157	6.244	389.8
70	302.774	272.657	901.629	1,174.286	.164584	6.076	379.3
72	304.669	274.597	900.269	1,174.866	.169003	5.917	369.4
74	306.526	276.493	898.938	1,175.431	.173417	5.767	360.0
76	308.344	278.350	897.635	1,175.985	.177825	5.624	351.1
78	310.123	280.170	896.359	1,176.529	.182229	5.488	342.6
80	311.866	281.952	895.108	1,177.060	.186627	5.358	334.5
82	313.576	283.701	893.879	1,177.580	.191017	5.235	326.8
84	315.250	285.414	892.677	1,178.091	.195401	5.118	319.5
86	316.893	287.096	891.496	1,178.592	.199781	5.006	312.5
88	318.510	288.750	890.335	1,179.085	.204155	4.898	305.8

TABLE—(Continued).

1	2	3	4	5	6	7	8
<i>p</i>	<i>t</i>	<i>q</i>	<i>L</i>	<i>H</i>	<i>W</i>	<i>V</i>	<i>R</i>
90	320.094	290.373	889.196	1,179.569	.208525	4.796	299.4
92	321.653	291.970	888.075	1,180.045	.212892	4.697	293.2
94	323.183	293.539	886.972	1,180.511	.217253	4.603	287.3
96	324.688	295.083	885.887	1,180.970	.221604	4.513	281.7
98	326.169	296.601	884.821	1,181.422	.225950	4.426	276.3
100	327.625	298.093	883.773	1,181.866	.230293	4.342	271.1
105	331.169	301.731	881.214	1,182.945	.241139	4.147	258.9
110	334.582	305.242	878.744	1,183.986	.251947	3.969	247.8
115	337.874	308.621	876.371	1,184.992	.262732	3.806	237.6
120	341.058	311.885	874.076	1,185.961	.273500	3.656	228.3
125	344.136	315.051	871.848	1,186.899	.284243	3.518	219.6
130	347.121	318.121	869.688	1,187.809	.294961	3.390	211.6
135	350.015	321.105	867.590	1,188.695	.305659	3.272	204.2
140	352.827	324.003	865.552	1,189.555	.316338	3.161	197.3
145	355.562	326.823	863.567	1,190.390	.326998	3.058	190.9
150	358.223	329.566	861.634	1,191.200	.337643	2.962	184.9
160	363.346	334.850	857.912	1,192.762	.358886	2.786	173.9
170	368.226	339.892	854.359	1,194.251	.380071	2.631	164.3
180	372.886	344.708	850.963	1,195.671	.401201	2.493	155.6
190	377.352	349.329	847.703	1,197.032	.422280	2.368	147.8
200	381.636	353.766	844.573	1,198.339	.443310	2.256	140.8
210	385.759	358.041	841.556	1,199.597	.464295	2.154	134.5
220	389.736	362.168	838.642	1,200.810	.485237	2.061	128.7
230	393.575	366.152	835.828	1,201.980	.506139	1.976	123.3
240	397.285	370.008	833.103	1,203.111	.527003	1.898	118.5
250	400.883	373.750	830.459	1,204.209	.547831	1.825	114.0
260	404.370	377.377	827.896	1,205.273	.568626	1.759	109.8
270	407.755	380.995	825.401	1,206.306	.589390	1.697	105.9
280	411.048	384.337	822.973	1,207.310	.610124	1.639	102.3
290	414.250	387.677	820.609	1,208.286	.630829	1.585	99.0
300	417.371	390.933	818.305	1,209.238	.651506	1.535	95.8

LOGARITHMS.

EXPONENTS.

By the use of logarithms, the processes of multiplication, division, involution, and evolution are greatly shortened, and some operations may be performed that would be impossible without them. Ordinary logarithms cannot be applied to addition and subtraction.

The *logarithm* of a number is that *exponent* by which some fixed number, called the *base*, must be affected in order to equal the number. Any number may be taken as the base. Suppose we choose 4. Then the logarithm of 16 is 2, because 2 is the exponent by which 4 (the base) must be affected in order to equal 16, since $4^2 = 16$. In this case, instead of reading 4^2 as 4 square, read it 4 exponent 2. With the same base, the logarithms of 64 and 8 would be 3 and 1.5, respectively, since $4^3 = 64$, and $4^{1.5} = 4^{\frac{3}{2}} = 8$. In these cases, as in the preceding, read 4^3 and $4^{1.5}$ as 4 exponent 3, and 4 exponent 1.5, respectively.

Although any positive number except 1 can be used as a base and a table of logarithms calculated, but two numbers have ever been employed. For all arithmetical operations (except addition and subtraction) the logarithms used are called the *Briggs*, or *common*, logarithms, and the base used is 10. In abstract mathematical analysis, the logarithms used are variously called *hyperbolic*, *Napierian*, or *natural* logarithms, and the base is $2.718281828+$. The common logarithm of any number may be converted into a Napierian logarithm by multiplying the common logarithm by $2.30258509+$, which is usually expressed as 2.3026, and sometimes as 2.3. Only the common system of logarithms will be considered here.

Since in the common system the base is 10, it follows that, since $10^1 = 10$, $10^2 = 100$, $10^3 = 1,000$, etc., the logarithm (exponent) of 10 is 1, of 100 is 2, of 1,000 is 3, etc. For the sake of brevity in writing, the words "logarithm of" are abbreviated to "log." Thus, instead of writing logarithm of $100 = 2$, write $\log 100 = 2$. When speaking, however, the words for which "log" stands should always be pronounced in full.

From the above it will be seen that, when the base is 10,
 since $10^0 = 1$, the exponent $0 = \log 1$;
 since $10^1 = 10$, the exponent $1 = \log 10$;
 since $10^2 = 100$, the exponent $2 = \log 100$;
 since $10^3 = 1,000$, the exponent $3 = \log 1,000$; etc.

Also,

since $10^{-1} = \frac{1}{10} = .1$, the exponent $-1 = \log .1$;
 since $10^{-2} = \frac{1}{100} = .01$, the exponent $-2 = \log .01$;
 since $10^{-3} = \frac{1}{1000} = .001$, the exponent $-3 = \log .001$; etc.

From this it will be seen that the logarithms of exact powers of 10 and of decimals like .1, .01, and .001 are the whole numbers 1, 2, 3, etc. and $-1, -2, -3$, etc., respectively. *Only numbers consisting of 1 and one or more ciphers have whole numbers for logarithms.*

Now, it is evident that, to produce a number between 1 and 10, the exponent of 10 must be a fraction; to produce a number between 10 and 100, it must be 1 plus a fraction; to produce a number between 100 and 1,000, it must be 2 plus a fraction; etc. Hence, the logarithm of any number between 1 and 10 is a fraction; of any number between 10 and 100, 1 plus a fraction; of any number between 100 and 1,000, 2 plus a fraction, etc. A logarithm, therefore, usually consists of two parts: a whole number, called the *characteristic*, and a fraction, called the *mantissa*. The mantissa is always expressed as a decimal. For example, to produce 20, 10 must have an exponent of *approximately* 1.30103, or $10^{1.30103} = 20$, very nearly, the degree of exactness depending on the number of decimal places used. Hence, $\log 20 = 1.30103$, 1 being the characteristic, and .30103, the mantissa.

Referring to the second part of the preceding table, it is clear that the logarithms of all numbers less than 1 are negative, the logarithms of those between 1 and .1 being -1 plus a fraction. For, since $\log .1 = -1$, the logarithms of .2, .3, etc. (which are all greater than .1, but less than 1) must be greater than -1 ; i. e., they must equal -1 *plus* a fraction. For the same reason, to produce a number between .1 and .01, the logarithm (exponent of 10) would be equal to -2 plus a fraction, and for a number between .01 and .001, it would be equal to -3 plus a fraction. Hence, the logarithm

of any number between 1 and .01 has a negative characteristic of 1 and a positive mantissa; of a number between .1 and .01, a negative characteristic of 2 and a positive mantissa; of a number between .01 and .001, a negative characteristic of 3 and a positive mantissa; of a number between .001 and .0001, a negative characteristic of 4 and a positive mantissa, etc. *The negative characteristics are distinguished from the positive by the — sign written over the characteristic.* Thus, $\bar{3}$ indicates that 3 is negative.

It must be remembered that in all cases the mantissa is positive. Thus, the logarithm 1.30103 means $+1 + .30103$, and the logarithm $\bar{1}.30103$ means $-1 + .30103$. Were the minus sign written in front of the characteristic, it would indicate that the entire logarithm was negative. Thus, $-1.30103 = -1 - .30103$.

Rule for Characteristic.—Starting from the unit figure, count the number of places to the first (left-hand) digit of the given number, calling unit's place zero; the number of places thus counted will be the required characteristic. If the first digit lies to the left of the unit figure, the characteristic is positive; if to the right, negative. If the first digit of the number is the unit figure, the characteristic is 0. Thus, the characteristic of the logarithm of 4,826 is 3, since the first digit, 4, lies in the 3d place to the left of the unit figure, 6. The characteristic of the logarithm of 0.0000072 is -6 or $\bar{6}$, since the first digit, 7, lies in the 6th place to the right of the unit figure. The characteristic of the logarithm of 4.391 is 0, since 4 is both the first digit of the number and also the unit figure.

TO FIND THE LOGARITHM OF A NUMBER.

To aid in obtaining the mantissas of logarithms, *tables of logarithms* have been calculated, some of which are very elaborate and convenient. In the Table of Logarithms, the mantissas of the logarithms of numbers from 1 to 9,999 are given to five places of decimals. The mantissas of logarithms of larger numbers can be found by interpolation. The table contains the *mantissas only*; the characteristics may be easily found by the preceding rule.

The table depends on the principle, which will be explained later, that all numbers having the same figures in the same order have the same mantissa, without regard to the position of the decimal point, which affects the characteristic only. To illustrate, if $\log 206 = 2.31387$, then,

$$\begin{array}{ll} \log 20.6 = 1.31387; & \log .206 = \bar{1}.31387; \\ \log 2.06 = .31387; & \log .0206 = \bar{2}.31387; \text{ etc.} \end{array}$$

To find the logarithm of a number not having more than four figures:

Rule.—Find the first three significant figures of the number whose logarithm is desired, in the left-hand column; find the fourth figure in the column at the top (or bottom) of the page; and in the column under (or above) this figure, and opposite the first three figures previously found, will be the mantissa or decimal part of the logarithm. The characteristic being found as previously described, write it at the left of the mantissa, and the resulting expression will be the logarithm of the required number.

EXAMPLE.—Find from the table the logarithm (a) of 476; (b) of 25.47; (c) of 1.073; (d) of .06313.

SOLUTION.—(a) In order to economize space and make the labor of finding the logarithms easier, the first two figures of the mantissa are given only in the column headed 0. The last three figures of the mantissa, opposite 476 in the column headed N (N stands for number), are 761, found in the column headed 0; glancing upwards, we find the first two figures of the mantissa, viz., 67. The characteristic is 2; hence, $\log 476 = 2.67761$.

NOTE.—Since all numbers in the table are decimal fractions, the decimal point is omitted throughout; this is customary in all tables of logarithms.

(b) To find the logarithm of 25.47, we find the first three figures, 254, in the column headed N, and on the same horizontal line, under the column headed 7 (the fourth figure of the given number), will be found the last three figures of the mantissa, viz., 603. The first two figures are evidently 40, and the characteristic is 1; hence, $\log 25.47 = 1.40603$.

(c) For 1.073; in the column headed 3, opposite 107 in the column headed N, the last three figures of the mantissa are found, in the usual manner, to be 060. It will be noticed

that these figures are printed *060, the star meaning that instead of glancing *upwards* in the column headed 0, and taking 02 for the first two figures, we must glance *downwards* and take the two figures opposite the number 108, in the left-hand column, i. e., 03. The characteristic being 0, $\log 1.073 = 0.03060$, or, more simply, .03060.

(d) For .06313; the last three figures of the mantissa are found opposite 631, in column headed 3, to be 024. In this case, the first two figures occur in the same row, and are 80. Since the characteristic is $\bar{2}$, $\log .06313 = \bar{2}.80024$.

If the original number contains but one digit (a cipher is not a digit), annex mentally two ciphers to the right of the digit; if the number contains but two digits (with no ciphers between, as in 4,008), annex mentally one cipher on the right before seeking the mantissa. Thus, if the logarithm of 7 is wanted, seek the mantissa for 700, which is .84510; or, if the logarithm of 48 is wanted, seek the mantissa for 480, which is .68124. Or, find the mantissas of logarithms of numbers between 0 and 100, on the first page of the tables.

The process of finding the logarithm of a number from the table is technically called *taking out the logarithm*.

To take out the logarithm of a number consisting of more than four figures, it is inexpedient to use more than five figures of the number when using five-place logarithms (the logarithms given in the accompanying table are five-place). Hence, if the number consists of more than five figures and the sixth figure is less than 5, replace all figures after the fifth with ciphers; if the sixth figure is 5 or greater, increase the fifth figure by 1 and replace the remaining figures with ciphers. Thus, if the number is 31,415,926, find the logarithm of 31,416,000; if 31,415,426, find the logarithm of 31,415,000.

EXAMPLE.—Find $\log 31,416$.

SOLUTION.—Find the mantissa of the logarithm of the first four figures, as explained above. This is, in the present case, .49707. Now, subtract the number in the column headed 1, opposite 314 (the first three figures of the given number), from the next greater consecutive number, in this case 721, in the column headed 2. $721 - 707 = 14$; this number is called the *difference*. At the extreme right of the page will be found a

secondary table headed P. P., and at the top of one of these columns, in this table, in bold-face type, will be found the difference. It will be noticed that each column is divided into two parts by a vertical line, and that the figures on the left of this line run in sequence from 1 to 9. Considering the difference column headed 14, we see opposite the number 6 (6 is the last or fifth figure of the number whose logarithm we are taking out) the number 8.4, and we add this number to the mantissa found above, disregarding the decimal point in the mantissa, obtaining $49,707 + 8.4 = 49,715.4$. Now, since 4 is less than 5, we reject it, and obtain for our complete mantissa .49715. Since the characteristic of the logarithm of 31,416 is 4, $\log 31,416 = 4.49715$.

EXAMPLE.—Find $\log 380.93$.

SOLUTION.—Proceeding in exactly the same manner as above, the mantissa for 3,809 is 58,081 (the star directs us to take 58 instead of 57 for the first two figures); the next greater mantissa is 58,092, found in the column headed 0, opposite 381 in column headed N. The difference is $092 - 081 = 11$. Looking in the section headed P. P. for column headed 11, we find opposite 3, 3.3; neglecting the .3, since it is less than 5, 3 is the amount to be added to the mantissa of the logarithm of 3,809 to form the logarithm of 38,093. Hence, $58,081 + 3 = 58,084$, and since the characteristic is 2, $\log 380.93 = 2.58084$.

EXAMPLE.—Find $\log 1,296,728$.

SOLUTION.—Since this number consists of more than five figures and the sixth figure is less than 5, we find the logarithm of 1,296,700 and call it the logarithm of 1,296,728. The mantissa of $\log 1,296$ is found to be 11,261. The difference is $294 - 261 = 33$. Looking in the P. P. section for column headed 33, we find opposite 7, on the extreme left, 23.1; neglecting the .1, the amount to be added to the above mantissa is 23. Hence, the mantissa of $\log 1,296,728 = 11,261 + 23 = 11,284$; since the characteristic is 6, $\log 1,296,728 = 6.11284$.

EXAMPLE.—Find $\log 89.126$.

SOLUTION.— $\log 89.12 = 1.94998$. Difference between this and $\log 80.13 = 1.95002 - 1.94998 = 4$. The P. P. (proportional part) for the fifth figure of the number 6 is 2.4, or 2.

Hence, $\log 89.126 = 1.94998 + .00002 = 1.95000$.

EXAMPLE.—Find log .096725.

SOLUTION.— Log .09672 = $\overline{2.98552}$. Difference = 4.

P. P. for 5 = $\frac{2}{2}$

Hence, log .096725 = $\overline{2.98554}$.

To find the logarithm of a number consisting of five or more figures:

Rule.—I. *If the number consists of more than five figures and the sixth figure is 5 or greater, increase the fifth figure by 1 and write ciphers in place of the sixth and remaining figures.*

II. *Find the mantissa corresponding to the logarithm of the first four figures, and subtract this mantissa from the next greater mantissa in the table; the remainder is the difference.*

III. *Find in the secondary table headed P. P. a column headed by the same number as that just found for the difference, and in this column, opposite the number corresponding to the fifth figure (or fifth figure increased by 1) of the given number (this figure is always situated at the left of the dividing line of the column), will be found the P. P. (proportional part) for that number. The P. P. thus found is to be added to the mantissa found in II, as in the preceding examples, and the result is the mantissa of the logarithm of the given number, as nearly as may be found with five-place tables.*

TO FIND A NUMBER WHOSE LOGARITHM IS GIVEN.

Rule.—I. *Consider the mantissa first. Glance along the different columns of the table which are headed 0, until the first two figures of the mantissa are found. Then, glance down the same column until the third figure is found (or 1 less than the third figure). Having found the first three figures, glance to the right along the row in which they are situated until the last three figures of the mantissa are found. Then, the number that heads the column in which the last three figures of the mantissa are found is the fourth figure of the required number, and the first three figures lie in the column headed N, and in the same row in which lie the last three figures of the mantissa.*

II. *If the mantissa cannot be found in the table, find the mantissa that is nearest to, but less than, the given mantissa, and which call the next less mantissa. Subtract the next less mantissa*

from the next greater mantissa in the table to obtain the difference. Also, subtract the next less mantissa from the mantissa of the given logarithm, and call the remainder the P. P. Looking in the secondary table headed P. P. for the column headed by the difference just found, find the number opposite the P. P. just found (or the P. P. corresponding most nearly to that just found); this number is the fifth figure of the required number; the fourth figure will be found at the top of the column containing the next less mantissa, and the first three figures in the column headed N and in the same row that contains the next less mantissa.

III. Having found the figures of the number as above directed, locate the decimal point by the rules for the characteristic, annexing ciphers to bring the number up to the required number of figures if the characteristic is greater than 4.

EXAMPLE.—Find the number whose logarithm is 3.56867.

SOLUTION.—The first two figures of the mantissa are 56; glancing down the column, we find the third figure, 8 (in connection with 820), opposite 370 in the N column. Glancing to the right along the row containing 820, the last three figures of the mantissa, 867, are found in the column headed 4; hence, the fourth figure of the required number is 4, and the first three figures are 370, making the figures of the required number 3,704. Since the characteristic is 3, there are three figures to the left of the unit figure, and the number whose logarithm is 3.56867 is 3,704.

EXAMPLE.—Find the number whose logarithm is 3.56871.

SOLUTION.—The mantissa is not found in the table. The next less mantissa is 56,867; the difference between this and the next greater mantissa is $879 - 867 = 12$, and the P. P. is $56,871 - 56,867 = 4$. Looking in the P. P. section for the column headed 12, we do not find 4, but we do find 3.6 and 4.8. Since 3.6 is nearer 4 than 4.8, we take the number opposite 3.6 for the fifth figure of the required number; this is 3. Hence, the fourth figure is 4; the first three figures 370, and the figures of the number are 37,043. The characteristic being 3, the number is 3,704.3.

EXAMPLE.—Find the number whose logarithm is 5.95424.

SOLUTION.—The mantissa is found in the column headed 0, opposite 900 in the column headed N. Hence, the fourth

figure is 0, and the number is 900,000, the characteristic being 5. Had the logarithm been $\bar{5}.95424$, the number would have been .00009.

EXAMPLE.—Find the number whose logarithm is .93036.

SOLUTION.—The first three figures of the mantissa, 930, are found in the 0 column, opposite 852 in the N column; but since the last two figures of all the mantissas in this row are greater than 36, we must seek the next less mantissa in the preceding row. We find it to be 93,034 (the star directing us to use 93 instead of 92 for the first two figures), in the column headed 8. The difference for this case is $039 - 034 = 5$, and the P. P. is $036 - 034 = 2$. Looking in the P. P. section for the column headed 5, we find the P. P., 2, opposite 4. Hence, the fifth figure is 4; the fourth figure is 8; the first three figures 851, and the number is 8.5184, the characteristic being 0.

EXAMPLE.—Find the number whose logarithm is $\bar{2}.05753$.

SOLUTION.—The next less mantissa is found in column headed 1, opposite 114 in the N column; hence, the first four figures are 1,141. The difference for this case is $767 - 729 = 38$, and the P. P. is $753 - 729 = 24$. Looking in the P. P. section for the column headed 38, we find that 24 falls between 22.8 and 26.6. The difference between 24 and 22.8 is 1.2, and between 24 and 26.6 is 2.6; hence, 24 is nearer 22.8 than it is to 26.6, and 6, opposite 22.8, is the fifth figure of the number. Hence, the number whose logarithm is $\bar{2}.05753$ is .011416.

In order to calculate by means of logarithms, a table is absolutely necessary. Hence, for this reason, we do not explain the method of calculating a logarithm. The work involved in calculating even a single logarithm is very great, and no method has yet been demonstrated, of which we are aware, by which the logarithm of a number like 121 can be calculated directly. Moreover, even if the logarithm could be readily obtained, it would be useless without a complete table, such as that which is here given, for the reason that after having used it, say to extract a root, the number corresponding to the logarithm of the result could not be found.

MULTIPLICATION BY LOGARITHMS.

The principle upon which the process is based may be illustrated as follows: Let X and Y represent two numbers whose logarithms are x and y . To find the logarithm of their product, we have, from the definition of a logarithm,

$$10^x = X, \quad (1)$$

and

$$10^y = Y. \quad (2)$$

Since both members of (1) may be multiplied by the same quantity without destroying the equality, they evidently may be multiplied by equal quantities like 10^y and Y . Hence, multiplying (1) by (2), member by member,

$$10^x \times 10^y = 10^{x+y} = XY,$$

or, by the definition of a logarithm, $x + y = \log XY$. But XY is the product of X and Y , and $x + y$ is the sum of their logarithms; from which it follows that the sum of the logarithms of two numbers is equal to the logarithm of their product. Hence,

To multiply two or more numbers by using logarithms:

Rule.—*Add the logarithms of the several numbers, and the sum will be the logarithm of the product. Find the number corresponding to this logarithm, and the result will be the number sought.*

EXAMPLE.—Multiply 4.38, 5.217, and 83 together.

SOLUTION.— $\text{Log } 4.38 = .64147$

$\text{Log } 5.217 = .71742$

$\text{Log } 83 = 1.91908$

Adding, $3.27797 = \log (4.38 \times 5.217 \times 83).$

Number corresponding to $3.27797 = 1,896.6$. Hence, $4.38 \times 5.217 \times 83 = 1,896.6$, nearly. By actual multiplication, the product is 1,896.5818, showing that the result obtained by using logarithms was correct to five figures.

When adding logarithms, their *algebraic* sum is always to be found. Hence, if some of their numbers multiplied together are wholly decimal, the algebraic sum of the characteristics will be the characteristic of the product. It must be remembered that the mantissas are always positive.

EXAMPLE.—Multiply 49.82, .00243, 17, and .97 together.

SOLUTION.—

$$\text{Log } 49.82 = 1.69740$$

$$\text{Log } .00243 = \bar{3}.38561$$

$$\text{Log } 17 = 1.23045$$

$$\text{Log } .97 = \bar{1}.98677$$

Adding, $0.30023 = \log (49.82 \times .00243 \times 17 \times .97)$.

Number corresponding to $0.30023 = 1.9963$. Hence, $49.82 \times .00243 \times 17 \times .97 = 1.9963$.

In this case the sum of the mantissas was 2.30023 . The integral 2 added to the positive characteristics makes their sum $= 2 + 1 + 1 = 4$; sum of negative characteristics $= \bar{3} + \bar{1} = \bar{4}$, whence $4 + (-4) = 0$. If, instead of 17, the number had been .17 in the above example, the logarithm of .17 would have been $\bar{1}.23045$, and the sum of the logarithms would have been $\bar{2}.30023$; the product would then have been .019963.

It can now be shown why all numbers with figures in the same order have the same mantissa, without regard to the decimal point. Thus, suppose it were known that $\log 2.06 = .31387$. Then, $\log 20.6 = \log (2.06 \times 10) = \log 2.06 + \log 10 = .31387 + 1 = 1.31387$. And so it might be proved with the decimal point in any other position.

DIVISION BY LOGARITHMS.

As before, let X and Y represent two numbers whose logarithms are x and y . To find the logarithm of their quotient, we have, from the definition of a logarithm,

$$10^x = X, \quad (1)$$

and

$$10^y = Y. \quad (2)$$

Dividing (1) by (2), $10^{x-y} = \frac{X}{Y}$, or, by the definition of a logarithm, $x - y = \log \frac{X}{Y}$. But $\frac{X}{Y}$ is the quotient of X by Y , and $x - y$ is the difference of their logarithms, from which it follows that *the difference between the logarithms of two numbers is equal to the logarithm of their quotient*. Hence, to divide one number by another by means of logarithms:

Rule.—*Subtract the logarithm of the divisor from the logarithm of the dividend, and the result will be the logarithm of the quotient.*

EXAMPLE.—Divide 6,784.2 by 27.42.

SOLUTION.— $\text{Log } 6,784.2 = 3.83150$

$\text{Log } 27.42 = 1.43807$

$\text{difference} = 2.39343 = \log (6,784.2 \div 27.42).$

Number corresponding to 2.39343 = 247.42. Hence, $6,784.2 \div 27.42 = 247.42$.

When subtracting logarithms, their *algebraic* difference is to be found. The operation may sometimes be confusing, because the mantissa is always positive, and the characteristic may be either positive or negative. *When the logarithm to be subtracted is greater than the logarithm from which it is to be taken, or when negative characteristics appear, subtract the mantissa first, and then the characteristic, by changing its sign and adding.*

EXAMPLE.—Divide 274.2 by 6,784.2.

SOLUTION.— $\text{Log } 274.2 = 2.43807$

$\text{Log } 6,784.2 = 3.83150$

$\underline{2.60657}$

First subtracting the mantissa .83150 gives .60657 for the mantissa of the quotient. In subtracting, 1 had to be taken from the characteristic of the minuend, leaving a characteristic of 1. Subtract the characteristic 3 from this, by changing its sign and adding $1 - 3 = \bar{2}$, the characteristic of the quotient. Number corresponding to $\bar{2}.60657 = .040418$. Hence, $274.2 \div 6,784.2 = .040418$.

EXAMPLE.—Divide .067842 by .002742.

SOLUTION.— $\text{Log } .067842 = \bar{2}.83150$

$\text{Log } .002742 = 3.43807$

$\text{difference} = 1.39343$

Since $.83150 - .43807 = .39343$ and $-2 + 3 = 1$, number corresponding to 1.39343 = 24.742. Hence, $.067842 \div .002742 = 24.742$.

The only case that is likely to cause trouble in subtracting is that in which the logarithm of the minuend has a negative characteristic, or none at all, and a mantissa less than the mantissa of the subtrahend. For example, let it be required to subtract the logarithm 3.74036 from the logarithm

$\bar{3}.55145$. The logarithm $\bar{3}.55145$ is equivalent to $-3 + .55145$. Now, if we add both $+1$ and -1 to this logarithm, it will not change its value. Hence, $\bar{3}.55145 = -3 - 1 + 1 + .55145 = \bar{4} + 1.55145$. Therefore, $\bar{3}.55145 - 3.74036 =$

$$\bar{4} + 1.55145$$

$$3 + .74036$$

$$\text{difference} = \bar{7} + .81109 = \bar{7}.81109.$$

Had the characteristic of the above logarithm been 0 instead of $\bar{3}$, the process would have been exactly the same. Thus, $.55145 = \bar{1} + 1.55145$; hence,

$$\bar{1} + 1.55145$$

$$3 + .74036$$

$$\text{difference} = \bar{4} + .81109 = \bar{4}.81109.$$

EXAMPLE.—Divide .02742 by 67.842.

SOLUTION.—Log .02742 = $\bar{2}.43807 = \bar{3} + 1.43807$

Log 67.842 = $1.83150 = 1 + .83150$

$$\text{difference} = \bar{4} + .60657 = \bar{4}.60657.$$

Number corresponding to $\bar{4}.60657 = .00040417$. Hence, $.02742 \div 67.842 = .00040417$.

EXAMPLE.—What is the reciprocal of 3.1416?

SOLUTION.—Reciprocal of 3.1416 = $\frac{1}{3.1416}$, and $\log \frac{1}{3.1416} = \log 1 - \log 3.1416 = 0 - .49715$. Since $0 = -1 + 1$,

$$\bar{1} + 1.00000$$

$$.49715$$

$$\text{difference} = \bar{1} + .50285 = \bar{1}.50285.$$

Number whose logarithm is $\bar{1}.50285 = .31831$.

INVOLUTION BY LOGARITHMS.

If X represents a number whose logarithm is x , we have, from the definition of a logarithm,

$$10^x = X.$$

Raising both numbers to some power, as the n th, the equation becomes

$$10^{xn} = X^n.$$

But X^n is the required power of X , and xn is its logarithm, from which it follows that the logarithm of a number

multiplied by the exponent of the power to which it is raised is equal to the logarithm of the power. Hence, to raise a number to any power by the use of logarithms:

Rule.—*Multiply the logarithm of the number by the exponent that denotes the power to which the number is to be raised, and the result will be the logarithm of the required power.*

EXAMPLE.—What is (a) the square of 7.92? (b) the cube of 94.7? (c) the 1.6 power of 512, that is, the value of $512^{1.6}$?

SOLUTION.—(a) $\text{Log } 7.92 = .89873$; exponent of power = 2. Hence, $.89873 \times 2 = 1.79746 = \log 7.92^2$. Number corresponding to 1.79746 = 62.727. Hence, $7.92^2 = 62.727$, nearly.

(b) $\text{Log } 94.7 = 1.97635$; $1.97635 \times 3 = 5.92905 = \log 94.7^3$. Number corresponding to 5.92905 = 849,280, nearly. Hence $94.7^3 = 849,280$, nearly.

(c) $\text{Log } 512^{1.6} = 1.6 \times \log 512 = 1.6 \times 2.70927 = 4.334832$ or 4.33483 (when using five-place logarithms) = $\log 21,616$. Hence, $512^{1.6} = 21,619$ nearly.

If the number is wholly decimal, so that the characteristic is negative, multiply the two parts of the logarithm separately by the exponent of the number. If, after multiplying the mantissa, the product has a characteristic, add it, algebraically, to the negative characteristic multiplied by the exponent, and the result will be the negative characteristic of the required power.

EXAMPLE.—Raise .0751 to the fourth power.

SOLUTION.— $\text{Log } .0751^4 = 4 \times \log .0751 = 4 \times \bar{2}.87564$. Multiplying the parts separately, $4 \times \bar{2} = \bar{8}$ and $4 \times .87564 = 3.50256$. Adding the 3 and 8, $3 + (-8) = -5$; therefore, $\log .0751^4 = \bar{5}.50256$. Number corresponding to this = .00003181. Hence, $.0751^4 = .00003181$.

A decimal may be raised to a power whose exponent contains a decimal as follows:

EXAMPLE.—Raise .8 to the 1.21 power.

SOLUTION.— $\text{Log } .8^{1.21} = 1.21 \times \bar{1}.90309$. There are several ways of performing the multiplication.

First Method.—Adding the characteristic and mantissa algebraically, the result is $-.09691$. Multiplying this by 1.21 gives $-.1172611$, or $-.11726$, when using five-place logarithms. To obtain a positive mantissa, add +1 and -1; whence, $\log .8^{1.21} = -1 + 1 - .11726 = \bar{1}.88274$.

Second Method.—Multiplying the characteristic and mantissa separately gives $-1.21 + 1.09274$. Adding characteristic and mantissa algebraically, gives $-.11726$; then, adding $+1$ and -1 , $\log .8^{1.21} = \bar{1}.88274$.

Third Method.—Multiplying the characteristic and mantissa separately gives $-1.21 + 1.09274$. Adding the decimal part of the characteristic to the mantissa gives $-1 + (-.21 + 1.09274) = \bar{1}.88274 = \log .8^{1.21}$. The number corresponding to the logarithm $\bar{1}.88274 = .76338$.

Any one of the above three methods may be used, but we recommend the first or the third. The third is the most elegant and saves figures, but requires the exercise of more caution than the first method does. Below will be found the entire work of multiplication for both $.8^{1.21}$ and $.8^{.21}$.

$\bar{1}.90309$	$\bar{1}.90309$
<u>1.21</u>	<u>.21</u>
90309	90309
180618	180618
<u>90309</u>	<u>+1.1896489</u>
1.0927389	<u>-1 - .21</u>
<u>-1.21</u>	$\bar{1}.9796489$, or $\bar{1}.97965$.
$\bar{1}.8827389$, or $\bar{1}.88274$.	

In the second case, the negative decimal obtained by multiplying -1 and $.21$ was greater than the positive decimal obtained by multiplying $.90309$ and $.21$; hence, $+1$ and -1 were added, as shown.

EVOLUTION BY LOGARITHMS.

If X represents a number whose logarithm is x , we have, from the definition of a logarithm,

$$10^x = X.$$

Extracting some root of both members, as the n th, the equation becomes

$$10^{\frac{x}{n}} = \sqrt[n]{X}.$$

But $\sqrt[n]{X}$ is the required root of X , and $\frac{x}{n}$ is its logarithm, from which it follows that the logarithm of a number divided

by the index of the root to be extracted is equal to the logarithm of the root. Hence, to extract any root of a number by means of logarithms:

Rule.—*Divide the logarithm of the number by the index of the root; the result will be the logarithm of the root.*

EXAMPLE.—Extract (a) the square root of 77,851; (b) the cube root of 698,970; (c) the 2.4 root of 8,964,300.

SOLUTION.—(a) $\text{Log } 77,851 = 4.89127$; the index of the root is 2; hence, $\log \sqrt{77,851} = 4.89127 \div 2 = 2.44564$; number corresponding to this = 279.02. Hence, $\sqrt{77,851} = 279.02$, nearly.

(b) $\text{Log } \sqrt[3]{698,970} = 5.84446 \div 3 = 1.94815 = \log 88.746$; or, $\sqrt[3]{698,970} = 88.747$, nearly.

(c) $\text{Log } \sqrt[2.4]{8,964,300} = 6.95251 \div 2.4 = 2.89688 = \log 788.64$; or, $\sqrt[2.4]{8,964,300} = 788.64$, nearly.

If it is required to extract a root of a number wholly decimal, and the negative characteristic will not exactly contain the index of the root, without a remainder, proceed as follows:

Separate the two parts of the logarithm; add as many units (or parts of a unit) to the negative characteristic as will make it exactly contain the index of the root. Add the same number to the mantissa, and divide both parts by the index. The result will be the characteristic and mantissa of the root.

EXAMPLE.—Extract the cube root of .0003181.

SOLUTION.— $\text{Log } \sqrt[3]{.0003181} = \frac{\log .0003181}{3} = \frac{4.50256}{3}$.

$$(\bar{4} + \bar{2} = \bar{6}) + (2 + .50256 = 2.50256).$$

$$(\bar{6} + 3 = \bar{2}) + (2.50256 \div 3 = .83419);$$

or, $\log \sqrt[3]{.0003181} = \bar{2}.83419 = \log .068263.$

Hence, $\sqrt[3]{.0003181} = .068263.$

EXAMPLE.—Find the value of $\sqrt[1.41]{.0003181}$.

SOLUTION.— $\text{Log } \sqrt[1.41]{.0003181} = \frac{\log .0003181}{1.41} = \frac{4.50256}{1.41}$.

If $-.23$ be added to the characteristic, it will contain 1.41 exactly 3 times. Hence,

$$[-4 + (-.23) = -4.23] + (.23 + .50256 = .73256).$$

$$(-4.23 + 1.41 = \bar{3}) + (.73256 + 1.41 = .51965);$$

or, $\log \sqrt[1.41]{.0003181} = \bar{3}.51965 = \log .0033079.$

Hence, $\sqrt[1.41]{.0003181} = .0033079.$

EXAMPLE.—Solve this expression by logarithms:

$$\frac{497 \times .0181 \times 762}{3,300 \times .6517} = ?$$

SOLUTION.—

Log	497	=	2.69636
Log	.0181	=	2.25768
Log	762	=	2.88195
Log product		=	3.83599
Log	3,300	=	3.51851
Log	.6517	=	1.81405
Log product		=	3.33256

$$3.83599 - 3.33256 = .50343 = \log 3.1874.$$

Hence, $\frac{497 \times .0181 \times 762}{3,300 \times .6517} = 3.1874.$

EXAMPLE.—Solve $\sqrt[3]{\frac{504,203 \times 507}{1.75 \times 71.4 \times 87}}$ by logarithms.

SOLUTION.—

Log	504,203	=	5.70260
Log	507	=	2.70501
Log product		=	8.40761
Log	1.75	=	.24304
Log	71.4	=	1.85370
Log	87	=	1.93952
Log product		=	4.03626

$$\frac{8.40761 - 4.03626}{3} = 1.45712 = \log 28.65.$$

Hence, $\sqrt[3]{\frac{504,203 \times 507}{1.75 \times 71.4 \times 87}} = 28.65.$

Logarithms can often be applied to the solution of equations.

EXAMPLE.—Solve the equation $2.43x^5 = \sqrt[5]{.0648}.$

SOLUTION.— $2.43x^5 = \sqrt[5]{.0648}.$

Dividing by 2.43, $x^5 = \frac{\sqrt[5]{.0648}}{2.43}.$

Taking the logarithm of both numbers,

$$5 \times \log x = \frac{\log .0648}{5} - \log 2.43;$$

$$\begin{aligned} \text{or} \quad 5 \log x &= \frac{2.81158}{6} - .38561 \\ &= 1.80193 - .38561 \\ &= 1.41632. \end{aligned}$$

Dividing by 5, $\log x = 1.88326$;

whence, $x = .7643$.

EXAMPLE.—Solve the equation $4.5^x = 8$.

SOLUTION.—Taking the logarithms of both numbers,

$$x \log 4.5 = \log 8,$$

$$\text{whence,} \quad x = \frac{\log 8}{\log 4.5} = \frac{.90309}{.65321}.$$

Taking logarithms again,

$$\begin{aligned} \log x &= \log .90309 - \log .65321 = 1.95573 - 1.81505 \\ &= .14068; \text{ whence, } x = 1.3825. \end{aligned}$$

REMARK.—Logarithms are particularly useful in those cases when the unknown quantity is an exponent, as in the last example, or when the exponent contains a decimal, as in several instances in the examples given on pages 45–49. Such examples can be solved without the use of logarithms, but the process is very long and somewhat involved, and the arithmetical work required is enormous. To solve the example last given without using the logarithmic table and obtain the value of x correct to five figures would require, perhaps, 100 times as many figures as were used in the solution given, and the resulting liability to error would be correspondingly increased; indeed, to confine the work to this number of figures would also require a good knowledge of short-cut methods in multiplication and division, and judgment and skill on the part of the calculator that can only be acquired by practice and experience.

Formulas containing quantities affected with decimal exponents are generally of an empirical nature; that is, the constants or exponents or both are given such values as will make the results obtained by the formulas agree with those obtained by experiment. Such formulas occur frequently in works treating on thermodynamics, strength of materials, machine design, etc.

COMMON LOGARITHMS.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.			
100	00 000	043	087	130	173	217	260	303	346	389				
101		432	475	518	561	604	647	689	732	775	1	4.4	4.3	4.2
102		860	903	945	988	*030	*072	*115	*157	*199	2	8.8	8.6	8.4
103	01 284	326	368	410	452	494	536	578	620	662	3	13.2	12.9	12.6
104	703	745	787	828	870	912	953	995	*036	*078	4	17.6	17.2	16.8
105	02 119	160	202	243	284	325	366	407	449	490	5	22.0	21.5	21.0
106		531	572	612	653	694	735	776	816	857	6	26.4	25.8	25.2
107		938	979	*019	*060	*100	*141	*181	*222	*262	7	30.8	30.1	29.4
108	03 342	383	423	463	503	543	583	623	663	703	8	35.2	34.4	33.6
109		743	782	822	862	902	941	981	*021	*060	9	39.6	38.7	37.8
110	04 139	179	218	258	297	336	376	415	454	493		41	40	39
111		532	571	610	650	689	727	766	805	844	1	4.1	4.0	3.9
112		922	961	999	*038	*077	*115	*154	*192	*231	2	8.2	8.0	7.8
113	05 308	346	385	423	461	500	538	576	614	652	3	12.3	12.0	11.7
114	690	729	767	805	843	881	918	956	994	*032	4	16.4	16.0	15.6
115	06 070	108	145	183	221	258	296	333	371	408	5	20.5	20.0	19.5
116		446	483	521	558	595	633	670	707	744	6	24.6	24.0	23.4
117		819	856	893	930	967	*004	*041	*078	*115	7	28.7	28.0	27.3
118	07 188	225	262	298	335	372	408	445	482	518	8	32.8	32.0	31.2
119		555	591	628	664	700	737	773	809	846	9	36.9	36.0	35.1
120		918	954	990	*027	*063	*099	*135	*171	*207		38	37	36
121	08 279	314	350	386	422	458	493	529	565	600	1	3.8	3.7	3.6
122		636	672	707	743	778	814	849	884	920	2	7.6	7.4	7.2
123		991	*026	*061	*096	*132	*167	*202	*237	*272	3	11.4	11.1	10.8
124	09 342	377	412	447	482	517	552	587	621	656	4	15.2	14.8	14.4
125		691	726	760	795	830	864	899	934	968	5	19.0	18.5	18.0
126	10 037	072	106	140	175	209	243	278	312	346	6	22.8	22.2	21.6
127		380	415	449	483	517	551	585	619	653	7	26.6	25.9	25.2
128		721	755	789	823	857	890	924	958	992	8	30.4	29.6	28.8
129	11 059	093	126	160	193	227	261	294	327	361	9	34.2	33.3	32.4
130		394	428	461	494	528	561	594	628	661		35	34	33
131		727	760	793	826	860	893	926	959	992	1	3.5	3.4	3.3
132	12 057	090	123	156	189	222	254	287	320	352	2	7.0	6.8	6.6
133		385	418	450	483	516	548	581	613	646	3	10.5	10.2	9.9
134		710	743	775	808	840	872	905	937	969	4	14.0	13.6	13.2
135	13 033	066	098	130	162	194	226	258	290	322	5	17.5	17.0	16.5
136		354	386	418	450	481	513	545	577	609	6	21.0	20.4	19.8
137		672	704	735	767	799	830	862	893	925	7	24.5	23.8	23.1
138		988	*019	*051	*082	*114	*145	*176	*208	*239	8	28.0	27.2	26.4
139	14 301	333	364	395	426	457	489	520	551	582	9	31.5	30.6	29.7
140		613	644	675	706	737	768	799	829	860		32	31	30
141		922	953	983	*014	*045	*076	*106	*137	*168	1	3.2	3.1	3.0
142	15 229	259	290	320	351	381	412	442	473	503	2	6.4	6.2	6.0
143		534	564	594	625	655	685	715	746	776	3	9.6	9.3	9.0
144		836	866	897	927	957	987	*017	*047	*077	4	12.8	12.4	12.0
145	16 137	167	197	227	256	286	316	346	376	406	5	16.0	15.5	15.0
146		435	465	495	524	554	584	613	643	673	6	19.2	18.6	18.0
147		732	761	791	820	850	879	909	938	967	7	22.4	21.7	21.0
148	17 026	056	085	114	143	173	202	231	260	289	8	25.6	24.8	24.0
149		319	348	377	406	435	464	493	522	551	9	28.8	27.9	27.0
150		609	638	667	696	725	754	782	811	840				
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.			

LOGARITHMS.

51

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.		
150	17 609	638	667	696	725	754	782	811	840	869			
151	898	926	955	984	*013	*041	*070	*099	*127	*156			
152	18 184	213	241	270	298	327	355	384	412	441	1	2.9	2.8
153	469	498	526	554	583	611	639	667	696	724	2	5.8	5.6
154	752	780	808	837	865	893	921	949	977	*005	3	8.7	8.4
155	19 033	061	089	117	145	173	201	229	257	285	4	11.6	11.2
156	312	340	368	396	424	451	479	507	535	562	5	14.5	14.0
157	590	618	645	673	700	728	756	783	811	838	6	17.4	16.8
158	866	893	921	948	976	*003	*030	*058	*085	*112	7	20.3	19.6
159	20 140	167	194	222	249	276	303	330	358	385	8	23.2	22.4
160	412	439	466	493	520	548	575	602	629	656	9	26.1	25.2
161	683	710	737	763	790	817	844	871	898	925			
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192	1	2.7	2.6
163	21 219	245	272	299	325	352	378	405	431	458	2	5.4	5.2
164	484	511	537	564	590	617	643	669	696	722	3	8.1	7.8
165	748	775	801	827	854	880	906	932	958	985	4	10.8	10.4
166	22 011	037	063	089	115	141	167	194	220	246	5	13.5	13.0
167	272	298	324	350	376	401	427	453	479	505	6	16.2	15.6
168	531	557	583	608	634	660	686	712	737	763	7	18.9	18.2
169	789	814	840	866	891	917	943	968	994	*019	8	21.6	20.8
170	23 045	070	096	121	147	172	198	223	249	274	9	24.3	23.4
171	300	325	350	376	401	426	452	477	502	528			
172	553	578	603	629	654	679	704	729	754	779	1	2.5	
173	805	830	855	880	905	930	955	980	*005	*030	2	5.0	
174	24 055	080	105	130	155	180	204	229	254	279	4	7.5	
175	304	329	353	378	403	428	452	477	502	527	4	10.0	
176	551	576	601	625	650	674	699	724	748	773	5	12.5	
177	797	822	846	871	895	920	944	969	993	*018	6	15.0	
178	25 042	066	091	115	139	164	188	212	237	261	7	17.5	
179	285	310	334	358	382	406	431	455	479	503	8	20.0	
180	527	551	575	600	624	648	672	696	720	744	9	22.5	
181	768	792	816	840	864	888	912	935	959	983			
182	26 007	031	055	079	102	126	150	174	198	221	1	2.4	2.3
183	245	269	293	316	340	364	387	411	435	458	2	4.8	4.6
184	482	505	529	553	576	600	623	647	670	694	3	7.2	6.9
185	717	741	764	788	811	834	858	881	905	928	4	9.6	9.2
186	951	975	998	*021	*045	*068	*091	*114	*138	*161	5	12.0	11.5
187	27 184	207	231	254	277	300	323	346	370	393	6	14.4	13.8
188	416	439	462	485	508	531	554	577	600	623	7	16.8	16.1
189	646	669	692	715	738	761	784	807	830	852	8	19.2	18.4
190	875	898	921	944	967	989	*012	*035	*058	*081	9	21.6	20.7
191	28 103	126	149	171	194	217	240	262	285	307			
192	330	353	375	398	421	443	466	488	511	533	1	2.2	2.1
193	556	578	601	623	646	668	691	713	735	758	2	4.4	4.2
194	780	803	825	847	870	892	914	937	959	981	3	6.6	6.3
195	29 003	026	048	070	092	115	137	159	181	203	4	8.8	8.4
196	226	248	270	292	314	336	358	380	403	425	5	11.0	10.5
197	447	469	491	513	535	557	579	601	623	645	6	13.2	12.6
198	667	688	710	732	754	776	798	820	842	863	7	15.4	14.7
199	885	907	929	951	973	994	*016	*038	*060	*081	8	17.6	16.8
200	30 103	125	146	168	190	211	233	255	276	298	9	19.8	18.9
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.		

USEFUL TABLES.

TABLE—(Continued).

N.	L. O	1	2	3	4	5	6	7	8	9	P. P.		
200	30 103	125	146	168	190	211	233	255	276	298			
201	320	341	363	384	406	428	449	471	492	514			
202	535	557	578	600	621	643	664	685	707	728	1	2.2	2.1
203	750	771	792	814	835	856	878	899	920	942	2	4.4	4.2
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154	3	6.6	6.3
205	31 175	197	218	239	260	281	302	323	345	366	4	8.8	8.4
206	387	408	429	450	471	492	513	534	555	576	5	11.0	10.5
207	597	618	639	660	681	702	723	744	765	785	6	13.2	12.6
208	806	827	848	869	890	911	931	952	973	994	7	15.4	14.7
209	32 015	035	056	077	098	118	139	160	181	201	8	17.6	16.8
210	222	243	263	284	305	325	346	366	387	408	9	19.8	18.9
211	428	449	469	490	510	531	552	572	593	613			20
212	634	654	675	695	715	736	756	777	797	818	1	2.0	
213	838	858	879	899	919	940	960	980	*001	*021	2	4.0	
214	33 041	062	082	102	122	143	163	183	203	224	3	6.0	
215	244	264	284	304	325	345	365	385	405	425	4	8.0	
216	445	465	486	506	526	546	566	586	606	626	5	10.0	
217	646	666	686	706	726	746	766	786	806	826	6	12.0	
218	846	866	885	905	925	945	965	985	*005	*025	7	14.0	
219	34 044	064	084	104	124	143	163	183	203	223	8	16.0	
220	242	262	282	301	321	341	361	380	400	420	9	18.0	
221	439	459	479	498	518	537	557	577	596	616			19
222	635	655	674	694	713	733	753	772	792	811	1	1.9	
223	830	850	869	889	908	928	947	967	986	*005	2	3.8	
224	35 025	044	064	083	102	122	141	160	180	199	3	5.7	
225	218	238	257	276	295	315	334	353	372	392	4	7.6	
226	411	430	449	468	488	507	526	545	564	583	5	9.5	
227	603	622	641	660	679	698	717	736	755	774	6	11.4	
228	793	813	832	851	870	889	908	927	946	965	7	13.3	
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154	8	15.2	
230	36 178	192	211	229	248	267	286	305	324	342	9	17.1	
231	361	380	399	418	436	455	474	493	511	530			18
232	549	568	586	605	624	642	661	680	698	717	1	1.8	
233	736	754	773	791	810	829	847	866	884	903	2	3.6	
234	922	940	959	977	996	*014	*033	*051	*070	*088	3	5.4	
235	37 107	125	144	162	181	199	218	236	254	273	4	7.2	
236	291	310	328	346	365	383	401	420	438	457	5	9.0	
237	475	493	511	530	548	566	585	603	621	639	6	10.8	
238	658	676	694	712	731	749	767	785	803	822	7	12.6	
239	840	858	876	894	912	931	949	967	985	*003	8	14.4	
240	38 021	039	057	075	093	112	130	148	166	184	9	16.2	
241	202	220	238	256	274	292	310	328	346	364			17
242	382	399	417	435	453	471	489	507	525	543	1	1.7	
243	561	578	596	614	632	650	668	686	703	721	2	3.4	
244	739	757	775	792	810	828	846	863	881	899	3	5.1	
245	917	934	952	970	987	*005	*023	*041	*058	*076	4	6.8	
246	39 094	111	129	146	164	182	199	217	235	252	5	8.5	
247	270	287	305	322	340	358	375	393	410	428	6	10.2	
248	445	463	480	498	515	533	550	568	585	602	7	11.9	
249	620	637	655	672	690	707	724	742	759	777	8	13.6	
250	794	811	829	846	863	881	898	915	933	950	9	15.3	
N.	L. O	1	2	3	4	5	6	7	8	9	P. P.		

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
250	39 794	811	829	846	863	881	898	915	933	950	
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	18
252	40 140	157	175	192	209	226	243	261	278	295	1 1.8
253	312	329	346	364	381	398	415	432	449	466	2 3.6
254	483	500	518	535	552	569	586	603	620	637	3 5.4
255	654	671	688	705	722	739	756	773	790	807	4 7.2
256	824	841	858	875	892	909	926	943	960	976	5 9.0
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	6 10.8
258	41 162	179	196	212	229	246	263	280	296	313	7 12.6
259	330	347	363	380	397	414	430	447	464	481	8 14.4
260	497	514	531	547	564	581	597	614	631	647	9 16.2
261	664	681	697	714	731	747	764	780	797	814	17
262	830	847	863	880	896	913	929	946	963	979	1 1.7
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	2 3.4
264	42 160	177	193	210	226	243	259	275	292	308	3 5.1
265	325	341	357	374	390	406	423	439	455	472	4 6.8
266	488	504	521	537	553	570	586	602	619	635	5 8.5
267	651	667	684	700	716	732	749	765	781	797	6 10.2
268	813	830	846	862	878	894	911	927	943	959	7 11.9
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	8 13.6
270	43 136	152	169	185	201	217	233	249	265	281	9 15.3
271	297	313	329	345	361	377	393	409	425	441	16
272	457	473	489	505	521	537	553	569	584	600	1 1.6
273	616	632	648	664	680	696	712	727	743	759	2 3.2
274	775	791	807	823	838	854	870	886	902	917	3 4.8
275	933	949	965	981	996	*012	*028	*044	*059	*075	4 6.4
276	44 091	107	122	138	154	170	185	201	217	232	5 8.0
277	248	264	279	295	311	326	342	358	373	389	6 9.6
278	404	420	436	451	467	483	498	514	529	545	7 11.2
279	560	576	592	607	623	638	654	669	685	700	8 12.8
280	716	731	747	762	778	793	809	824	840	855	9 14.4
281	871	886	902	917	932	948	963	979	994	*010	15
282	45 025	040	056	071	086	102	117	133	148	163	1 1.5
283	179	194	209	225	240	255	271	286	301	317	2 3.0
284	332	347	362	378	393	408	423	439	454	469	3 4.5
285	484	500	515	530	545	561	576	591	606	621	4 6.0
286	637	652	667	682	697	712	728	743	758	773	5 7.5
287	788	803	818	834	849	864	879	894	909	924	6 9.0
288	939	954	969	984	*000	*015	*030	*045	*060	*075	7 10.5
289	46 090	105	120	135	150	165	180	195	210	225	8 12.0
290	240	255	270	285	300	315	330	345	359	374	9 13.5
291	389	404	419	434	449	464	479	494	509	523	14
292	538	553	568	583	598	613	627	642	657	672	1 1.4
293	687	702	716	731	746	761	776	790	805	820	2 2.8
294	835	850	864	879	894	909	923	938	953	967	3 4.2
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	4 5.6
296	47 129	144	159	173	188	202	217	232	246	261	5 7.0
297	276	290	305	319	334	349	363	378	392	407	6 8.4
298	422	436	451	465	480	494	509	524	538	553	7 9.8
299	567	582	596	611	625	640	654	669	683	698	8 11.2
300	712	727	741	756	770	784	799	813	828	842	9 12.6
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	15
303	144	159	173	187	202	216	230	244	259	273	1
304	287	302	316	330	344	359	373	387	401	416	1.5
305	430	444	458	473	487	501	515	530	544	558	2
306	572	586	601	615	629	643	657	671	686	700	3
307	714	728	742	756	770	785	799	813	827	841	4
308	855	869	883	897	911	926	940	954	968	982	5
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	6
310	49 136	150	164	178	192	206	220	234	248	262	7
311	276	290	304	318	332	346	360	374	388	402	8
312	415	429	443	457	471	485	499	513	527	541	9
313	554	568	582	596	610	624	638	651	665	679	14
314	693	707	721	734	748	762	776	790	803	817	1
315	831	845	859	872	886	900	914	927	941	955	1.4
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	2
317	50 106	120	133	147	161	174	188	202	215	229	2.8
318	243	256	270	284	297	311	325	338	352	365	3
319	379	393	406	420	433	447	461	474	488	501	4
320	515	529	542	556	569	583	596	610	623	637	5
321	651	664	678	691	705	718	732	745	759	772	6
322	786	799	813	826	840	853	866	880	893	907	7
323	920	934	947	961	974	987	*001	*014	*028	*041	8
324	51 055	068	081	095	108	121	135	148	162	175	9
325	188	202	215	228	242	255	268	282	295	308	13
326	322	335	348	362	375	388	402	415	428	441	1
327	455	468	481	495	508	521	534	548	561	574	2
328	587	601	614	627	640	654	667	680	693	706	3
329	720	733	746	759	772	786	799	812	825	838	4
330	851	865	878	891	904	917	930	943	957	970	5
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	6
332	52 114	127	140	153	166	179	192	205	218	231	7
333	244	257	270	284	297	310	323	336	349	362	8
334	375	388	401	414	427	440	453	466	479	492	9
335	504	517	530	543	556	569	582	595	608	621	10.4
336	634	647	660	673	686	699	711	724	737	750	11.7
337	763	776	789	802	815	827	840	853	866	879	
338	892	905	917	930	943	956	969	982	994	*007	
339	53 020	033	046	058	071	084	097	110	122	135	12
340	148	161	173	186	199	212	224	237	250	263	1
341	275	288	301	314	326	339	352	364	377	390	2
342	403	415	428	441	453	466	479	491	504	517	3
343	529	542	555	567	580	593	605	618	631	643	4
344	656	668	681	694	706	719	732	744	757	769	5
345	782	794	807	820	832	845	857	870	882	895	6
346	908	920	933	945	958	970	983	995	*008	*020	7
347	54 033	045	058	070	083	095	108	120	133	145	8
348	158	170	183	195	208	220	233	245	258	270	9
349	283	295	307	320	332	345	357	370	382	394	10.8
350	407	419	432	444	456	469	481	494	506	518	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
350	54 407	419	432	444	456	469	481	494	506	518	
351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	
354	900	913	925	937	949	962	974	986	998	*011	
355	55 023	035	047	060	072	084	096	108	121	133	
356	145	157	169	182	194	206	218	230	242	255	
357	267	279	291	303	315	328	340	352	364	376	
358	388	400	413	425	437	449	461	473	485	497	
359	509	522	534	546	558	570	582	594	606	618	
360	630	642	654	666	678	691	703	715	727	739	
361	751	763	775	787	799	811	823	835	847	859	
362	871	883	895	907	919	931	943	955	967	979	
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	
366	348	360	372	384	396	407	419	431	443	455	
367	467	478	490	502	514	526	538	549	561	573	
368	585	597	608	620	632	644	656	667	679	691	
369	703	714	726	738	750	761	773	785	797	808	
370	820	832	844	855	867	879	891	902	914	926	
371	937	949	961	972	984	996	*008	*019	*031	*043	
372	57 054	066	078	089	101	113	124	136	148	159	
373	171	183	194	206	217	229	241	252	264	276	
374	287	299	310	322	334	345	357	368	380	392	
375	403	415	426	438	449	461	473	484	496	507	
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	
378	749	761	772	784	795	807	818	830	841	852	
379	864	875	887	898	910	921	933	944	955	967	
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	
381	58 092	104	115	127	138	149	161	172	184	195	
382	206	218	229	240	252	263	274	286	297	309	
383	320	331	343	354	365	377	388	399	410	422	
384	433	444	456	467	478	490	501	512	524	535	
385	546	557	569	580	591	602	614	625	636	647	
386	659	670	681	692	704	715	726	737	749	760	
387	771	782	794	805	816	827	838	850	861	872	
388	883	894	906	917	928	939	950	961	973	984	
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	
391	218	229	240	251	262	273	284	295	306	318	
392	329	340	351	362	373	384	395	406	417	428	
393	439	450	461	472	483	494	506	517	528	539	
394	550	561	572	583	594	605	616	627	638	649	
395	660	671	682	693	704	715	726	737	748	759	
396	770	780	791	802	813	824	835	846	857	868	
397	879	890	901	912	923	934	945	956	966	977	
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	
399	60 097	108	119	130	141	152	163	173	184	195	
400	206	217	228	239	249	260	271	282	293	304	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLE—(Continued).

N.	L.O	1	2	3	4	5	6	7	8	9	P.P.
400	60 206	217	228	239	249	260	271	282	295	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	
407	959	970	981	991	*002	*013	*023	*034	*045	*055	
408	61 066	077	087	098	109	119	130	140	151	162	
409	172	183	194	204	215	225	236	247	257	268	
410	278	289	300	310	321	331	342	352	363	374	
411	384	395	405	416	426	437	448	458	469	479	
412	490	500	511	521	532	542	553	563	574	584	
413	595	606	616	627	637	648	658	669	679	690	
414	700	711	721	731	742	752	763	773	784	794	
415	805	815	826	836	847	857	868	878	888	899	
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	
422	531	542	552	562	572	583	593	603	613	624	
423	634	644	655	665	675	685	696	706	716	726	
424	737	747	757	767	778	788	798	808	818	829	
425	839	849	859	870	880	890	900	910	921	931	
426	941	951	961	972	982	992	*002	*012	*022	*033	
427	63 043	053	063	073	083	094	104	114	124	134	
428	144	155	165	175	185	195	205	215	225	236	
429	246	256	266	276	286	296	306	317	327	337	
430	347	357	367	377	387	397	407	417	428	438	
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	988	998	*008	*018	*028	*038	
437	64 048	058	068	078	088	098	108	118	128	137	
438	147	157	167	177	187	197	207	217	227	237	
439	246	256	266	276	286	296	306	316	326	335	
440	345	355	365	375	385	395	404	414	424	434	
441	444	454	464	473	483	493	503	513	523	532	
442	542	552	562	572	582	591	601	611	621	631	
443	640	650	660	670	680	689	699	709	719	729	
444	738	748	758	768	777	787	797	807	816	826	
445	836	846	856	865	875	885	895	904	914	924	
446	933	943	953	963	972	982	992	*002	*011	*021	
447	65 031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	L.O	1	2	3	4	5	6	7	8	9	P.P.

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	10
458	66 087	096	106	115	124	134	143	153	162	172	1 1.0
459	181	191	200	210	219	229	238	247	257	266	2 2.0
460	276	285	295	304	314	323	332	342	351	361	3 3.0
461	370	380	389	398	408	417	427	436	445	455	4 4.0
462	464	474	483	492	502	511	521	530	539	549	5 5.0
463	558	567	577	586	596	605	614	624	633	642	6 6.0
464	652	661	671	680	689	699	708	717	727	736	7 7.0
465	745	755	764	773	783	792	801	811	820	829	8 8.0
466	839	848	857	867	876	885	894	904	913	922	9 9.0
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	9
472	394	403	413	422	431	440	449	459	468	477	1 0.9
473	486	495	504	514	523	532	541	550	560	569	2 1.8
474	578	587	596	605	614	624	633	642	651	660	3 2.7
475	669	679	688	697	706	715	724	733	742	752	4 3.6
476	761	770	779	788	797	806	815	825	834	843	5 4.5
477	852	861	870	879	888	897	906	916	925	934	6 5.4
478	943	952	961	970	979	988	997	*006	*015	*024	7 6.3
479	68 034	043	052	061	070	079	088	097	106	115	8 7.2
480	124	133	142	151	160	169	178	187	196	205	9 8.1
481	215	224	233	242	251	260	269	278	287	296	
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	
488	842	851	860	869	878	886	895	904	913	922	8
489	931	940	949	958	966	975	984	993	*002	*011	1 0.8
490	69 020	028	037	046	055	064	073	082	090	099	2 1.6
491	108	117	126	135	144	152	161	170	179	188	3 2.4
492	197	205	214	223	232	241	249	258	267	276	4 3.2
493	285	294	302	311	320	329	338	346	355	364	5 4.0
494	373	381	390	399	408	417	425	434	443	452	6 4.8
495	461	469	478	487	496	504	513	522	531	539	7 5.6
496	548	557	566	574	583	592	601	609	618	627	8 6.4
497	636	644	653	662	671	679	688	697	705	714	9 7.2
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLE—(Continued).

N.	L. O	1	2	3	4	5	6	7	8	9	P. P.
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	
508	586	595	603	612	621	629	638	646	655	663	
509	672	680	689	697	706	714	723	731	740	749	
510	757	766	774	783	791	800	808	817	825	834	
511	842	851	859	868	876	885	893	902	910	919	
512	927	935	944	952	961	969	978	986	995	*003	
513	71 012	020	029	037	046	054	063	071	079	088	
514	096	105	113	122	130	139	147	155	164	172	
515	181	189	198	206	214	223	231	240	248	257	
516	265	273	282	290	299	307	315	324	332	341	
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	
522	767	775	784	792	800	809	817	825	834	842	
523	850	858	867	875	883	892	900	908	917	925	
524	933	941	950	958	966	975	983	991	999	*008	
525	72 016	024	032	041	049	057	066	074	082	090	
526	099	107	115	123	132	140	148	156	165	173	
527	181	189	198	206	214	222	230	239	247	255	
528	263	272	280	288	296	304	313	321	329	337	
529	346	354	362	370	378	387	395	403	411	419	
530	428	436	444	452	460	469	477	485	493	501	
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73 078	086	094	102	111	119	127	135	143	151	
539	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	272	280	288	296	304	312	
541	320	328	336	344	352	360	368	376	384	392	
542	400	408	416	424	432	440	448	456	464	472	
543	480	488	496	504	512	520	528	536	544	552	
544	560	568	576	584	592	600	608	616	624	632	
545	640	648	656	664	672	679	687	695	703	711	
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	L. O	1	2	3	4	5	6	7	8	9	P. P.

9

1 0.9

2 1.8

3 2.7

4 3.6

5 4.5

6 5.4

7 6.3

8 7.2

9 8.1

8

1 0.8

2 1.6

3 2.4

4 3.2

5 4.0

6 4.8

7 5.6

8 6.4

9 7.2

7

1 0.7

2 1.4

3 2.1

4 2.8

5 3.5

6 4.2

7 4.9

8 5.6

9 6.3

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
550	74 086	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	8
562	974	981	989	997	*005	*012	*020	*028	*035	*043	1 0.8
563	75 051	059	066	074	082	089	097	105	113	120	2 1.6
564	128	136	143	151	159	166	174	182	189	197	3 2.4
565	205	213	220	228	236	243	251	259	266	274	4 3.2
566	282	289	297	305	312	320	328	335	343	351	5 4.0
567	358	366	374	381	389	397	404	412	420	427	6 4.8
568	435	442	450	458	465	473	481	488	496	504	7 5.6
569	511	519	526	534	542	549	557	565	572	580	8 6.4
570	587	595	603	610	618	626	633	641	648	656	9 7.2
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	1 0.7
583	567	574	582	589	597	604	612	619	626	634	2 1.4
584	641	649	656	664	671	678	686	693	701	708	3 2.1
585	716	723	730	738	745	753	760	768	775	782	4 2.8
586	790	797	805	812	819	827	834	842	849	856	5 3.5
587	864	871	879	886	893	901	908	916	923	930	6 4.2
588	938	945	953	960	967	975	982	989	997	*004	7 4.9
589	77 012	019	026	034	041	048	056	063	070	078	8 5.6
590	085	093	100	107	115	122	129	137	144	151	9 6.3
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

USEFUL TABLES.

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	
607	319	326	333	340	347	355	362	369	376	383	
608	390	398	405	412	419	426	433	440	447	455	
609	462	469	476	483	490	497	504	512	519	526	
610	533	540	547	554	561	569	576	583	590	597	
611	604	611	618	625	633	640	647	654	661	668	
612	675	682	689	696	704	711	718	725	732	739	
613	746	753	760	767	774	781	789	796	803	810	
614	817	824	831	838	845	852	859	866	873	880	
615	888	895	902	909	916	923	930	937	944	951	
616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	
622	379	386	393	400	407	414	421	428	435	442	
623	449	456	463	470	477	484	491	498	505	511	
624	518	525	532	539	546	553	560	567	574	581	
625	588	595	602	609	616	623	630	637	644	650	
626	657	664	671	678	685	692	699	706	713	720	
627	727	734	741	748	754	761	768	775	782	789	
628	796	803	810	817	824	831	837	844	851	858	
629	865	872	879	885	893	900	906	913	920	927	
630	934	941	948	955	962	969	975	982	989	996	
631	80 003	010	017	024	030	037	044	051	058	065	
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	
637	414	421	428	434	441	448	455	462	468	475	
638	482	489	496	502	509	516	523	530	536	543	
639	550	557	564	570	577	584	591	598	604	611	
640	618	625	632	638	645	652	659	665	672	679	
641	686	693	699	706	713	720	726	733	740	747	
642	754	760	767	774	781	787	794	801	808	814	
643	821	828	835	841	848	855	862	868	875	882	
644	889	895	902	909	916	922	929	936	943	949	
645	956	963	969	976	983	990	996	*003	*010	*017	
646	81 023	030	037	043	050	057	064	070	077	084	
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

8

1 0.8
2 1.6
3 2.4
4 3.2
5 4.0
6 4.8
7 5.6
8 6.4
9 7.2

7

1 0.7
2 1.4
3 2.1
4 2.8
5 3.5
6 4.2
7 4.9
8 5.6
9 6.8

6

1 0.6
2 1.2
3 1.8
4 2.4
5 3.0
6 3.6
7 4.2
8 4.8
9 5.4

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
661	82 020	027	033	040	046	053	060	066	073	079	7
662	086	092	099	105	112	119	125	132	138	145	1 0.7
663	151	158	164	171	178	184	191	197	204	210	2 1.4
664	217	223	230	236	243	249	256	263	269	276	3 2.1
665	282	289	295	302	308	315	321	328	334	341	4 2.8
666	347	354	360	367	373	380	387	393	400	406	5 3.5
667	413	419	426	432	439	445	452	458	465	471	6 4.2
668	478	484	491	497	504	510	517	523	530	536	7 4.9
669	543	549	556	562	569	575	582	588	595	601	8 5.6
670	607	614	620	627	633	640	646	653	659	666	9 6.3
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	8
682	378	385	391	398	404	410	417	423	429	436	1 0.6
683	442	448	455	461	467	474	480	487	493	499	2 1.2
684	506	512	518	525	531	537	544	550	556	563	3 1.8
685	569	575	582	588	594	601	607	613	620	626	4 2.4
686	632	639	645	651	658	664	670	677	683	689	5 3.0
687	696	702	708	715	721	727	734	740	746	753	6 3.6
688	759	765	771	778	784	790	797	803	809	816	7 4.2
689	822	828	835	841	847	853	860	866	872	879	8 4.8
690	885	891	897	904	910	916	923	929	935	942	9 5.4
691	948	954	960	967	973	979	985	992	998	*004	
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
700	84 510	516	522	528	535	541	547	553	559	566	
701	572	578	584	590	597	603	609	615	621	628	
702	634	640	646	652	658	665	671	677	683	689	
703	696	702	708	714	720	726	733	739	745	751	
704	757	763	770	776	782	788	794	800	807	813	
705	819	825	831	837	844	850	856	862	868	874	
706	880	887	893	899	905	911	917	924	930	936	
707	942	948	954	960	967	973	979	985	991	997	
708	85 003	009	016	022	028	034	040	046	052	058	
709	065	071	077	083	089	095	101	107	114	120	
710	126	132	138	144	150	156	163	169	175	181	
711	187	193	199	205	211	217	224	230	236	242	
712	248	254	260	266	272	278	285	291	297	303	
713	309	315	321	327	333	339	345	352	358	364	
714	370	376	382	388	394	400	406	412	418	425	
715	431	437	443	449	455	461	467	473	479	485	
716	491	497	503	509	516	522	528	534	540	546	
717	552	558	564	570	576	582	588	594	600	606	
718	612	618	625	631	637	643	649	655	661	667	
719	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
721	794	800	806	812	818	824	830	836	842	848	
722	854	860	866	872	878	884	890	896	902	908	
723	914	920	926	932	938	944	950	956	962	968	
724	974	980	986	992	998	*004	*010	*016	*022	*028	
725	86 034	040	046	052	058	064	070	076	082	088	
726	094	100	106	112	118	124	130	136	141	147	
727	153	159	165	171	177	183	189	195	201	207	
728	213	219	225	231	237	243	249	255	261	267	
729	273	279	285	291	297	303	308	314	320	326	
730	332	338	344	350	356	362	368	374	380	386	
731	392	398	404	410	415	421	427	433	439	445	
732	451	457	463	469	475	481	487	493	499	504	
733	510	516	522	528	534	540	546	552	558	564	
734	570	576	581	587	593	599	605	611	617	623	
735	629	635	641	646	652	658	664	670	676	682	
736	688	694	700	705	711	717	723	729	735	741	
737	747	753	759	764	770	776	782	788	794	800	
738	806	812	817	823	829	835	841	847	853	859	
739	864	870	876	882	888	894	900	906	911	917	
740	923	929	935	941	947	953	958	964	970	976	
741	982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87 040	046	052	058	064	070	075	081	087	093	
743	099	105	111	116	122	128	134	140	146	151	
744	157	163	169	175	181	186	192	198	204	210	
745	216	221	227	233	239	245	251	256	262	268	
746	274	280	286	291	297	303	309	315	320	326	
747	332	338	344	349	355	361	367	373	379	384	
748	390	396	402	408	413	419	425	431	437	442	
749	448	454	460	466	471	477	483	489	495	500	
750	506	512	518	523	529	535	541	547	552	558	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
761	138	144	150	156	161	167	173	178	184	190	
762	195	201	207	213	218	224	230	235	241	247	1 0.6
763	252	258	264	270	275	281	287	292	298	304	2 1.2
764	309	315	321	326	332	338	343	349	355	360	3 1.8
765	366	372	377	383	389	395	400	406	412	417	4 2.4
766	423	429	434	440	446	451	457	463	468	474	5 3.0
767	480	485	491	497	502	508	513	519	525	530	6 3.6
768	536	542	547	553	559	564	570	576	581	587	7 4.2
769	593	598	604	610	615	621	627	632	638	643	8 4.8
770	649	655	660	666	672	677	683	689	694	700	9 5.4
771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
781	265	271	276	282	287	293	298	304	310	315	
782	321	326	332	337	343	348	354	360	365	371	1 5
783	376	382	387	393	398	404	409	415	421	426	2 1.0
784	432	437	443	448	454	459	465	470	476	481	3 1.5
785	487	492	498	504	509	515	520	526	531	537	4 2.0
786	542	548	553	559	564	570	575	581	586	592	5 2.5
787	597	603	609	614	620	625	631	636	642	647	6 3.0
788	653	658	664	669	675	680	686	691	697	702	7 3.5
789	708	713	719	724	730	735	741	746	752	757	8 4.0
790	763	768	774	779	785	790	796	801	807	812	9 4.5
791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
799	255	260	266	271	276	282	287	293	298	304	
800	309	314	320	325	331	336	342	347	352	358	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
800	90	309	314	320	325	331	336	342	347	352	358
801		363	369	374	380	385	390	396	401	407	412
802		417	423	428	434	439	445	450	455	461	466
803		472	477	482	488	493	499	504	509	515	520
804		526	531	536	542	547	553	558	563	569	574
805		580	585	590	596	601	607	612	617	623	628
806		634	639	644	650	655	660	666	671	677	682
807		687	693	698	703	709	714	720	725	730	736
808		741	747	752	757	763	768	773	779	784	789
809		795	800	806	811	816	822	827	832	838	843
810		849	854	859	865	870	875	881	886	891	897
811		902	907	913	918	924	929	934	940	945	950
812		956	961	966	972	977	982	988	993	998	*004
813	91	009	014	020	025	030	036	041	046	052	057
814		062	068	073	078	084	089	094	100	105	110
815		116	121	126	132	137	142	148	153	158	164
816		169	174	180	185	190	196	201	206	212	217
817		222	228	233	238	243	249	254	259	265	270
818		275	281	286	291	297	302	307	312	318	323
819		328	334	339	344	350	355	360	365	371	376
820		381	387	392	397	403	408	413	418	424	429
821		434	440	445	450	455	461	466	471	477	482
822		487	492	498	503	508	514	519	524	529	535
823		540	545	551	556	561	566	572	577	582	587
824		593	598	603	609	614	619	624	630	635	640
825		645	651	656	661	666	672	677	682	687	693
826		698	703	709	714	719	724	730	735	740	745
827		751	756	761	766	772	777	782	787	793	798
828		803	808	814	819	824	829	834	840	845	850
829		855	861	866	871	876	882	887	892	897	903
830		908	913	918	924	929	934	939	944	950	955
831		960	965	971	976	981	986	991	997	*002	*007
832	92	012	018	023	028	033	038	044	049	054	059
833		065	070	075	080	085	091	096	101	106	111
834		117	122	127	132	137	143	148	153	158	163
835		169	174	179	184	189	195	200	205	210	215
836		221	226	231	236	241	247	252	257	262	267
837		273	278	283	288	293	298	304	309	314	319
838		324	330	335	340	345	350	355	361	366	371
839		376	381	387	392	397	402	407	412	418	423
840		428	433	438	443	449	454	459	464	469	474
841		480	485	490	495	500	505	511	516	521	526
842		531	536	542	547	552	557	562	567	572	578
843		583	588	593	598	603	609	614	619	624	629
844		634	639	645	650	655	660	665	670	675	681
845		686	691	696	701	706	711	716	722	727	732
846		737	742	747	752	758	763	768	773	778	783
847		788	793	799	804	809	814	819	824	829	834
848		840	845	850	855	860	865	870	875	881	886
849		891	896	901	906	911	916	921	927	932	937
850		942	947	952	957	962	967	973	978	983	988
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

6

1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

5

1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
850	92 942	947	952	957	962	967	973	978	983	988	
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93 044	049	054	059	064	069	075	080	085	090	
853	095	100	105	110	115	120	125	131	136	141	
854	146	151	156	161	166	171	176	181	186	192	
855	197	202	207	212	217	222	227	232	237	242	
856	247	252	258	263	268	273	278	283	288	293	
857	298	303	308	313	318	323	328	334	339	344	
858	349	354	359	364	369	374	379	384	389	394	
859	399	404	409	414	420	425	430	435	440	445	
860	450	455	460	465	470	475	480	485	490	495	
861	500	505	510	515	520	526	531	536	541	546	
862	551	556	561	566	571	576	581	586	591	596	
863	601	606	611	616	621	626	631	636	641	646	
864	651	656	661	666	671	676	682	687	692	697	
865	702	707	712	717	722	727	732	737	742	747	
866	752	757	762	767	772	777	782	787	792	797	
867	802	807	812	817	822	827	832	837	842	847	
868	852	857	862	867	872	877	882	887	892	897	
869	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
871	94 002	007	012	017	022	027	032	037	042	047	
872	052	057	062	067	072	077	082	086	091	096	
873	101	106	111	116	121	126	131	136	141	146	
874	151	156	161	166	171	176	181	186	191	196	
875	201	206	211	216	221	226	231	236	240	245	
876	250	255	260	265	270	275	280	285	290	295	
877	300	305	310	315	320	325	330	335	340	345	
878	349	354	359	364	369	374	379	384	389	394	
879	399	404	409	414	419	424	429	433	438	443	
880	448	453	458	463	468	473	478	483	488	493	
881	498	503	507	512	517	522	527	532	537	542	
882	547	552	557	562	567	571	576	581	586	591	
883	596	601	606	611	616	621	626	630	635	640	
884	645	650	655	660	665	670	675	680	685	689	
885	694	699	704	709	714	719	724	729	734	738	
886	743	748	753	758	763	768	773	778	783	787	
887	792	797	802	807	812	817	822	827	832	836	
888	841	846	851	856	861	866	871	876	880	885	
889	890	895	900	905	910	915	919	924	929	934	
890	939	944	949	954	959	963	968	973	978	983	
891	988	993	998	*002	*007	*012	*017	*022	*027	*032	
892	95 036	041	046	051	056	061	066	071	075	080	
893	085	090	095	100	105	109	114	119	124	129	
894	134	139	143	148	153	158	163	168	173	177	
895	182	187	192	197	202	207	211	216	221	226	
896	231	236	240	245	250	255	260	265	270	274	
897	279	284	289	294	299	303	308	313	318	323	
898	328	332	337	342	347	352	357	361	366	371	
899	376	381	386	390	395	400	405	410	415	419	
900	424	429	434	439	444	448	453	458	463	468	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

8
1 0.6
2 1.2
3 1.8
4 2.4
5 3.0
6 3.6
7 4.2
8 4.8
9 5.4

5
1 0.5
2 1.0
3 1.5
4 2.0
5 2.5
6 3.0
7 3.5
8 4.0
9 4.5

4
1 0.4
2 0.8
3 1.2
4 1.6
5 2.0
6 2.4
7 2.8
8 3.2
9 3.6

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
911	952	957	961	966	971	976	980	985	990	995	
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	1 3
913	96 047	052	057	061	066	071	076	080	085	090	2 0.5
914	095	099	104	109	114	118	123	128	133	137	3 1.0
915	142	147	152	156	161	166	171	175	180	185	4 1.5
916	190	194	199	204	209	213	218	223	227	232	5 2.0
917	237	242	246	251	256	261	265	270	275	280	6 2.5
918	284	289	294	298	303	308	313	317	322	327	7 3.0
919	332	336	341	346	350	355	360	365	369	374	8 3.5
920	379	384	388	393	398	402	407	412	417	421	9 4.0
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
931	895	900	904	909	914	918	923	928	932	937	4
932	942	946	951	956	960	965	970	974	979	984	1 0.4
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	2 0.3
934	97 035	039	044	049	053	058	063	067	072	077	3 1.2
935	081	086	090	095	100	104	109	114	118	123	4 1.6
936	128	132	137	142	146	151	155	160	165	169	5 2.0
937	174	179	183	188	192	197	202	206	211	216	6 2.4
938	220	225	230	234	239	243	248	253	257	262	7 2.8
939	267	271	276	280	285	290	294	299	304	308	8 3.2
940	313	317	322	327	331	336	340	345	350	354	9 3.6
941	359	364	368	373	377	382	387	391	396	400	
942	405	410	414	419	424	428	433	437	442	447	
943	451	456	460	465	470	474	479	483	488	493	
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLE—(Continued).

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.	
950	97 772	777	782	786	791	795	800	804	809	813		
951	818	823	827	832	836	841	845	850	855	859		
952	864	868	873	877	882	886	891	896	900	905		
953	909	914	918	923	928	932	937	941	946	950		
954	955	959	964	968	973	978	982	987	991	996		
955	98 000	005	009	014	019	023	028	032	037	041		
956	046	050	055	059	064	068	073	078	082	087		
957	091	096	100	105	109	114	118	123	127	132		
958	137	141	146	150	155	159	164	168	173	177		
959	182	186	191	195	200	204	209	214	218	223		
960	227	232	236	241	245	250	254	259	263	268		
961	272	277	281	286	290	295	299	304	308	313		
962	318	322	327	331	336	340	345	349	354	358		
963	363	367	372	376	381	385	390	394	399	403		
964	408	412	417	421	426	430	435	439	444	448		
965	453	457	462	466	471	475	480	484	489	493		
966	498	502	507	511	516	520	525	529	534	538		
967	543	547	552	556	561	565	570	574	579	583		
968	588	592	597	601	605	610	614	619	623	628		
969	632	637	641	646	650	655	659	664	668	673		
970	677	682	686	691	695	700	704	709	713	717		
971	722	726	731	735	740	744	749	753	758	762		
972	767	771	776	780	784	789	793	798	802	807		
973	811	816	820	825	829	834	838	843	847	851		
974	856	860	865	869	874	878	883	887	892	896		
975	900	905	909	914	918	923	927	932	936	941		
976	945	949	954	958	963	967	972	976	981	985		
977	989	994	998	*003	*007	*012	*016	*021	*025	*029		
978	99 034	038	043	047	052	056	061	065	069	074		
979	078	083	087	092	096	100	105	109	114	118		
980	123	127	131	136	140	145	149	154	158	162		
981	167	171	176	180	185	189	193	198	202	207		
982	211	216	220	224	229	233	238	242	247	251		
983	255	260	264	269	273	277	282	286	291	295		
984	300	304	308	313	317	322	326	330	335	339		
985	344	348	352	357	361	366	370	374	379	383		
986	388	392	396	401	405	410	414	419	423	427		
987	432	436	441	445	449	454	458	463	467	471		
988	476	480	484	489	493	498	502	506	511	515		
989	520	524	528	533	537	542	546	550	555	559		
990	564	568	572	577	581	585	590	594	599	603		
991	607	612	616	621	625	629	634	638	642	647		
992	651	656	660	664	669	673	677	682	686	691		
993	695	699	704	708	712	717	721	726	730	734		
994	739	743	747	752	756	760	765	769	774	778		
995	782	787	791	795	800	804	808	813	817	822		
996	826	830	835	839	843	848	852	856	861	865		
997	870	874	878	883	887	891	896	900	904	909		
998	913	917	922	926	930	935	939	944	948	952		
999	957	961	965	970	974	978	983	987	991	996		
1000	00 000	004	009	013	017	022	026	030	035	039		
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.	

5
1 0.5
2 1.0
3 1.5
4 2.0
5 2.5
6 3.0
7 3.5
8 4.0
9 4.5

4
1 0.4
2 0.8
3 1.2
4 1.6
5 2.0
6 2.4
7 2.8
8 3.2
9 3.6

TRIGONOMETRIC FUNCTIONS.

DIRECTIONS FOR USING THE TABLE.

The table given on pages 74-78 contains the natural sines, cosines, tangents, and cotangents of angles from 0° to 90° . Angles less than 45° are given in the first column at the left-hand side of the page, and the names of the functions are given at the top of the page; angles greater than 45° appear at the right-hand side of the page, and the names of the functions are given at the bottom. Thus, the second column contains the sines of angles less than 45° and the cosines of angles greater than 45° ; the sixth column contains the cotangents of angles less than 45° and the tangents of angles greater than 45° . To find the function of an angle less than 45° , look in the column of angles at the left of the page for the angle, and at the top of the page for the name of the function; to find a function of an angle greater than 45° , look in the column at the right of the page for the angle and at the bottom of the page for the name of the function. The successive angles differ by an interval of $10'$; they increase downwards in the left-hand column and upwards in the right-hand column. Thus, for angles less than 45° read down from top of page, and for angles greater than 45° read up from bottom of page.

The third, fifth, seventh, and ninth columns, headed *d*, contain the differences between the successive functions; for example, in the second column we find that the sine of $32^\circ 10'$ is .5324 and that the sine of $32^\circ 20'$ is .5348; the difference is $.5348 - .5324 = .0024$, and the 24 is written in the third column, just opposite the space between .5324 and .5348. In like manner the differences between the successive tabular values of the tangents are given in the fifth column, those between the cotangents in the seventh column, and those for the cosines in the ninth column. These differences in the functions correspond to a difference of $10'$ in the angle; thus, when the angle $32^\circ 10'$ is increased by $10'$, that is, to $32^\circ 20'$, the increase of the sine is .0024, or, as given in the table, 24. It will be observed that in the tabular difference no attention is paid to the decimal point, it being understood that the difference is

merely the number obtained by subtracting the last two or three figures of the smaller function from those of the larger. These differences are used to obtain the sines, cosines, etc. of angles not given in the table; the method employed may be illustrated by an example. Required, the tangent of $27^{\circ} 34'$. Looking in the table, we see that the tangent of $27^{\circ} 30'$ is .5206, and (in column 5) the difference for $10'$ is 37. Difference for $1'$ is $37 \div 10 = 3.7$, and difference for $4'$ is $3.7 \times 4 = 14.8$. Adding this difference to the value of the $\tan 27^{\circ} 30'$, we have

$$\begin{array}{r} \tan 27^{\circ} 30' = .5206 \\ \text{difference for } 4' = \quad 14.8 \\ \hline \end{array}$$

$$\tan 27^{\circ} 34' = .5220.8 \text{ or } .5221, \text{ to four places.}$$

Since only four decimal places are retained, the 8 in the fifth place is dropped and the figure in the fourth place is increased by 1, because 8 is greater than 5.

To avoid multiplication, the column of proportional parts, headed P. P., at the extreme right of the page, is used. At the head of each table in this column is the difference for $10'$, and below are the differences for any intermediate number of minutes from $1'$ to $9'$. In the above example, the difference for $10'$ was 37; looking in the table with 37 at the head, the difference opposite 4 is 14.8; that opposite 7 is 25.9; and so on. For want of space, the differences for the cotangents for angles less than 45° (or the tangents of angles greater than 45°) have been omitted from the tables of proportional parts. The use of these functions should be avoided, if possible, since the differences change very rapidly, and the computation is therefore likely to be inexact. The method to be employed when dealing with these functions may be shown by an example: Required, the tangent of $76^{\circ} 34'$. Since this angle is greater than 45° , we look for it in the column at the right, and read up; opposite the $76^{\circ} 30'$, we find, in sixth column, the number 4.1653, and corresponding to it in seventh column is the difference 540. Since 540 is the difference for $10'$, the difference for $4'$ is $540 \times \frac{4}{10} = 216$. Adding this difference:

$$\begin{array}{r} \tan 76^{\circ} 30' = 4.1653 \\ \text{difference for } 4' = \quad 216 \\ \hline \end{array}$$

$$\tan 76^{\circ} 34' = 4.1869$$

When the angle contains a certain number of seconds, divide the number by 6, and take the whole number nearest to the quotient; look out this number in the table of proportional parts (under the proper *difference*), and take out the number that is opposite to it. Shift the decimal point one place to the left, and then add it to the partial function already found.

Find the sine of $34^{\circ} 26' 44''$.

$$\begin{array}{rcl} \text{sine } 34^{\circ} 20' & = & .5640 \\ \text{difference for } 6' & = & 14.4 \\ \text{difference for } 44'' & = & 1.7 \\ \hline \text{sine } 34^{\circ} 26' 44'' & = & .5656 \end{array}$$

Difference for $10' = 24$.

$\frac{4}{3} = 7\frac{1}{3}$. Look out in the P. P. table the number under 24 and opposite 7. It is 16.8. Shifting the decimal point one place to the left, we get 1.68, or, say, 1.7.

The tangent is found in the same way as the sine.

To find the cosine of an angle:

As the angle increases, the value of the cosine decreases, so that, instead of adding the values corresponding to $6'$ and $44''$ to the function already found, we subtract them from it.

Thus, find $\cos 34^{\circ} 26' 44''$.

$$\begin{array}{rcl} \cos 34^{\circ} 20' & = & .8258 \\ \text{difference for } 6' & = & 10.2 \\ \text{difference for } 44'' & = & 1.2 \\ \hline \text{total difference} & = & 11.4 \\ \hline & & .8247 \end{array}$$

Difference for $10' = 17$.

The number under the 17 and opposite the 7, in the P. P. table, is 11.9. Therefore, take 1.19, or, say, 1.2.

Therefore, $\cos 34^{\circ} 26' 44'' = .8258 - .0011 = .824^{\circ}$.

Only four decimal places are kept; therefore, the figure of the difference following the decimal point is dropped before subtracting.

The cotangent is found in the same manner.

We will now consider angles greater than 45° .

Find the sine of $68^{\circ} 47' 22''$.

In obtaining the *difference*, it must be remembered to choose the one between the sine of $68^{\circ} 40'$ and the next angle above it, namely, $68^{\circ} 50'$.

sine $68^{\circ} 40'$ = .9315	Difference for $10'$ = 10.
difference for $7'$ = 7	
difference for $19''$ = .4	$\frac{7}{10} = 3\frac{1}{2}$, say 4. Under the 10
sine $68^{\circ} 47' 22''$ = .9322	and opposite the 4 is the
	number 4.0; shifting the deci-
	mal point, we get .4.

As usual, only four decimal places are kept.

The tangent is found in the same manner.

Find $\cos 68^{\circ} 47' 22''$.

As before, the cosine decreases as the angle increases; therefore, we subtract the successive sine values corresponding to the increments in the angle.

$\cos 68^{\circ} 40'$ = .3638	Difference for $10'$ = 27.
difference for $7'$ = 18.9	
difference for $22''$ = 1.1	Under the 27 and opposite the
total difference = 20	4 is the number 10.8; there-
.3618	fore, take 1.08 in this case,
	or, say, 1.1.

Therefore, $\cos 68^{\circ} 47' 22'' = .3638 - .002 = .3618$.

The cotangent is found in the same way.

In finding the functions of an angle, the only difficulty likely to be encountered is to determine whether the difference obtained from the table of proportional parts is to be added or subtracted. This can be told in every case by observing whether the function is increasing or decreasing as the angle increases. For example, take the angle 21° ; its sine is .3584, and the following sines, reading downwards, are .3611, .3638, etc. It is plain, therefore, that the sine of say $21^{\circ} 6'$ is greater than that of 21° , and that the difference for $6'$ must be added. On the other hand, the cosine of 21° is .9336, and the following cosines, reading downwards, are .9325, .9315, etc.; that is, as the angle grows larger the cosine decreases. The cosine of an angle between 21° and $21^{\circ} 10'$, say $21^{\circ} 6'$, must therefore lie between .9325 and .9315; that is, it must be smaller than .9325, which shows that in this case the difference for $6'$ must be subtracted from the cosine of 21° .

We will now consider the case in which the function, i. e. the sine, cosine, tangent, or cotangent, is given and the corresponding angle is to be found.

Find the angle whose sine is .4943. The operation is arranged as follows:

$$\begin{array}{rcl}
 .4943 & & \text{Difference for } 10' = 26. \\
 .4924 & = \sin 29^\circ 30'. & \\
 \hline
 \text{1st remainder} & 19 & \\
 & 18.2 & = \text{difference for } 7'. \\
 \hline
 \text{2d remainder} & .8 & \\
 & .78 & = \text{difference for } .3' \text{ or } 18''.
 \end{array}$$

$$.4943 = \sin 29^\circ 37' 18''.$$

Looking down the second column, we find the sine next *smaller* than .4943 to be .4924, and the difference for 10' to be 26. The angle corresponding to .4924 is $29^\circ 30'$. Subtracting the .4924 from .4943, the first remainder is 19; looking in the table of proportional parts, the part next lower than this difference is 18.2, opposite which is 7'. Subtracting this difference from the remainder, we get .8, and, looking in the table, we see that 7.8 with its decimal point moved one place to the left is nearest to the second difference. This is the difference for .3' or 18''. Hence, the angle is $29^\circ 30' + 7' + 18'' = 29^\circ 37' 18''$.

Find the angle whose tangent is .8824.

$$\begin{array}{rcl}
 .8824 & & \text{Difference for } 10' = 51. \\
 .8796 & = \tan 41^\circ 20'. & \\
 \hline
 \text{1st remainder} & 28 & \\
 & 25.5 & = \text{difference for } 5'. \\
 \hline
 \text{2d remainder} & 2.5 & \\
 & 2.55 & = \text{difference for } .5' \text{ or } 30''.
 \end{array}$$

$$.8824 = \tan 41^\circ 25' 30''.$$

In the two examples just given, the minutes and seconds corresponding to the 1st and 2d remainders are added to the angle taken from the table. Thus, in the first example, an inspection of the table shows that the angle increases as the sine increases; hence, the angle whose sine is .4943 must be greater than $29^\circ 30'$, whose sine is .4924. For this reason the correction must be *added* to $29^\circ 30'$. The same reasoning applies to the second example.

Find the angle whose cosine is .7742.

$$\begin{array}{rcl} .7742 & & \text{Difference for } 10' = 18. \\ .7735 & = & \cos 39^\circ 20'. \end{array}$$

$$\begin{array}{rcl} \text{1st remainder} & 7 & \\ & \underline{5.4} & = \text{difference for } 3'. \end{array}$$

$$\begin{array}{rcl} \text{2d remainder} & 1.6 & \\ & \underline{1.62} & = \text{difference for } .9' \text{ or } 54''. \end{array}$$

$39^\circ 20' - 3' 54'' = 39^\circ 16' 6''$, which is the angle whose cosine is .7742.

Looking down the eighth column, headed cos, the next smaller cosine is .7735, to which corresponds the angle $39^\circ 20'$. The difference for $10'$ is 18. Subtracting, the remainder is 7, and the next lower number in the table of proportional parts is 5.4, which is the difference for $3'$. Subtracting this from 1st remainder, 2d remainder is 1.6, which is nearest 16.2 of table of proportional parts, if the decimal point of the latter is moved to the left one place. Since 16.2 corresponds to a difference of $9'$, 1.62 corresponds to a difference of $.9'$, or $54''$. Hence, the correction for the angle $39^\circ 20'$ is $3' 54''$. From the table, it appears that, as the cosine increases, the angle grows smaller; therefore, the angle whose cosine is .7742 must be smaller than the angle whose cosine is .7735, and the correction for the angle must be subtracted.

Find the angle whose cotangent is .9847.

$$\begin{array}{rcl} .9847 & & \text{Difference for } 10' = 57. \\ .9827 & = & \cos 45^\circ 30'. \end{array}$$

$$\begin{array}{rcl} \text{1st remainder} & 20 & \\ & \underline{17.1} & = \text{difference for } 3'. \end{array}$$

$$\begin{array}{rcl} \text{2d remainder} & 2.9 & \\ & \underline{2.85} & = \text{difference for } .5' \text{ or } 30''. \end{array}$$

$45^\circ 30' - 3' 30'' = 45^\circ 26' 30''$, the angle whose cotangent is .9847.

In finding the angle corresponding to a function, as in the above examples, the angles obtained may vary from the true angle by 2 or 3 seconds; in order to obtain the number of seconds accurately, the functions should contain six or seven decimal places.

°	'	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.		P. P.
0	0	0.0000		0.0000		infin.		1.0000		0 90	
	10	0.0029	29	0.0029	29	343.7737		1.0000	0	50	
	20	0.0058	29	0.0058	29	171.8854		1.0000	0	40	30
	30	0.0087	29	0.0087	29	114.5887		1.0000	0	30	1 3.0
	40	0.0116	29	0.0116	29	85.9398		0.9999	1	20	2 6.0
	50	0.0145	29	0.0145	30	68.7501		0.9999	0	10	3 9.0
1	0	0.0175	30	0.0175	30	57.2900		0.9998	1	4	4 12.0
	10	0.0204	29	0.0204	29	49.1039	81861	0.9998	0	50	5 15.0
	20	0.0233	29	0.0233	29	42.9641	61398	0.9997	1	40	6 18.0
	30	0.0262	29	0.0262	29	38.1885	47756	0.9997	0	30	7 21.0
	40	0.0291	29	0.0291	29	34.3678	38207	0.9996	1	20	8 24.0
	50	0.0320	29	0.0320	29	31.2416	31262	0.9995	1	10	9 27.0
2	0	0.0349	29	0.0349	29	28.6363	26053	0.9994	1	0 88	
	10	0.0378	29	0.0378	29	26.4316	22047	0.9993	1	50	29
	20	0.0407	29	0.0407	30	24.5418	18898	0.9992	1	40	-1 2.9
	30	0.0436	29	0.0437	29	22.9038	16380	0.9990	2	30	2 5.8
	40	0.0465	29	0.0466	29	21.4704	14334	0.9989	1	20	3 8.7
	50	0.0494	29	0.0495	29	20.2056	12648	0.9988	1	10	4 11.6
3	0	0.0523	29	0.0524	29	19.0811	11245	0.9986	2	0 87	
	10	0.0552	29	0.0553	29	18.07 0	10061	0.9985	1	50	5 14.5
	20	0.0581	29	0.0582	30	17.1693	9057	0.9983	2	40	6 17.4
	30	0.0610	30	0.0612	29	16.3499	8194	0.9981	2	30	7 20.3
	40	0.0640	29	0.0641	29	15.6048	7451	0.9980	1	20	8 23.2
	50	0.0669	29	0.0670	29	14.9244	6804	0.9978	2	10	9 26.1
4	0	0.0698	29	0.0699	30	14.3007	6237	0.9976	2	0 86	
	10	0.0727	29	0.0729	29	13.7267	5740	0.9974	2	50	28
	20	0.0756	29	0.0758	29	13.1969	5298	0.9971	3	40	1 2.8
	30	0.0785	29	0.0787	29	12.7062	4907	0.9969	3	30	2 5.6
	40	0.0814	29	0.0816	30	12.2505	4557	0.9967	2	20	3 8.4
	50	0.0843	29	0.0846	30	11.8262	4243	0.9964	3	10	4 11.2
5	0	0.0872	29	0.0875	29	11.4301	3961	0.9962	3	0 85	
	10	0.0901	28	0.0904	29	11.0594	3707	0.9959	2	50	5 16.8
	20	0.0929	29	0.0934	30	10.7119	3475	0.9957	3	40	6 19.6
	30	0.0958	29	0.0963	29	10.3854	3265	0.9954	3	30	7 22.4
	40	0.0987	29	0.0992	30	10.0780	3074	0.9951	3	20	8 25.2
	50	0.1016	29	0.1022	30	9.7882	2898	0.9948	3	10	5
6	0	0.1045	29	0.1051	29	9.5144	2738	0.9945	3	0 84	
	10	0.1074	29	0.1080	30	9.2553	2591	0.9942	3	50	1 0.5
	20	0.1103	29	0.1110	30	9.0098	2455	0.9939	3	40	2 1.0
	30	0.1132	29	0.1139	30	8.7769	2329	0.9936	3	30	3 1.5
	40	0.1161	29	0.1169	29	8.5555	2214	0.9932	4	20	4 2.0
	50	0.1190	29	0.1198	30	8.34 0	2105	0.9929	3	10	5 2.5
7	0	0.1219	29	0.1228	30	8.1443	2007	0.9925	4	0 83	
	10	0.1248	28	0.1257	30	7.9530	1913	0.9922	3	50	6 3.0
	20	0.1276	29	0.1287	30	7.7704	1826	0.9918	4	40	7 3.5
	30	0.1305	29	0.1317	29	7.5958	1746	0.9914	4	30	8 4.0
	40	0.1334	29	0.1346	30	7.4287	1671	0.9911	3	20	9 4.5
	50	0.1363	29	0.1376	30	7.2687	1600	0.9907	4	10	
8	0	0.1392	29	0.1405	30	7.1154	1533	0.9903	4	0 82	
	10	0.1421	28	0.1435	30	6.9682	1472	0.9899	4	50	4
	20	0.1449	29	0.1465	30	6.8269	1413	0.9894	5	40	1 0.4
	30	0.1478	29	0.1495	30	6.6912	1357	0.9890	4	30	2 0.8
	40	0.1507	29	0.1524	30	6.5606	1306	0.9886	4	20	3 1.2
	50	0.1536	28	0.1554	30	6.4348	1258	0.9881	5	10	4 1.6
9	0	0.1564	28	0.1584	30	6.3138	1210	0.9877	4	0 81	
		Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	°	P. P.

°	'	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.		P. P.
9	0	0.1564		0.1584		6.8138		0.9877		0 81	
	10	0.1593	29	0.1614	30	6.1970	1168	0.9872	5	50	
	20	0.1622	29	0.1644	30	6.0844	1126	0.9868	4	40	
	30	0.1650	28	0.1673	29	5.9758	1086	0.9863	5	30	
	40	0.1679	29	0.1703	30	5.8708	1050	0.9858	5	20	
	50	0.1708	29	0.1733	30	5.7694	1014	0.9853	5	10	
			28		30		981		5	0 80	
10	0	0.1736		0.1763		5.6713		0.9848			
	10	0.1765	29	0.1793	30	5.5764	949	0.9843	5	50	
	20	0.1794	28	0.1823	30	5.4845	919	0.9838	5	40	
	30	0.1822	29	0.1853	30	5.3955	890	0.9833	5	30	
	40	0.1851	29	0.1883	31	5.3093	862	0.9827	6	20	
	50	0.1880	29	0.1914	30	5.2257	836	0.9822	5	10	
			28		30		811		6	0 79	
11	0	0.1908		0.1944		5.1446		0.9816			
	10	0.1937	28	0.1974	30	5.0658	788	0.9811	5	50	
	20	0.1965	29	0.2004	31	4.9894	764	0.9805	6	40	
	30	0.1994	28	0.2035	30	4.9152	742	0.9799	6	30	
	40	0.2022	29	0.2065	30	4.8430	722	0.9793	6	20	
	50	0.2051	29	0.2095	30	4.7729	701	0.9787	6	10	
			28		31		683		6	0 78	
12	0	0.2079		0.2126		4.7046		0.9781			
	10	0.2108	29	0.2156	30	4.6382	664	0.9775	6	50	
	20	0.2136	28	0.2186	31	4.5736	646	0.9769	6	40	
	30	0.2164	29	0.2217	30	4.5107	629	0.9763	6	30	
	40	0.2193	28	0.2247	31	4.4494	613	0.9757	6	20	
	50	0.2221	28	0.2278	31	4.3897	597	0.9750	6	10	
			29		31		582		6	0 77	
13	0	0.2250		0.2309		4.3315		0.9744			
	10	0.2278	28	0.2339	30	4.2747	568	0.9737	7	50	
	20	0.2306	28	0.2370	31	4.2193	554	0.9730	7	40	
	30	0.2334	29	0.2401	31	4.1653	540	0.9724	6	30	
	40	0.2363	28	0.2432	30	4.1126	527	0.9717	7	20	
	50	0.2391	28	0.2462	31	4.0611	515	0.9710	7	10	
			29		31		503		7	0 76	
14	0	0.2419		0.2493		4.0108		0.9703			
	10	0.2447	29	0.2524	31	3.9617	491	0.9696	7	50	
	20	0.2476	28	0.2555	31	3.9136	481	0.9689	7	40	
	30	0.2504	28	0.2586	31	3.8667	469	0.9681	8	30	
	40	0.2532	28	0.2617	31	3.8208	459	0.9674	7	20	
	50	0.2560	28	0.2648	31	3.7760	448	0.9667	7	10	
			29		31		439		8	0 75	
15	0	0.2588		0.2679		3.7321		0.9659			
	10	0.2616	28	0.2711	32	3.6891	430	0.9652	7	50	
	20	0.2644	28	0.2742	31	3.6470	421	0.9644	8	40	
	30	0.2672	28	0.2773	32	3.6059	411	0.9636	8	30	
	40	0.2700	28	0.2805	32	3.5656	403	0.9628	8	20	
	50	0.2728	28	0.2836	31	3.5261	395	0.9621	7	10	
			29		31		387		8	0 74	
16	0	0.2756		0.2867		3.4874		0.9613			
	10	0.2784	28	0.2899	32	3.4495	379	0.9605	8	50	
	20	0.2812	28	0.2931	31	3.4124	371	0.9596	9	40	
	30	0.2840	28	0.2962	32	3.3759	365	0.9588	8	30	
	40	0.2868	28	0.2994	32	3.3402	357	0.9580	8	20	
	50	0.2896	28	0.3026	32	3.3052	350	0.9572	8	10	
			29		31		343		9	0 73	
17	0	0.2924		0.3057		3.2709		0.9563			
	10	0.2952	28	0.3089	32	3.2371	338	0.9555	8	50	
	20	0.2979	28	0.3121	32	3.2041	330	0.9546	9	40	
	30	0.3007	28	0.3153	32	3.1716	325	0.9537	9	30	
	40	0.3035	27	0.3185	32	3.1397	319	0.9528	8	20	
	50	0.3062	27	0.3217	32	3.1084	313	0.9520	8	10	
			28		32		307		9	0 72	
18	0	0.3090		0.3249		3.0777		0.9511			
		Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	°	P. P.

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	28	27
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

										P. P.		
°	'	Sin.	d.	Tan	d.	Cot.	d.	Cos.	d.			
18	0	0.3090		0.3249		3.0777		0.9511		0 72		
	10	0.3118	28	0.3281	32	3.0475	302	0.9502	9	50		
	20	0.3145	27	0.3314	33	3.0178	297	0.9492	10	40		
	30	0.3173	28	0.3346	32	2.9887	291	0.9483	9	30		
	40	0.3201	27	0.3378	33	2.9600	287	0.9474	9	20		
	50	0.3228	28	0.3411	32	2.9319	281	0.9465	9	10		
19	0	0.3256		0.3443		2.9042		0.9455	10	0 71		
	10	0.3283	27	0.3476	33	2.8770	272	0.9446	9	50		
	20	0.3311	27	0.3508	33	2.8502	268	0.9436	10	40		
	30	0.3338	28	0.3541	33	2.8239	263	0.9426	9	30		
	40	0.3365	27	0.3574	33	2.7980	259	0.9417	9	20		
	50	0.3393	28	0.3607	33	2.7725	255	0.9407	10	10		
20	0	0.3420		0.3640		2.7471		0.9397	10	0 70		
	10	0.3448	28	0.3673	33	2.7228	247	0.9387	10	50		
	20	0.3475	27	0.3706	33	2.6985	243	0.9377	10	40		
	30	0.3502	27	0.3739	33	2.6746	239	0.9367	11	30		
	40	0.3529	28	0.3772	33	2.6511	235	0.9356	10	20		
	50	0.3557	27	0.3805	34	2.6279	232	0.9346	10	10		
21	0	0.3584		0.3839		2.6051		0.9336	10	0 69		
	10	0.3611	27	0.3872	33	2.5826	228	0.9336	11	50		
	20	0.3638	27	0.3906	33	2.5607	225	0.9325	11	40		
	30	0.3665	27	0.3939	34	2.5386	221	0.9315	11	30		
	40	0.3692	27	0.3973	33	2.5172	219	0.9304	11	20		
	50	0.3719	27	0.4006	34	2.4960	214	0.9293	10	10		
22	0	0.3746		0.4040		2.4751		0.9283	11	0 68		
	10	0.3773	27	0.4074	34	2.4545	209	0.9272	11	50		
	20	0.3800	27	0.4108	34	2.4342	206	0.9261	11	40		
	30	0.3827	27	0.4142	34	2.4142	203	0.9250	11	30		
	40	0.3854	27	0.4176	34	2.3947	200	0.9239	11	20		
	50	0.3881	26	0.4210	35	2.3750	197	0.9228	12	10		
23	0	0.3907		0.4244		2.3559		0.9216	11	0 67		
	10	0.3934	27	0.4279	34	2.3369	191	0.9205	11	50		
	20	0.3961	26	0.4314	34	2.3183	186	0.9194	12	40		
	30	0.3987	27	0.4348	35	2.2998	185	0.9182	12	30		
	40	0.4014	27	0.4383	34	2.2817	181	0.9171	12	20		
	50	0.4041	26	0.4417	35	2.2637	180	0.9159	12	10		
24	0	0.4067		0.4452		2.2460		0.9147	12	0 66		
	10	0.4094	26	0.4487	35	2.2286	177	0.9135	12	50		
	20	0.4120	26	0.4522	35	2.2113	174	0.9124	11	40		
	30	0.4147	26	0.4557	35	2.1943	173	0.9112	12	30		
	40	0.4173	26	0.4592	36	2.1777	170	0.9100	12	20		
	50	0.4200	26	0.4628	36	2.1609	168	0.9088	13	10		
25	0	0.4226		0.4663		2.1445		0.9075	12	0 65		
	10	0.4253	26	0.4699	36	2.1283	164	0.9063	12	50		
	20	0.4279	26	0.4734	36	2.1123	162	0.9051	13	40		
	30	0.4305	26	0.4770	36	2.0965	160	0.9038	13	30		
	40	0.4331	26	0.4806	36	2.0809	158	0.9026	13	20		
	50	0.4358	26	0.4841	35	2.0655	156	0.9013	12	10		
26	0	0.4384		0.4877		2.0503		0.9001	13	0 64		
	10	0.4410	26	0.4913	36	2.0353	152	0.8988	13	50		
	20	0.4436	26	0.4950	36	2.0204	150	0.8975	13	40		
	30	0.4462	26	0.4986	36	2.0057	149	0.8962	13	30		
	40	0.4488	26	0.5022	37	1.9912	147	0.8949	13	20		
	50	0.4514	26	0.5059	36	1.9768	145	0.8936	13	10		
27	0	0.4540		0.5095		1.9626		0.8923	13	0 63		
		Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.			
										P. P.		

	37	36	35
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	33	32
1	3.4	3.3	3.2
2	6.8	6.6	6.4
3	10.2	9.9	9.6
4	13.6	13.2	12.8
5	17.0	16.5	16.0
6	20.4	19.8	19.2
7	23.8	23.1	22.4
8	27.2	26.4	25.6
9	30.6	29.7	28.8

	28	27	26
1	2.8	2.7	2.6
2	5.6	5.4	5.2
3	8.4	8.1	7.8
4	11.2	10.8	10.4
5	14.0	13.5	13.0
6	16.8	16.2	15.6
7	19.6	18.9	18.2
8	22.4	21.6	20.8
9	25.2	24.3	23.4

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

	11	10	9
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

°	'	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.		P. P.
27	0	0.4540		0.5095		1.9626		0.8910		0 63	44 43 42
	10	0.4566	26	0.5132	37	1.9486	140	0.8897	13	50	1 4.4 4.3 4.2
	20	0.4592	25	0.5169	37	1.9347	139	0.8884	14	40	2 8.8 8.6 8.4
	30	0.4617	26	0.5206	37	1.9210	137	0.8870	14	30	3 13.2 12.9 12.6
	40	0.4643	26	0.5243	37	1.9074	136	0.8857	13	20	4 17.6 17.2 16.8
	50	0.4669	26	0.5280	37	1.8940	134	0.8843	14	10	5 22.0 21.5 21.0
28	0	0.4695	26	0.5317	37	1.8807	133	0.8829	14	0 62	6 26.4 25.8 25.2
	10	0.4720	25	0.5354	38	1.8676	131	0.8816	13	50	7 30.8 30.1 29.4
	20	0.4746	26	0.5392	38	1.8546	130	0.8802	14	40	8 35.2 34.4 33.6
	30	0.4772	25	0.5430	37	1.8418	128	0.8788	14	30	9 39.6 38.7 37.8
	40	0.4797	26	0.5467	38	1.8291	127	0.8774	14	20	41 40 39
	50	0.4823	25	0.5505	38	1.8165	126	0.8760	14	10	1 4.1 4.0 3.9
29	0	0.4848	25	0.5543	38	1.8040	125	0.8746	14	0 61	2 8.2 8.0 7.8
	10	0.4874	26	0.5581	38	1.7917	123	0.8732	14	50	3 12.3 12.0 11.7
	20	0.4899	25	0.5619	39	1.7796	121	0.8718	14	40	4 16.4 16.0 15.6
	30	0.4924	26	0.5658	38	1.7675	119	0.8704	15	30	5 20.5 20.0 19.5
	40	0.4950	25	0.5696	39	1.7556	119	0.8689	14	20	6 24.6 24.0 23.4
	50	0.4975	25	0.5735	39	1.7437	116	0.8675	14	10	7 28.7 28.0 27.3
30	0	0.5000	25	0.5774	38	1.7321	116	0.8660	15	0 60	8 32.8 32.0 31.2
	10	0.5025	25	0.5812	39	1.7205	115	0.8646	14	50	9 36.9 36.0 35.1
	20	0.5050	25	0.5851	39	1.7090	113	0.8631	15	40	38 37
	30	0.5075	25	0.5890	40	1.6977	113	0.8616	15	30	1 3.8 3.7
	40	0.5100	25	0.5930	39	1.6864	111	0.8601	14	20	2 7.6 7.4
	50	0.5125	25	0.5969	40	1.6753	110	0.8587	14	10	3 11.4 11.1
31	0	0.5150	25	0.6009	39	1.6643	109	0.8572	15	0 59	4 15.2 14.8
	10	0.5175	25	0.6048	40	1.6534	108	0.8557	15	50	5 19.0 18.5
	20	0.5200	25	0.6088	40	1.6426	107	0.8542	16	40	6 22.8 22.2
	30	0.5225	25	0.6128	40	1.6319	107	0.8526	15	30	7 26.6 25.9
	40	0.5250	25	0.6168	40	1.6212	105	0.8511	15	20	8 30.4 29.6
	50	0.5275	24	0.6208	41	1.6107	104	0.8496	16	10	9 34.2 33.3
32	0	0.5299	25	0.6249	40	1.6003	103	0.8480	15	0 58	26 25 24
	10	0.5324	24	0.6289	41	1.5900	102	0.8465	15	50	1 2.6 2.5 2.4
	20	0.5348	25	0.6330	41	1.5798	101	0.8450	16	40	2 5.2 5.0 4.8
	30	0.5373	25	0.6371	41	1.5697	100	0.8434	16	30	3 7.8 7.5 7.2
	40	0.5398	24	0.6412	41	1.5597	100	0.8418	15	20	4 10.4 10.0 9.6
	50	0.5422	24	0.6453	41	1.5497	98	0.8403	16	10	5 13.0 12.5 12.0
33	0	0.5446	24	0.6494	42	1.5399	98	0.8387	16	0 57	6 15.6 15.0 14.4
	10	0.5471	25	0.6536	41	1.5301	97	0.8371	16	50	7 18.2 17.5 16.8
	20	0.5495	24	0.6577	42	1.5204	96	0.8355	16	40	8 20.8 20.0 19.2
	30	0.5519	25	0.6619	42	1.5108	95	0.8339	16	30	9 23.4 22.5 21.6
	40	0.5544	24	0.6661	42	1.5013	94	0.8323	16	20	23 17 16
	50	0.5568	24	0.6703	42	1.4919	93	0.8307	17	10	1 2.3 1.7 1.6
34	0	0.5592	24	0.6745	42	1.4826	93	0.8290	17	0 56	2 4.6 3.4 3.2
	10	0.5616	24	0.6787	43	1.4733	92	0.8274	16	50	3 6.9 5.1 4.8
	20	0.5640	24	0.6830	43	1.4641	91	0.8258	17	40	4 9.2 6.8 6.4
	30	0.5664	24	0.6873	43	1.4550	90	0.8241	16	30	5 11.5 8.5 8.0
	40	0.5688	24	0.6916	43	1.4460	90	0.8225	17	20	6 13.8 10.2 9.6
	50	0.5712	24	0.6959	43	1.4370	89	0.8208	16	10	7 16.1 11.9 11.2
35	0	0.5736	24	0.7002	44	1.4281	88	0.8192	17	0 55	8 18.4 13.6 12.8
	10	0.5760	23	0.7046	43	1.4193	87	0.8175	17	50	9 20.7 15.3 14.4
	20	0.5783	24	0.7089	44	1.4106	87	0.8158	17	40	15 14 13
	30	0.5807	24	0.7133	44	1.4019	85	0.8141	17	30	1 1.5 1.4 1.3
	40	0.5831	23	0.7177	44	1.3934	86	0.8124	17	20	2 3.0 2.8 2.6
	50	0.5854	24	0.7221	44	1.3848	84	0.8107	17	10	3 4.5 4.2 3.9
36	0	0.5878	24	0.7265	44	1.3764	84	0.8090	17	0 54	4 6.0 5.6 5.2
											5 7.5 7.0 6.5
											6 9.0 8.4 7.8
											7 10.5 9.8 9.1
											8 12.0 11.2 10.4
											9 13.5 12.6 11.7
		Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	°	P. P.

°	'	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.		P. P.
36	0	0.5878		0.7265		1.3764		0.8090		0 54	58 57 56 55
10		0.5901	23	0.7310	45	1.3680	84	0.8073	17	50	1 5.8 5.7 5.6 5.5
20		0.5925	24	0.7355	45	1.3597	83	0.8056	17	40	2 11.6 11.4 11.2 11.0
30		0.5948	24	0.7400	45	1.3514	82	0.8039	17	30	3 17.4 17.1 16.8 16.5
40		0.5972	23	0.7445	45	1.3432	81	0.8021	18	20	4 23.2 22.8 22.4 22.0
50		0.5995	23	0.7490	45	1.3351	81	0.8004	17	10	5 29.0 28.5 28.0 27.5
37	0	0.6018		0.7536		1.3270		0.7986		0 53	6 34.8 34.2 33.6 33.0
10		0.6041	23	0.7581	45	1.3190	80	0.7969	17	50	7 40.6 39.9 39.2 38.5
20		0.6065	23	0.7627	46	1.3111	79	0.7951	18	40	8 46.4 45.6 44.8 44.0
30		0.6088	23	0.7673	47	1.3032	78	0.7934	17	30	9 52.2 51.3 50.4 49.5
40		0.6111	23	0.7720	46	1.2954	78	0.7916	18	20	54 53 52 51
50		0.6134	23	0.7766	47	1.2876	77	0.7898	18	10	1 5.4 5.3 5.2 5.1
38	0	0.6157		0.7813		1.2799		0.7880		0 52	2 10.8 10.6 10.4 10.2
10		0.6180	22	0.7860	47	1.2723	76	0.7862	18	50	3 16.2 15.9 15.6 15.3
20		0.6202	23	0.7907	47	1.2647	75	0.7844	18	40	4 21.6 21.2 20.8 20.4
30		0.6225	23	0.7954	48	1.2572	75	0.7826	18	30	5 27.0 26.5 26.0 25.5
40		0.6248	23	0.8002	48	1.2497	74	0.7808	18	20	6 32.4 31.8 31.2 30.6
50		0.6271	22	0.8050	48	1.2423	74	0.7790	19	10	7 37.8 37.1 36.4 35.7
39	0	0.6293		0.8098		1.2349		0.7771		0 51	8 43.2 42.4 41.6 40.8
10		0.6316	22	0.8146	49	1.2276	73	0.7753	18	50	9 48.6 47.7 46.8 45.9
20		0.6338	22	0.8193	48	1.2203	72	0.7735	18	40	50 49 48
30		0.6361	22	0.8243	49	1.2131	72	0.7716	19	30	1 5.0 4.9 4.8
40		0.6383	23	0.8292	50	1.2059	71	0.7698	19	20	2 10.0 9.8 9.6
50		0.6406	22	0.8342	49	1.1988	70	0.7679	19	10	3 15.0 14.7 14.4
40	0	0.6428		0.8391		1.1918		0.7660		0 50	4 20.0 19.6 19.2
10		0.6450	22	0.8441	50	1.1847	69	0.7642	18	50	5 25.0 24.5 24.0
20		0.6472	22	0.8491	50	1.1778	70	0.7623	19	40	6 30.0 29.4 28.8
30		0.6494	23	0.8541	50	1.1708	68	0.7604	19	30	7 35.0 34.3 33.6
40		0.6517	22	0.8591	51	1.1640	69	0.7585	19	20	8 40.0 39.2 38.4
50		0.6539	22	0.8642	51	1.1571	67	0.7566	19	10	9 45.0 44.1 43.2
41	0	0.6561		0.8693		1.1504		0.7547		0 49	47 46 45
10		0.6583	21	0.8744	52	1.1436	68	0.7528	19	50	1 4.7 4.6 4.5
20		0.6604	22	0.8796	51	1.1369	66	0.7509	19	40	2 9.4 9.2 9.0
30		0.6626	22	0.8847	52	1.1303	66	0.7490	20	30	3 14.1 13.8 13.5
40		0.6648	22	0.8899	53	1.1237	66	0.7470	20	20	4 18.8 18.4 18.0
50		0.6670	21	0.8952	52	1.1171	65	0.7451	20	10	5 23.5 23.0 22.5
42	0	0.6691		0.9004		1.1106		0.7431		0 48	6 28.2 27.6 27.0
10		0.6713	21	0.9057	53	1.1041	64	0.7412	20	50	7 32.9 32.2 31.5
20		0.6734	22	0.9110	53	1.0977	64	0.7392	20	40	8 37.6 36.8 36.0
30		0.6756	21	0.9163	54	1.0913	63	0.7373	20	30	9 42.3 41.4 40.5
40		0.6777	22	0.9217	54	1.0850	64	0.7353	20	20	24 23 22 21
50		0.6799	21	0.9271	54	1.0786	62	0.7333	19	10	1 2.4 2.3 2.2 2.1
43	0	0.6820		0.9325		1.0724		0.7314		0 47	2 4.8 4.6 4.4 4.2
10		0.6841	21	0.9380	55	1.0661	63	0.7294	20	50	3 7.2 6.9 6.6 6.3
20		0.6862	22	0.9435	55	1.0599	61	0.7274	20	40	4 9.6 9.2 8.8 8.4
30		0.6884	21	0.9490	55	1.0538	61	0.7254	20	30	5 12.0 11.5 11.0 10.5
40		0.6905	21	0.9545	56	1.0477	61	0.7234	20	20	6 14.4 13.8 13.2 12.6
50		0.6926	21	0.9601	56	1.0416	61	0.7214	21	10	7 16.8 16.1 15.4 14.7
44	0	0.6947		0.9657		1.0355		0.7193		0 46	8 19.2 18.4 17.6 16.8
10		0.6967	21	0.9713	57	1.0295	60	0.7173	20	50	9 21.6 20.7 19.8 18.9
20		0.6988	21	0.9770	57	1.0235	59	0.7153	20	40	20 19 18 17
30		0.7009	21	0.9827	57	1.0176	59	0.7133	20	30	1 2.0 1.9 1.8 1.7
40		0.7030	20	0.9884	58	1.0117	59	0.7112	20	20	2 4.0 3.8 3.6 3.4
50		0.7050	21	0.9942	58	1.0058	58	0.7092	21	10	3 6.0 5.7 5.4 5.1
45	0	0.7071		1.0000		1.0000		0.7071		0 45	4 8.0 7.6 7.2 6.8
		Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.		P. P.

PRIME NUMBERS.

Every prime number is an odd number and has for its unit figure 1, 3, 7, or 9; any odd number that has 5 for its unit figure is divisible by 5, and is not a prime number. The prime factors of any number less than 1,000 may be found from the following table. If the number is odd and does not end with 5, the factors are given directly; thus, the prime factors of 357 are 3, 7, and 17; those of 931 are 7, 7, and 19, the exponent 2 of the 7 indicating that 7 is used twice as a factor. If a number is a prime number, the space beside it is blank; thus, 817 and 859 are prime numbers. To find the prime factors of an odd number that has 5 for the unit figure, divide by 5 until a quotient is obtained which does not have 5 for a unit figure; the factors of this quotient are then found from the table, and with the 5's already used as divisors constitute the prime factors. For example, to find the prime factors of 5,775 proceed as follows: $5,775 \div 5 = 1,155$; $1,155 \div 5 = 231$; from the table, $231 = 3 \times 7 \times 11$; hence, $5,775 = 3 \times 5 \times 5 \times 7 \times 11$. If the number is even, divide it by 2, the quotient by 2, and so on until an odd quotient is reached; then find the prime factors of the quotient from the table. The process of finding the prime factors of 936 is as follows:

$936 \div 2 = 468$; $468 \div 2 = 234$; $234 \div 2 = 117$; $117 = 3^2 \times 13$, from table. Hence, $936 = 2^3 \times 3^2 \times 13 = 2 \times 2 \times 2 \times 3 \times 3 \times 13$.

FACTORS OF 3.1416.

NOT REGARDING DECIMAL POINT, 3.1416 =

2×15708	22×1428	68×462
3×10472	24×1309	77×408
4×7854	28×1122	84×374
6×5236	33×952	88×357
7×4488	34×924	102×308
8×3927	42×748	119×264
11×2856	44×714	132×238
12×2618	51×616	136×231
14×2244	56×561	154×204
17×1848	66×476	168×187
21×1496		

PRIME FACTORS.

PRIME FACTORS OF ALL ODD NUMBERS FROM 1 TO 1,000
THAT ARE NOT DIVISIBLE BY 5.

1		101		201	3·67	301	7·43	401	
3		103		203	7·29	303	3·101	403	13·31
7		107		207	3 ² ·23	307		407	11·37
9	3 ²	109		209	11·19	309	3·103	409	
11		111	3·37	211		311		411	3·137
13		113		213	3·71	313		413	7·59
17		117	3 ² ·13	217	7·31	317		417	3·139
19		119	7·17	219	3·73	319	11·29	419	
21	3·7	121	11 ²	221	13·17	321	3·107	421	
23		123	3·41	223		323	17·19	423	3 ² ·47
27	3 ³	127		227		327	3·109	427	7·61
29		129	3·43	229		329	7·47	429	3·11·13
31		131		231	3·7·11	331		431	
33	3·11	133	7·19	233		333	3 ² ·37	433	
37		137		237	3·79	337		437	19·23
39	3·13	139		239		339	3·113	439	
41		141	3·47	241		341	11·31	441	3 ² ·7 ²
43		143	11·13	243	3 ⁵	343	7 ³	443	
47		147	3·7 ²	247	13·19	347		447	3·149
49	7 ²	149		249	3·83	349		449	
51	3·17	151		251		351	3 ² ·13	451	11·41
53		153	3 ² ·17	253	11·23	353		453	3·151
57	3·19	157		257		357	3·7·17	457	
59		159	3·53	259	7·37	359		459	3 ² ·17
61		161	7·23	261	3 ² ·29	361	19 ²	461	
63	3 ² ·7	163		263		363	3·11 ²	463	
67		167		267	3·89	367		467	
69	3·23	169	13 ²	269		369	3 ² ·41	469	7·67
71		171	3 ² ·19	271		371	7·53	471	3·157
73		173		273	3·7·13	373		473	11·43
77	7·11	177	3·59	277		377	13·29	477	3 ² ·53
79		179		279	3 ² ·31	379		479	
81	3 ⁴	181		281		381	3·127	481	13·37
83		183	3·61	283		383		483	3·7·23
87	3·29	187	11·17	287	7·41	387	3 ² ·43	487	
89		189	3 ³ ·7	289	17 ²	389		489	3·163
91	7·13	191		291	3·97	391	17·23	491	
93	3·31	193		293		393	3·131	493	17·29
97		197		297	3 ³ ·11	397		497	7·71
99	3 ² ·11	199		299	13·23	399	3·7·19	499	

PRIME FACTORS OF ALL ODD NUMBERS FROM 1 TO 1,000
THAT ARE NOT DIVISIBLE BY 5.

(Continued).

501	3·167	601		701		801	3 ² ·89	901	17·53
503		603	3 ² ·67	703	19·37	803	11·73	903	3·7·43
507	3·13 ²	607		707	7·101	807	3·269	907	
509		609	3·7·29	709		809		909	3 ² ·101
511	7·73	611	13·47	711	3 ² ·79	811		911	
513	3 ² ·19	613		713	23·31	813	3·271	913	11·83
517	11·47	617		717	3·239	817	19·43	917	7·131
519	3·173	619		719		819	3 ² ·7·13	919	
521		621	3 ³ ·23	721	7·103	821		921	3·307
523		623	7·89	723	3·241	823		923	13·71
527	17·31	627	3·11·19	727		827		927	3 ² ·103
529	23 ²	629	17·37	729	3 ⁶	829		929	
531	3 ² ·59	631		731	17·43	831	3·277	931	7 ² ·19
533	13·41	633	3·211	733		833	7 ² ·17	933	3·311
537	3·179	637	7 ² ·13	737	11·67	837	3 ² ·31	937	
539	7 ² ·11	639	3 ² ·71	739		839		939	3·313
541		641		741	3·13·19	841	29 ²	941	
543	3·181	643		743		843	3·281	943	23·41
547		647		747	3 ² ·83	847	7·11 ²	947	
549	3 ² ·61	649	11·59	749	7·107	849	3·233	949	13·73
551	19·29	651	3·7·31	751		851	23·37	951	3·317
553	7·79	653		753	3·251	853		953	
557		657	3 ² ·73	757		857		957	3·11·29
559	13·43	659		759	3·11·23	859		959	7·137
561	3·11·17	661		761		861	3·7·41	961	31 ²
563		663	3·13·17	763	7·109	863		963	3 ² ·107
567	3 ⁴ ·7	667	23·29	767	13·59	867	3·17 ²	967	
569		669	3·223	769		869	11·79	969	3·17·19
571		671	11·61	771	3·257	871	13·67	971	
573	3·191	673		773		873	3 ² ·97	973	7·139
577		677		777	3·7·37	877		977	
579	3·193	679	7·97	779	19·41	879	3·293	979	11·89
581	7·83	681	3·227	781	11·71	881		981	3 ² ·109
583	11·53	683		783	3 ³ ·29	883		983	
587		687	3·229	787		887		987	3·7·47
589	19·31	689	13·53	789	3·263	889	7·127	989	23·43
591	3·197	691		791	7·113	891	3 ⁴ ·11	991	
593		693	3 ² ·7·11	793	13·61	893	19·47	993	3·331
597	3·199	697	17·41	797		897	3·13·23	997	
599		699	3·233	799	17·47	899	29·31	999	3 ³ ·37

CIRCUMFERENCES AND AREAS OF CIRCLES FROM 1-64 TO 100.

Diam.	Circum.	Area.	Diam.	Circum.	Area.
$\frac{1}{8}$	.0491	.0002	$\frac{48}{8}$	13.7445	15.0390
$\frac{1}{4}$	.0982	.0008	$\frac{49}{8}$	14.1872	15.9043
$\frac{3}{8}$	.1963	.0031	$\frac{50}{8}$	14.5299	16.8002
$\frac{1}{2}$	.3927	.0123	$\frac{51}{8}$	14.9226	17.7206
$\frac{5}{8}$	.5890	.0276	$\frac{52}{8}$	15.3153	18.6555
$\frac{3}{4}$	.7854	.0491	$\frac{53}{8}$	15.7080	19.6350
$\frac{7}{8}$	.9817	.0767	$\frac{54}{8}$	16.1007	20.6290
$\frac{1}{2}$	1.1781	.1104	$\frac{55}{8}$	16.4934	21.6476
$\frac{1}{2}$	1.3744	.1503	$\frac{56}{8}$	16.8861	22.6907
$\frac{1}{2}$	1.5708	.1963	$\frac{57}{8}$	17.2788	23.7583
$\frac{1}{2}$	1.7671	.2485	$\frac{58}{8}$	17.6715	24.8506
$\frac{1}{2}$	1.9635	.3068	$\frac{59}{8}$	18.0642	25.9673
$\frac{1}{2}$	2.1598	.3712	$\frac{60}{8}$	18.4569	27.1086
$\frac{1}{2}$	2.3562	.4418	6	18.8496	28.2744
$\frac{1}{2}$	2.5525	.5185	$\frac{61}{8}$	19.2423	29.4648
$\frac{1}{2}$	2.7489	.6013	$\frac{62}{8}$	19.6350	30.6797
$\frac{1}{2}$	2.9452	.6903	$\frac{63}{8}$	20.0277	31.9191
1	3.1416	.7854	$\frac{64}{8}$	20.4204	33.1831
$\frac{1}{2}$	3.5343	.9940	$\frac{65}{8}$	20.8131	34.4717
$\frac{1}{2}$	3.9270	1.2272	$\frac{66}{8}$	21.2058	35.7848
$\frac{1}{2}$	4.3197	1.4849	$\frac{67}{8}$	21.5985	37.1224
$\frac{1}{2}$	4.7124	1.7671	7	21.9912	38.4846
$\frac{1}{2}$	5.1051	2.0739	$\frac{71}{8}$	22.3839	39.8713
$\frac{1}{2}$	5.4978	2.4053	$\frac{72}{8}$	22.7766	41.2826
$\frac{1}{2}$	5.8905	2.7612	$\frac{73}{8}$	23.1693	42.7184
2	6.2832	3.1416	$\frac{74}{8}$	23.5620	44.1787
$\frac{2}{2}$	6.6759	3.5466	$\frac{75}{8}$	23.9547	45.6636
$\frac{2}{2}$	7.0686	3.9761	$\frac{76}{8}$	24.3474	47.1731
$\frac{2}{2}$	7.4613	4.4301	$\frac{77}{8}$	24.7401	48.7071
$\frac{2}{2}$	7.8540	4.9087	8	25.1328	50.2656
$\frac{2}{2}$	8.2467	5.4119	$\frac{81}{8}$	25.5255	51.8487
$\frac{2}{2}$	8.6394	5.9396	$\frac{82}{8}$	25.9182	53.4563
$\frac{2}{2}$	9.0321	6.4918	$\frac{83}{8}$	26.3109	55.0884
3	9.4248	7.0686	$\frac{84}{8}$	26.7036	56.7451
$\frac{3}{2}$	9.8175	7.6699	$\frac{85}{8}$	27.0963	58.4264
$\frac{3}{2}$	10.2102	8.2958	$\frac{86}{8}$	27.4890	60.1322
$\frac{3}{2}$	10.6029	8.9462	$\frac{87}{8}$	27.8817	61.8625
$\frac{3}{2}$	10.9956	9.6211	9	28.2744	63.6174
$\frac{3}{2}$	11.3883	10.3206	$\frac{91}{8}$	28.6671	65.3968
$\frac{3}{2}$	11.7810	11.0447	$\frac{92}{8}$	29.0598	67.2008
$\frac{3}{2}$	12.1737	11.7933	$\frac{93}{8}$	29.4525	69.0293
4	12.5664	12.5664	$\frac{94}{8}$	29.8452	70.8823
$\frac{4}{2}$	12.9591	13.3641	$\frac{95}{8}$	30.2379	72.7599
$\frac{4}{2}$	13.3518	14.1863	$\frac{96}{8}$	30.6306	74.6621

TABLE—(Continued).

Diam.	Circum.	Area.	Diam.	Circum.	Area.
9 $\frac{7}{8}$	31.0233	76.589	15 $\frac{5}{8}$	49.0875	191.748
10	31.4160	78.540	15 $\frac{3}{4}$	49.4802	194.828
10 $\frac{1}{8}$	31.8087	80.516	15 $\frac{1}{2}$	49.8729	197.933
10 $\frac{1}{4}$	32.2014	82.516	16	50.2656	201.062
10 $\frac{3}{8}$	32.5941	84.541	16 $\frac{1}{8}$	50.6583	204.216
10 $\frac{1}{2}$	32.9868	86.590	16 $\frac{1}{4}$	51.0510	207.396
10 $\frac{3}{4}$	33.3795	88.664	16 $\frac{3}{8}$	51.4437	210.598
10 $\frac{7}{8}$	33.7722	90.763	16 $\frac{1}{2}$	51.8364	213.825
11	34.1649	92.886	16 $\frac{3}{4}$	52.2291	217.077
11 $\frac{1}{8}$	34.5576	95.033	16 $\frac{5}{8}$	52.6218	220.354
11 $\frac{1}{4}$	34.9503	97.205	16 $\frac{3}{4}$	53.0145	223.655
11 $\frac{3}{8}$	35.3430	99.402	17	53.4072	226.981
11 $\frac{1}{2}$	35.7357	101.623	17 $\frac{1}{8}$	53.7999	230.331
11 $\frac{3}{4}$	36.1284	103.869	17 $\frac{1}{4}$	54.1926	233.706
11 $\frac{7}{8}$	36.5211	106.139	17 $\frac{3}{8}$	54.5853	237.106
12	36.9138	108.434	17 $\frac{1}{2}$	54.9780	240.529
12 $\frac{1}{8}$	37.3065	110.754	17 $\frac{3}{4}$	55.3707	243.977
12 $\frac{1}{4}$	37.6992	113.098	17 $\frac{5}{8}$	55.7634	247.450
12 $\frac{3}{8}$	38.0919	115.466	17 $\frac{1}{2}$	56.1561	250.948
12 $\frac{1}{2}$	38.4846	117.859	18	56.5488	254.470
12 $\frac{3}{4}$	38.8773	120.277	18 $\frac{1}{8}$	56.9415	258.016
12 $\frac{7}{8}$	39.2700	122.719	18 $\frac{1}{4}$	57.3342	261.587
13	39.6627	125.185	18 $\frac{3}{8}$	57.7269	265.183
13 $\frac{1}{8}$	40.0554	127.677	18 $\frac{1}{2}$	58.1196	268.803
13 $\frac{1}{4}$	40.4481	130.192	18 $\frac{3}{4}$	58.5123	272.448
13 $\frac{3}{8}$	40.8408	132.733	18 $\frac{5}{8}$	58.9050	276.117
13 $\frac{1}{2}$	41.2335	135.297	18 $\frac{3}{4}$	59.2977	279.811
13 $\frac{3}{4}$	41.6262	137.887	19	59.6904	283.529
14	42.0189	140.501	19 $\frac{1}{8}$	60.0831	287.272
14 $\frac{1}{8}$	42.4116	143.139	19 $\frac{1}{4}$	60.4758	291.040
14 $\frac{1}{4}$	42.8043	145.802	19 $\frac{3}{8}$	60.8685	294.832
14 $\frac{3}{8}$	43.1970	148.490	19 $\frac{1}{2}$	61.2612	298.648
14 $\frac{1}{2}$	43.5897	151.202	19 $\frac{3}{4}$	61.6539	302.489
14 $\frac{3}{4}$	43.9824	153.938	19 $\frac{5}{8}$	62.0466	306.355
15	44.3751	156.700	19 $\frac{1}{2}$	62.4393	310.245
15 $\frac{1}{8}$	44.7678	159.485	20	62.8320	314.160
15 $\frac{1}{4}$	45.1605	162.296	20 $\frac{1}{8}$	63.2247	318.099
15 $\frac{3}{8}$	45.5532	165.130	20 $\frac{1}{4}$	63.6174	322.063
15 $\frac{1}{2}$	45.9459	167.990	20 $\frac{3}{8}$	64.0101	326.051
15 $\frac{3}{4}$	46.3386	170.874	20 $\frac{1}{2}$	64.4028	330.064
16	46.7313	173.782	20 $\frac{3}{4}$	64.7955	334.102
16 $\frac{1}{8}$	47.1240	176.715	20 $\frac{5}{8}$	65.1882	338.164
16 $\frac{1}{4}$	47.5167	179.673	20 $\frac{3}{4}$	65.5809	342.250
16 $\frac{3}{8}$	47.9094	182.655	21	65.9736	346.361
16 $\frac{1}{2}$	48.3021	185.661	21 $\frac{1}{8}$	66.3663	350.497
16 $\frac{3}{4}$	48.6948	188.692	21 $\frac{1}{4}$	66.7590	354.657

TABLE—(Continued).

Diam.	Circum.	Area.	Diam.	Circum.	Area.
21 $\frac{3}{8}$	67.1517	358.842	27 $\frac{1}{8}$	85.2159	577.870
21 $\frac{1}{2}$	67.5444	363.051	27 $\frac{1}{4}$	85.6086	583.209
21 $\frac{5}{8}$	67.9371	367.285	27 $\frac{3}{8}$	86.0013	588.571
21 $\frac{7}{8}$	68.3298	371.543	27 $\frac{1}{2}$	86.3940	593.959
22	68.7225	375.826	27 $\frac{5}{8}$	86.7867	599.371
22 $\frac{1}{8}$	69.1152	380.134	27 $\frac{3}{4}$	87.1794	604.807
22 $\frac{1}{4}$	69.5079	384.466	27 $\frac{7}{8}$	87.5721	610.268
22 $\frac{3}{8}$	69.9006	388.822	28	87.9648	615.754
22 $\frac{5}{8}$	70.2933	393.203	28 $\frac{1}{8}$	88.3575	621.264
22 $\frac{7}{8}$	70.6860	397.609	28 $\frac{1}{4}$	88.7502	626.798
22 $\frac{9}{8}$	71.0787	402.038	28 $\frac{3}{8}$	89.1429	632.357
22 $\frac{5}{4}$	71.4714	406.494	28 $\frac{1}{2}$	89.5356	637.941
22 $\frac{7}{4}$	71.8641	410.973	28 $\frac{3}{4}$	89.9283	643.549
23	72.2568	415.477	28 $\frac{5}{8}$	90.3210	649.182
23 $\frac{1}{8}$	72.6495	420.004	28 $\frac{7}{8}$	90.7137	654.840
23 $\frac{1}{4}$	73.0422	424.558	29	91.1064	660.521
23 $\frac{3}{8}$	73.4349	429.135	29 $\frac{1}{8}$	91.4991	666.228
23 $\frac{5}{8}$	73.8276	433.737	29 $\frac{1}{4}$	91.8918	671.959
23 $\frac{7}{8}$	74.2203	438.364	29 $\frac{3}{8}$	92.2845	677.714
23 $\frac{9}{8}$	74.6130	443.015	29 $\frac{1}{2}$	92.6772	683.494
23 $\frac{5}{4}$	75.0057	447.690	29 $\frac{5}{8}$	93.0699	689.299
24	75.3984	452.390	29 $\frac{3}{4}$	93.4626	695.128
24 $\frac{1}{8}$	75.7911	457.115	29 $\frac{7}{8}$	93.8553	700.982
24 $\frac{1}{4}$	76.1838	461.864	30	94.2480	706.860
24 $\frac{3}{8}$	76.5765	466.638	30 $\frac{1}{8}$	94.6407	712.763
24 $\frac{5}{8}$	76.9692	471.436	30 $\frac{1}{4}$	95.0334	718.690
24 $\frac{7}{8}$	77.3619	476.259	30 $\frac{3}{8}$	95.4261	724.642
24 $\frac{9}{8}$	77.7546	481.107	30 $\frac{1}{2}$	95.8188	730.618
24 $\frac{5}{4}$	78.1473	485.979	30 $\frac{5}{8}$	96.2115	736.619
25	78.5400	490.875	30 $\frac{3}{4}$	96.6042	742.645
25 $\frac{1}{8}$	78.9327	495.796	30 $\frac{7}{8}$	96.9969	748.695
25 $\frac{1}{4}$	79.3254	500.742	31	97.3896	754.769
25 $\frac{3}{8}$	79.7181	505.712	31 $\frac{1}{8}$	97.7823	760.869
25 $\frac{5}{8}$	80.1108	510.706	31 $\frac{1}{4}$	98.1750	766.992
25 $\frac{7}{8}$	80.5035	515.726	31 $\frac{3}{8}$	98.5677	773.140
25 $\frac{9}{8}$	80.8962	520.769	31 $\frac{1}{2}$	98.9604	779.313
25 $\frac{5}{4}$	81.2889	525.838	31 $\frac{5}{8}$	99.3531	785.510
26	81.6816	530.930	31 $\frac{3}{4}$	99.7458	791.732
26 $\frac{1}{8}$	82.0743	536.048	31 $\frac{7}{8}$	100.1385	797.979
26 $\frac{1}{4}$	82.4670	541.190	32	100.5312	804.250
26 $\frac{3}{8}$	82.8597	546.356	32 $\frac{1}{8}$	100.9239	810.545
26 $\frac{5}{8}$	83.2524	551.547	32 $\frac{1}{4}$	101.3166	816.865
26 $\frac{7}{8}$	83.6451	556.763	32 $\frac{3}{8}$	101.7093	823.210
26 $\frac{9}{8}$	84.0378	562.003	32 $\frac{1}{2}$	102.1020	829.579
26 $\frac{5}{4}$	84.4305	567.267	32 $\frac{5}{8}$	102.4947	835.972
27	84.8232	572.557	32 $\frac{3}{4}$	102.8874	842.391

TABLE—(Continued).

Diam.	Circum.	Area.	Diam.	Circum.	Area.
$32\frac{7}{8}$	103.290	848.833	$38\frac{5}{8}$	121.344	1,171.731
33	103.673	855.301	$38\frac{3}{4}$	121.737	1,179.327
$33\frac{1}{8}$	104.065	861.792	$38\frac{1}{2}$	122.130	1,186.948
$33\frac{1}{4}$	104.458	868.309	39	122.522	1,194.593
$33\frac{3}{8}$	104.851	874.850	$39\frac{1}{8}$	122.915	1,202.263
$33\frac{1}{2}$	105.244	881.415	$39\frac{1}{4}$	123.308	1,209.958
$33\frac{5}{8}$	105.636	888.005	$39\frac{3}{8}$	123.700	1,217.677
$33\frac{3}{4}$	106.029	894.620	$39\frac{1}{2}$	124.093	1,225.420
$33\frac{7}{8}$	106.422	901.259	$39\frac{3}{4}$	124.486	1,233.188
34	106.814	907.922	$39\frac{5}{8}$	124.879	1,240.981
$34\frac{1}{8}$	107.207	914.611	$39\frac{3}{2}$	125.271	1,248.798
$34\frac{1}{4}$	107.600	921.323	40	125.664	1,256.640
$34\frac{3}{8}$	107.992	928.061	$40\frac{1}{8}$	126.057	1,264.510
$34\frac{1}{2}$	108.385	934.822	$40\frac{1}{4}$	126.449	1,272.400
$34\frac{5}{8}$	108.778	941.609	$40\frac{3}{8}$	126.842	1,280.310
$34\frac{3}{4}$	109.171	948.420	$40\frac{1}{2}$	127.235	1,288.250
$34\frac{7}{8}$	109.563	955.255	$40\frac{3}{4}$	127.627	1,296.220
35	109.956	962.115	$40\frac{5}{8}$	128.020	1,304.210
$35\frac{1}{8}$	110.349	969.000	$40\frac{3}{2}$	128.413	1,312.220
$35\frac{1}{4}$	110.741	975.909	41	128.806	1,320.260
$35\frac{3}{8}$	111.134	982.842	$41\frac{1}{8}$	129.198	1,328.320
$35\frac{1}{2}$	111.527	989.800	$41\frac{1}{4}$	129.591	1,336.410
$35\frac{5}{8}$	111.919	996.783	$41\frac{3}{8}$	129.984	1,344.520
$35\frac{3}{4}$	112.312	1,003.790	$41\frac{1}{2}$	130.376	1,352.660
$35\frac{7}{8}$	112.705	1,010.822	$41\frac{3}{4}$	130.769	1,360.820
36	113.098	1,017.878	$41\frac{5}{8}$	131.162	1,369.000
$36\frac{1}{8}$	113.490	1,024.960	$41\frac{3}{2}$	131.554	1,377.210
$36\frac{1}{4}$	113.883	1,032.065	42	131.947	1,385.450
$36\frac{3}{8}$	114.276	1,039.195	$42\frac{1}{8}$	132.340	1,393.700
$36\frac{1}{2}$	114.668	1,046.349	$42\frac{1}{4}$	132.733	1,401.990
$36\frac{5}{8}$	115.061	1,053.528	$42\frac{3}{8}$	133.125	1,410.300
$36\frac{3}{4}$	115.454	1,060.732	$42\frac{1}{2}$	133.518	1,418.630
$36\frac{7}{8}$	115.846	1,067.960	$42\frac{3}{4}$	133.911	1,426.990
37	116.239	1,075.213	$42\frac{5}{8}$	134.303	1,435.370
$37\frac{1}{8}$	116.632	1,082.490	$42\frac{3}{2}$	134.696	1,443.770
$37\frac{1}{4}$	117.025	1,089.792	43	135.089	1,452.200
$37\frac{3}{8}$	117.417	1,097.118	$43\frac{1}{8}$	135.481	1,460.660
$37\frac{1}{2}$	117.810	1,104.469	$43\frac{1}{4}$	135.874	1,469.140
$37\frac{5}{8}$	118.203	1,111.844	$43\frac{3}{8}$	136.267	1,477.640
$37\frac{3}{4}$	118.595	1,119.244	$43\frac{1}{2}$	136.660	1,486.170
$37\frac{7}{8}$	118.988	1,126.669	$43\frac{3}{4}$	137.052	1,494.730
38	119.381	1,134.118	$43\frac{5}{8}$	137.445	1,503.300
$38\frac{1}{8}$	119.773	1,141.591	$43\frac{3}{2}$	137.838	1,511.910
$38\frac{1}{4}$	120.166	1,149.089	44	138.230	1,520.530
$38\frac{3}{8}$	120.559	1,156.612	$44\frac{1}{8}$	138.623	1,529.190
$38\frac{1}{2}$	120.952	1,164.159	$44\frac{1}{4}$	139.016	1,537.860

TABLE—(Continued).

Diam.	Circum.	Area.	Diam.	Circum.	Area.
44 $\frac{3}{8}$	139.408	1,546.56	50 $\frac{1}{8}$	157.473	1,978.33
44 $\frac{1}{2}$	139.801	1,555.29	50 $\frac{1}{4}$	157.865	1,983.18
44 $\frac{5}{8}$	140.194	1,564.04	50 $\frac{3}{8}$	158.258	1,993.06
44 $\frac{7}{8}$	140.587	1,572.81	50 $\frac{1}{2}$	158.651	2,002.97
45	140.979	1,581.61	50 $\frac{5}{8}$	159.043	2,012.89
45 $\frac{1}{8}$	141.372	1,590.43	50 $\frac{3}{4}$	159.436	2,022.85
45 $\frac{1}{4}$	141.765	1,599.28	50 $\frac{7}{8}$	159.829	2,032.82
45 $\frac{3}{8}$	142.157	1,608.16	51	160.222	2,042.83
45 $\frac{1}{2}$	142.550	1,617.05	51 $\frac{1}{8}$	160.614	2,052.85
45 $\frac{3}{4}$	142.943	1,625.97	51 $\frac{1}{4}$	161.007	2,062.90
45 $\frac{5}{8}$	143.335	1,634.92	51 $\frac{3}{8}$	161.400	2,072.98
45 $\frac{7}{8}$	143.728	1,643.89	51 $\frac{1}{2}$	161.792	2,083.08
46	144.121	1,652.89	51 $\frac{3}{4}$	162.185	2,093.20
46 $\frac{1}{8}$	144.514	1,661.91	51 $\frac{5}{8}$	162.578	2,103.35
46 $\frac{1}{4}$	144.906	1,670.95	51 $\frac{7}{8}$	162.970	2,113.52
46 $\frac{3}{8}$	145.299	1,680.02	52	163.363	2,123.72
46 $\frac{1}{2}$	145.692	1,689.11	52 $\frac{1}{8}$	163.756	2,133.94
46 $\frac{3}{4}$	146.084	1,698.23	52 $\frac{1}{4}$	164.149	2,144.19
46 $\frac{5}{8}$	146.477	1,707.37	52 $\frac{3}{8}$	164.541	2,154.46
46 $\frac{7}{8}$	146.870	1,716.54	52 $\frac{1}{2}$	164.934	2,164.76
47	147.262	1,725.73	52 $\frac{3}{4}$	165.327	2,175.08
47 $\frac{1}{8}$	147.655	1,734.95	52 $\frac{5}{8}$	165.719	2,185.42
47 $\frac{1}{4}$	148.048	1,744.19	52 $\frac{7}{8}$	166.112	2,195.79
47 $\frac{3}{8}$	148.441	1,753.45	53	166.505	2,206.19
47 $\frac{1}{2}$	148.833	1,762.74	53 $\frac{1}{8}$	166.897	2,216.61
47 $\frac{3}{4}$	149.226	1,772.06	53 $\frac{1}{4}$	167.290	2,227.05
47 $\frac{5}{8}$	149.619	1,781.40	53 $\frac{3}{8}$	167.683	2,237.52
47 $\frac{7}{8}$	150.011	1,790.76	53 $\frac{1}{2}$	168.076	2,248.01
48	150.404	1,800.15	53 $\frac{3}{4}$	168.468	2,258.53
48 $\frac{1}{8}$	150.797	1,809.56	53 $\frac{5}{8}$	168.861	2,269.07
48 $\frac{1}{4}$	151.189	1,819.00	53 $\frac{7}{8}$	169.254	2,279.64
48 $\frac{3}{8}$	151.582	1,828.46	54	169.646	2,290.23
48 $\frac{1}{2}$	151.975	1,837.95	54 $\frac{1}{8}$	170.039	2,300.84
48 $\frac{3}{4}$	152.368	1,847.46	54 $\frac{1}{4}$	170.432	2,311.48
48 $\frac{5}{8}$	152.760	1,856.99	54 $\frac{3}{8}$	170.824	2,322.15
48 $\frac{7}{8}$	153.153	1,866.55	54 $\frac{1}{2}$	171.217	2,332.83
49	153.546	1,876.14	54 $\frac{3}{4}$	171.610	2,343.55
49 $\frac{1}{8}$	153.938	1,885.75	54 $\frac{5}{8}$	172.003	2,354.29
49 $\frac{1}{4}$	154.331	1,895.38	54 $\frac{7}{8}$	172.395	2,365.05
49 $\frac{3}{8}$	154.724	1,905.04	55	172.788	2,375.83
49 $\frac{1}{2}$	155.116	1,914.72	55 $\frac{1}{8}$	173.181	2,386.65
49 $\frac{3}{4}$	155.509	1,924.43	55 $\frac{1}{4}$	173.573	2,397.48
49 $\frac{5}{8}$	155.902	1,934.16	55 $\frac{3}{8}$	173.966	2,408.34
49 $\frac{7}{8}$	156.295	1,943.91	55 $\frac{1}{2}$	174.359	2,419.23
50	156.687	1,953.69	55 $\frac{3}{4}$	174.751	2,430.14
	157.080	1,963.50	55 $\frac{5}{8}$	175.144	2,441.07

TABLE—(Continued).

Diam.	Circum.	Area.	Diam.	Circum.	Area.
55 $\frac{7}{8}$	175.537	2,452.03	61 $\frac{5}{8}$	193.601	2,982.67
56	175.930	2,463.01	61 $\frac{3}{4}$	193.994	2,994.78
56 $\frac{1}{8}$	176.322	2,474.02	61 $\frac{1}{2}$	194.386	3,006.92
56 $\frac{1}{4}$	176.715	2,485.05	62	194.779	3,019.08
56 $\frac{3}{8}$	177.108	2,496.11	62 $\frac{1}{8}$	195.172	3,031.26
56 $\frac{1}{2}$	177.500	2,507.19	62 $\frac{1}{4}$	195.565	3,043.47
56 $\frac{5}{8}$	177.893	2,518.30	62 $\frac{3}{8}$	195.957	3,055.71
56 $\frac{3}{4}$	178.286	2,529.43	62 $\frac{1}{2}$	196.350	3,067.97
56 $\frac{7}{8}$	178.678	2,540.58	62 $\frac{5}{8}$	196.743	3,080.25
57	179.071	2,551.76	62 $\frac{3}{4}$	197.135	3,092.56
57 $\frac{1}{8}$	179.464	2,562.97	62 $\frac{7}{8}$	197.528	3,104.89
57 $\frac{1}{4}$	179.857	2,574.20	63	197.921	3,117.25
57 $\frac{3}{8}$	180.249	2,585.45	63 $\frac{1}{8}$	198.313	3,129.64
57 $\frac{1}{2}$	180.642	2,596.73	63 $\frac{1}{4}$	198.706	3,142.04
57 $\frac{5}{8}$	181.035	2,608.03	63 $\frac{3}{8}$	199.099	3,154.47
57 $\frac{3}{4}$	181.427	2,619.36	63 $\frac{1}{2}$	199.492	3,166.93
57 $\frac{7}{8}$	181.820	2,630.71	63 $\frac{5}{8}$	199.884	3,179.41
58	182.213	2,642.09	63 $\frac{3}{4}$	200.277	3,191.91
58 $\frac{1}{8}$	182.605	2,653.49	63 $\frac{7}{8}$	200.670	3,204.44
58 $\frac{1}{4}$	182.998	2,664.91	64	201.062	3,217.00
58 $\frac{3}{8}$	183.391	2,676.36	64 $\frac{1}{8}$	201.455	3,229.58
58 $\frac{1}{2}$	183.784	2,687.84	64 $\frac{1}{4}$	201.848	3,242.18
58 $\frac{5}{8}$	184.176	2,699.33	64 $\frac{3}{8}$	202.240	3,254.81
58 $\frac{3}{4}$	184.569	2,710.86	64 $\frac{1}{2}$	202.633	3,267.46
58 $\frac{7}{8}$	184.962	2,722.41	64 $\frac{5}{8}$	203.026	3,280.14
59	185.354	2,733.98	64 $\frac{3}{4}$	203.419	3,292.84
59 $\frac{1}{8}$	185.747	2,745.57	64 $\frac{7}{8}$	203.811	3,305.56
59 $\frac{1}{4}$	186.140	2,757.20	65	204.204	3,318.31
59 $\frac{3}{8}$	186.532	2,768.84	65 $\frac{1}{8}$	204.597	3,331.09
59 $\frac{1}{2}$	186.925	2,780.51	65 $\frac{1}{4}$	204.989	3,343.89
59 $\frac{5}{8}$	187.318	2,792.21	65 $\frac{3}{8}$	205.382	3,356.71
59 $\frac{3}{4}$	187.711	2,803.93	65 $\frac{1}{2}$	205.775	3,369.56
59 $\frac{7}{8}$	188.103	2,815.67	65 $\frac{5}{8}$	206.167	3,382.44
60	188.496	2,827.44	65 $\frac{3}{4}$	206.560	3,395.33
60 $\frac{1}{8}$	188.889	2,839.23	65 $\frac{7}{8}$	206.953	3,408.26
60 $\frac{1}{4}$	189.281	2,851.05	66	207.346	3,421.20
60 $\frac{3}{8}$	189.674	2,862.89	66 $\frac{1}{8}$	207.738	3,434.17
60 $\frac{1}{2}$	190.067	2,874.76	66 $\frac{1}{4}$	208.131	3,447.17
60 $\frac{5}{8}$	190.459	2,886.65	66 $\frac{3}{8}$	208.524	3,460.19
60 $\frac{3}{4}$	190.852	2,898.57	66 $\frac{1}{2}$	208.916	3,473.24
60 $\frac{7}{8}$	191.245	2,910.51	66 $\frac{5}{8}$	209.309	3,486.30
61	191.638	2,922.47	66 $\frac{3}{4}$	209.702	3,499.40
61 $\frac{1}{8}$	192.030	2,934.46	66 $\frac{7}{8}$	210.094	3,512.52
61 $\frac{1}{4}$	192.423	2,946.48	67	210.487	3,525.66
61 $\frac{3}{8}$	192.816	2,958.52	67 $\frac{1}{8}$	210.880	3,538.83
61 $\frac{1}{2}$	193.208	2,970.58	67 $\frac{1}{4}$	211.273	3,552.02

TABLE—(Continued).

Diam.	Circum.	Area.	Diam.	Circum.	Area.
67 $\frac{3}{8}$	211.665	3,565.24	73 $\frac{1}{8}$	229.729	4,199.74
67 $\frac{1}{2}$	212.058	3,578.48	73 $\frac{1}{4}$	230.122	4,214.11
67 $\frac{5}{8}$	212.451	3,591.74	73 $\frac{3}{8}$	230.515	4,228.51
67 $\frac{7}{8}$	212.843	3,605.04	73 $\frac{1}{2}$	230.908	4,242.93
68	213.236	3,618.35	73 $\frac{3}{4}$	231.300	4,257.37
68 $\frac{1}{8}$	213.629	3,631.69	73 $\frac{5}{8}$	231.693	4,271.84
68 $\frac{1}{4}$	214.021	3,645.05	73 $\frac{7}{8}$	232.086	4,286.33
68 $\frac{3}{8}$	214.414	3,658.44	74	232.478	4,300.85
68 $\frac{1}{2}$	214.807	3,671.86	74 $\frac{1}{8}$	232.871	4,315.39
68 $\frac{3}{4}$	215.200	3,685.29	74 $\frac{1}{4}$	233.264	4,329.96
68 $\frac{5}{8}$	215.592	3,698.76	74 $\frac{3}{8}$	233.656	4,344.55
68 $\frac{3}{4}$	215.985	3,712.24	74 $\frac{1}{2}$	234.049	4,359.17
68 $\frac{7}{8}$	216.378	3,725.75	74 $\frac{3}{4}$	234.442	4,373.81
69	216.770	3,739.29	74 $\frac{5}{8}$	234.835	4,388.47
69 $\frac{1}{8}$	217.163	3,752.85	74 $\frac{3}{4}$	235.227	4,403.16
69 $\frac{1}{4}$	217.556	3,766.43	75	235.620	4,417.87
69 $\frac{3}{8}$	217.948	3,780.04	75 $\frac{1}{8}$	236.013	4,432.61
69 $\frac{1}{2}$	218.341	3,793.68	75 $\frac{1}{4}$	236.405	4,447.38
69 $\frac{3}{4}$	218.734	3,807.34	75 $\frac{3}{8}$	236.798	4,462.16
69 $\frac{5}{8}$	219.127	3,821.02	75 $\frac{1}{2}$	237.191	4,476.98
69 $\frac{7}{8}$	219.519	3,834.73	75 $\frac{3}{4}$	237.583	4,491.81
70	219.912	3,848.46	75 $\frac{5}{8}$	237.976	4,506.67
70 $\frac{1}{8}$	220.305	3,862.22	75 $\frac{3}{4}$	238.369	4,521.56
70 $\frac{1}{4}$	220.697	3,876.00	76	238.762	4,536.47
70 $\frac{3}{8}$	221.090	3,889.80	76 $\frac{1}{8}$	239.154	4,551.41
70 $\frac{1}{2}$	221.483	3,903.63	76 $\frac{1}{4}$	239.547	4,566.36
70 $\frac{3}{4}$	221.875	3,917.49	76 $\frac{3}{8}$	239.940	4,581.35
70 $\frac{5}{8}$	222.268	3,931.37	76 $\frac{1}{2}$	240.332	4,596.36
70 $\frac{7}{8}$	222.661	3,945.27	76 $\frac{3}{4}$	240.725	4,611.39
71	223.054	3,959.20	76 $\frac{5}{8}$	241.118	4,626.45
71 $\frac{1}{8}$	223.446	3,973.15	76 $\frac{3}{4}$	241.510	4,641.53
71 $\frac{1}{4}$	223.839	3,987.13	77	241.903	4,656.64
71 $\frac{3}{8}$	224.232	4,001.13	77 $\frac{1}{8}$	242.296	4,671.77
71 $\frac{1}{2}$	224.624	4,015.16	77 $\frac{1}{4}$	242.689	4,686.92
71 $\frac{3}{4}$	225.017	4,029.21	77 $\frac{3}{8}$	243.081	4,702.10
71 $\frac{5}{8}$	225.410	4,043.29	77 $\frac{1}{2}$	243.474	4,717.31
71 $\frac{7}{8}$	225.802	4,057.39	77 $\frac{3}{4}$	243.867	4,732.54
72	226.195	4,071.51	77 $\frac{5}{8}$	244.259	4,747.79
72 $\frac{1}{8}$	226.588	4,085.66	77 $\frac{3}{4}$	244.652	4,763.07
72 $\frac{1}{4}$	226.981	4,099.84	78	245.045	4,778.37
72 $\frac{3}{8}$	227.373	4,114.04	78 $\frac{1}{8}$	245.437	4,793.70
72 $\frac{1}{2}$	227.766	4,128.26	78 $\frac{1}{4}$	245.830	4,809.05
72 $\frac{3}{4}$	228.159	4,142.51	78 $\frac{3}{8}$	246.223	4,824.43
72 $\frac{5}{8}$	228.551	4,156.78	78 $\frac{1}{2}$	246.616	4,839.83
72 $\frac{7}{8}$	228.944	4,171.08	78 $\frac{3}{4}$	247.008	4,855.26
73	229.337	4,185.40	78 $\frac{5}{8}$	247.401	4,870.71

TABLE OF CIRCLES.

89

TABLE—(Continued).

Diam.	Circum.	Area.	Diam.	Circum.	Area.
787 $\frac{1}{8}$	247.794	4,886.18	845 $\frac{1}{8}$	265.858	5,624.56
79	248.186	4,901.68	845 $\frac{1}{4}$	266.251	5,641.18
791 $\frac{1}{8}$	248.579	4,917.21	845 $\frac{3}{8}$	266.643	5,657.84
791 $\frac{1}{4}$	248.972	4,932.75	85	267.036	5,674.51
793 $\frac{1}{8}$	249.364	4,948.33	851 $\frac{1}{8}$	267.429	5,691.22
793 $\frac{1}{4}$	249.757	4,963.92	851 $\frac{1}{4}$	267.821	5,707.94
795 $\frac{1}{8}$	250.150	4,979.55	853 $\frac{1}{8}$	268.214	5,724.69
795 $\frac{1}{4}$	250.543	4,995.19	853 $\frac{1}{4}$	268.607	5,741.47
797 $\frac{1}{8}$	250.935	5,010.86	853 $\frac{3}{8}$	268.999	5,758.27
80	251.328	5,026.56	853 $\frac{1}{2}$	269.392	5,775.10
801 $\frac{1}{8}$	251.721	5,042.28	855 $\frac{1}{8}$	269.785	5,791.94
801 $\frac{1}{4}$	252.113	5,058.03	86	270.178	5,808.82
803 $\frac{1}{8}$	252.506	5,073.79	861 $\frac{1}{8}$	270.570	5,825.72
803 $\frac{1}{4}$	252.899	5,089.59	861 $\frac{1}{4}$	270.963	5,842.64
805 $\frac{1}{8}$	253.291	5,105.41	863 $\frac{1}{8}$	271.356	5,859.59
805 $\frac{1}{4}$	253.684	5,121.25	863 $\frac{1}{4}$	271.748	5,876.56
807 $\frac{1}{8}$	254.077	5,137.12	863 $\frac{3}{8}$	272.141	5,893.55
81	254.470	5,153.01	863 $\frac{1}{2}$	272.534	5,910.58
811 $\frac{1}{8}$	254.862	5,168.93	867 $\frac{1}{8}$	272.926	5,927.62
811 $\frac{1}{4}$	255.255	5,184.87	87	273.319	5,944.69
813 $\frac{1}{8}$	255.648	5,200.83	871 $\frac{1}{8}$	273.712	5,961.79
813 $\frac{1}{4}$	256.040	5,216.82	871 $\frac{1}{4}$	274.105	5,978.91
815 $\frac{1}{8}$	256.433	5,232.84	873 $\frac{1}{8}$	274.497	5,996.05
815 $\frac{1}{4}$	256.826	5,248.88	873 $\frac{1}{4}$	274.890	6,013.22
817 $\frac{1}{8}$	257.218	5,264.94	873 $\frac{3}{8}$	275.283	6,030.41
82	257.611	5,281.03	873 $\frac{1}{2}$	275.675	6,047.63
821 $\frac{1}{8}$	258.004	5,297.14	877 $\frac{1}{8}$	276.068	6,064.87
821 $\frac{1}{4}$	258.397	5,313.28	88	276.461	6,082.14
823 $\frac{1}{8}$	258.789	5,329.44	881 $\frac{1}{8}$	276.853	6,099.43
823 $\frac{1}{4}$	259.182	5,345.63	881 $\frac{1}{4}$	277.246	6,116.74
825 $\frac{1}{8}$	259.575	5,361.84	883 $\frac{1}{8}$	277.629	6,134.08
825 $\frac{1}{4}$	259.967	5,378.08	883 $\frac{1}{4}$	278.032	6,151.45
827 $\frac{1}{8}$	260.360	5,394.34	883 $\frac{3}{8}$	278.424	6,168.84
83	260.753	5,410.62	883 $\frac{1}{2}$	278.817	6,186.25
831 $\frac{1}{8}$	261.145	5,426.93	887 $\frac{1}{8}$	279.210	6,203.69
831 $\frac{1}{4}$	261.538	5,443.26	89	279.602	6,221.15
833 $\frac{1}{8}$	261.931	5,459.62	891 $\frac{1}{8}$	279.995	6,238.64
833 $\frac{1}{4}$	262.324	5,476.01	891 $\frac{1}{4}$	280.388	6,256.15
835 $\frac{1}{8}$	262.716	5,492.41	893 $\frac{1}{8}$	280.780	6,273.69
835 $\frac{1}{4}$	263.109	5,508.84	893 $\frac{1}{4}$	281.173	6,291.25
837 $\frac{1}{8}$	263.502	5,525.30	893 $\frac{3}{8}$	281.566	6,308.84
84	263.894	5,541.78	893 $\frac{1}{2}$	281.959	6,326.45
841 $\frac{1}{8}$	264.287	5,558.29	897 $\frac{1}{8}$	282.351	6,344.08
841 $\frac{1}{4}$	264.680	5,574.82	90	282.744	6,361.74
843 $\frac{1}{8}$	265.072	5,591.37	901 $\frac{1}{8}$	283.137	6,379.42
843 $\frac{1}{4}$	265.465	5,607.95	901 $\frac{1}{4}$	283.529	6,397.13

TABLE—(Continued).

Diam.	Circum.	Area.	Diam.	Circum.	Area.
90 $\frac{3}{8}$	283.922	6,414.86	95 $\frac{1}{8}$	299.237	7,125.59
90 $\frac{1}{2}$	284.315	6,432.62	95 $\frac{3}{8}$	299.630	7,144.31
90 $\frac{5}{8}$	284.707	6,450.40	95 $\frac{1}{2}$	300.023	7,163.04
90 $\frac{3}{4}$	285.100	6,468.21	95 $\frac{3}{4}$	300.415	7,181.81
90 $\frac{7}{8}$	285.493	6,486.04	95 $\frac{5}{8}$	300.808	7,200.60
91	285.886	6,503.90	95 $\frac{7}{8}$	301.201	7,219.41
91 $\frac{1}{8}$	286.278	6,521.78	96	301.594	7,238.25
91 $\frac{1}{4}$	286.671	6,539.68	96 $\frac{1}{8}$	301.986	7,257.11
91 $\frac{3}{8}$	287.064	6,557.61	96 $\frac{1}{4}$	302.379	7,275.99
91 $\frac{1}{2}$	287.456	6,575.56	96 $\frac{3}{8}$	302.772	7,294.91
91 $\frac{3}{4}$	287.849	6,593.54	96 $\frac{1}{2}$	303.164	7,313.84
91 $\frac{5}{8}$	288.242	6,611.55	96 $\frac{3}{4}$	303.557	7,332.80
91 $\frac{7}{8}$	288.634	6,629.57	96 $\frac{5}{8}$	303.950	7,351.79
92	289.027	6,647.63	96 $\frac{7}{8}$	304.342	7,370.79
92 $\frac{1}{8}$	289.420	6,665.70	97	304.735	7,389.83
92 $\frac{1}{4}$	289.813	6,683.80	97 $\frac{1}{8}$	305.128	7,408.89
92 $\frac{3}{8}$	290.205	6,701.93	97 $\frac{1}{4}$	305.521	7,427.97
92 $\frac{1}{2}$	290.598	6,720.08	97 $\frac{3}{8}$	305.913	7,447.08
92 $\frac{3}{4}$	290.991	6,738.25	97 $\frac{1}{2}$	306.306	7,466.21
92 $\frac{5}{8}$	291.383	6,756.45	97 $\frac{3}{4}$	306.699	7,485.37
92 $\frac{7}{8}$	291.776	6,774.68	97 $\frac{5}{8}$	307.091	7,504.55
93	292.169	6,792.92	97 $\frac{7}{8}$	307.484	7,523.75
93 $\frac{1}{8}$	292.562	6,811.20	98	307.877	7,542.98
93 $\frac{1}{4}$	292.954	6,829.49	98 $\frac{1}{8}$	308.270	7,562.24
93 $\frac{3}{8}$	293.347	6,847.82	98 $\frac{1}{4}$	308.662	7,581.52
93 $\frac{1}{2}$	293.740	6,866.16	98 $\frac{3}{8}$	309.055	7,600.82
93 $\frac{3}{4}$	294.132	6,884.53	98 $\frac{1}{2}$	309.448	7,620.15
93 $\frac{5}{8}$	294.525	6,902.93	98 $\frac{3}{4}$	309.840	7,639.50
93 $\frac{7}{8}$	294.918	6,921.35	98 $\frac{5}{8}$	310.233	7,658.88
94	295.310	6,939.79	98 $\frac{7}{8}$	310.626	7,678.28
94 $\frac{1}{8}$	295.703	6,958.26	99	311.018	7,697.71
94 $\frac{1}{4}$	296.096	6,976.76	99 $\frac{1}{8}$	311.411	7,717.16
94 $\frac{3}{8}$	296.488	6,995.28	99 $\frac{1}{4}$	311.804	7,736.63
94 $\frac{1}{2}$	296.881	7,013.82	99 $\frac{3}{8}$	312.196	7,756.13
94 $\frac{3}{4}$	297.274	7,032.39	99 $\frac{1}{2}$	312.589	7,775.66
94 $\frac{5}{8}$	297.667	7,050.98	99 $\frac{3}{4}$	312.982	7,795.21
94 $\frac{7}{8}$	298.059	7,069.59	99 $\frac{5}{8}$	313.375	7,814.78
95	298.452	7,088.24	99 $\frac{7}{8}$	313.767	7,834.38
95 $\frac{1}{8}$	298.845	7,106.90	100	314.160	7,854.00

The preceding table may be used to determine the diameter when the circumference or area is known. Thus, the diameter of a circle having an area of 7,200 sq. in. is, approximately, 95 $\frac{1}{4}$ in.

DECIMAL EQUIVALENTS OF PARTS OF ONE INCH.

1-64	.015625	17-64	.265625	33-64	.515625	49-64	.765625
1-32	.031250	9-32	.281250	17-32	.531250	25-32	.781250
3-64	.046875	19-64	.296875	35-64	.546875	51-64	.796875
1-16	.062500	5-16	.312500	9-16	.562500	13-16	.812500
5-64	.078125	21-64	.328125	37-64	.578125	53-64	.828125
3-32	.093750	11-32	.343750	19-32	.593750	27-32	.843750
7-64	.109375	23-64	.359375	39-64	.609375	55-64	.859375
1-8	.125000	3-8	.375000	5-8	.625000	7-8	.875000
9-64	.140625	25-64	.390625	41-64	.640625	57-64	.890625
5-32	.156250	13-32	.406250	21-32	.656250	29-32	.906250
11-64	.171875	27-64	.421875	43-64	.671875	59-64	.921875
3-16	.187500	7-16	.437500	11-16	.687500	15-16	.937500
13-64	.203125	29-64	.453125	45-64	.703125	61-64	.953125
7-32	.218750	15-32	.468750	23-32	.718750	31-32	.968750
15-64	.234375	31-64	.484375	47-64	.734375	63-64	.984375
1-4	.250000	1-2	.500000	3-4	.750000	1	1

DECIMALS OF A FOOT FOR EACH 1-32 OF AN INCH.

Inch.	0'	1'	2'	3'	4'	5'
0	0	.0833	.1667	.2500	.3333	.4167
1/32	.0026	.0859	.1693	.2526	.3359	.4193
2/32	.0052	.0885	.1719	.2552	.3385	.4219
3/32	.0078	.0911	.1745	.2578	.3411	.4245
4/32	.0104	.0937	.1771	.2604	.3437	.4271
5/32	.0130	.0964	.1797	.2630	.3464	.4297
6/32	.0156	.0990	.1823	.2656	.3490	.4323
7/32	.0182	.1016	.1849	.2682	.3516	.4349
8/32	.0208	.1042	.1875	.2708	.3542	.4375
9/32	.0234	.1068	.1901	.2734	.3568	.4401
10/32	.0260	.1094	.1927	.2760	.3594	.4427
11/32	.0286	.1120	.1953	.2786	.3620	.4453
12/32	.0312	.1146	.1979	.2812	.3646	.4479
13/32	.0339	.1172	.2005	.2839	.3672	.4505
14/32	.0365	.1198	.2031	.2865	.3698	.4531
15/32	.0391	.1224	.2057	.2891	.3724	.4557
16/32	.0417	.1250	.2083	.2917	.3750	.4583
17/32	.0443	.1276	.2109	.2943	.3776	.4609
18/32	.0469	.1302	.2135	.2969	.3802	.4635
19/32	.0495	.1328	.2161	.2995	.3828	.4661
20/32	.0521	.1354	.2188	.3021	.3854	.4688
21/32	.0547	.1380	.2214	.3047	.3880	.4714
22/32	.0573	.1406	.2240	.3073	.3906	.4740
23/32	.0599	.1432	.2266	.3099	.3932	.4766

USEFUL TABLES.

TABLE—(Continued).

Inch.	0''	1''	2''	3''	4''	5''
$\frac{3}{16}$	.0625	.1458	.2292	.3125	.3958	.4792
$\frac{1}{4}$	.0651	.1484	.2318	.3151	.3984	.4818
$\frac{5}{16}$	.0677	.1510	.2344	.3177	.4010	.4844
$\frac{3}{8}$	.0703	.1536	.2370	.3203	.4036	.4870
$\frac{7}{16}$	.0729	.1562	.2396	.3229	.4063	.4896
$\frac{1}{2}$	.0755	.1589	.2422	.3255	.4089	.4922
$\frac{9}{16}$	.0781	.1615	.2448	.3281	.4115	.4948
$\frac{5}{8}$	.0807	.1641	.2474	.3307	.4141	.4974

DECIMALS OF A FOOT FOR EACH 1-32 OF AN INCH.

Inch.	6''	7''	8''	9''	10''	11''
0	.5000	.5833	.6667	.7500	.8333	.9167
$\frac{1}{32}$	.5026	.5859	.6693	.7526	.8359	.9193
$\frac{1}{16}$	.5052	.5885	.6719	.7552	.8385	.9219
$\frac{3}{32}$	.5078	.5911	.6745	.7578	.8411	.9245
$\frac{1}{8}$	.5104	.5937	.6771	.7604	.8437	.9271
$\frac{5}{32}$	.5130	.5964	.6797	.7630	.8464	.9297
$\frac{3}{16}$	.5156	.5990	.6823	.7656	.8490	.9323
$\frac{7}{32}$	.5182	.6016	.6849	.7682	.8516	.9349
$\frac{1}{4}$	.5208	.6042	.6875	.7708	.8542	.9375
$\frac{9}{32}$	.5234	.6068	.6901	.7734	.8568	.9401
$\frac{5}{16}$	.5260	.6094	.6927	.7760	.8594	.9427
$\frac{11}{32}$	.5286	.6120	.6953	.7786	.8620	.9453
$\frac{3}{8}$	.5312	.6146	.6979	.7812	.8646	.9479
$\frac{13}{32}$	.5339	.6172	.7005	.7839	.8672	.9505
$\frac{7}{16}$	.5365	.6198	.7031	.7865	.8698	.9531
$\frac{15}{32}$	.5391	.6224	.7057	.7891	.8724	.9557
$\frac{1}{2}$	.5417	.6250	.7083	.7917	.8750	.9583
$\frac{17}{32}$	.5443	.6276	.7109	.7943	.8776	.9609
$\frac{9}{16}$	.5469	.6302	.7135	.7969	.8802	.9635
$\frac{19}{32}$	.5495	.6328	.7161	.7995	.8828	.9661
$\frac{5}{8}$	.5521	.6354	.7188	.8021	.8854	.9688
$\frac{21}{32}$	.5547	.6380	.7214	.8047	.8880	.9714
$\frac{11}{16}$	.5573	.6406	.7240	.8073	.8906	.9740
$\frac{23}{32}$	.5599	.6432	.7266	.8099	.8932	.9766
$\frac{3}{4}$	.5625	.6458	.7292	.8125	.8958	.9792
$\frac{25}{32}$	.5651	.6484	.7318	.8151	.8984	.9818
$\frac{13}{16}$	.5677	.6510	.7344	.8177	.9010	.9844
$\frac{27}{32}$	.5703	.6536	.7370	.8203	.9036	.9870
$\frac{7}{8}$	.5729	.6562	.7396	.8229	.9062	.9896
$\frac{29}{32}$	.5755	.6589	.7422	.8255	.9089	.9922
$\frac{15}{16}$	.5781	.6615	.7448	.8281	.9115	.9948
1	.5807	.6641	.7474	.8307	.9141	.9974

FORMULAS.

$$= \{ + [- : (\sqrt{\times / +}) : -] \} =$$

The term *formula*, as used in mathematics and in technical books, may be defined as a *rule in which symbols are used instead of words*; in fact, a formula may be regarded as a shorthand method of expressing a rule.

Most people having no knowledge of algebra regard formulas with distrust; they think that a person must be a good algebraic scholar in order to be able to use formulas. This idea, however, is erroneous. As a rule, no knowledge of any branch of mathematics except arithmetic is required to enable one to use a formula. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred. In the following pages we purpose to show the reader how to use such formulas as he is likely to encounter in "pocket-books," or other works of like nature.

The signs used in formulas are the ordinary signs indicative of operations and the signs of aggregation. All these signs are used in arithmetic, but, to refresh the reader's memory, we will explain their nature and uses before proceeding further.

The signs indicative of operations are six in number, viz.: $+$, $-$, $\times$, $\div$, $|$, $\sqrt{}$.

The sign $(+)$ indicates addition, and is called *plus*; when placed between two quantities, it indicates that the two quantities are to be added. Thus, in the expression $25 + 17$, the sign $(+)$ shows that 17 is to be added to 25.

The sign $(-)$ indicates subtraction, and is called *minus*; when placed between two quantities, it indicates that the

quantity on the right is to be subtracted from that on the left. Thus, in the expression $25 - 17$, the sign ($-$) shows that 17 is to be subtracted from 25.

The sign ($\times$) indicates multiplication, and is read *times*, or *multiplied by*; when placed between two quantities, it indicates that the quantity on the left is to be multiplied by that on the right. Thus, in the expression 25×17 , the sign ($\times$) shows that 25 is to be multiplied by 17.

The sign ($\div$) indicates division, and is read *divided by*; when placed between two quantities, it indicates that the quantity on the left is to be divided by that on the right. Thus, in the expression $25 \div 17$, the sign ($\div$) shows that 25 is to be divided by 17.

Division is also indicated by placing a straight line between the two quantities. Thus, $25 | 17$, $25 / 17$, and $25 \over 17$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\sqrt{}$) indicates that some root of the quantity to the right is to be taken; it is called the *radical sign*. To indicate what root is to be taken, a small figure, called the *index*, is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\sqrt{25}$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken, etc.

NOTE.—As the term “quantity” is a very convenient one to use, we will define it. In mathematics the word *quantity* is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say “two or more numbers,” or “two or more quantities.” The word *quantity* is more general in its meaning than the word *number*.

The signs of aggregation are four in number, viz.: —, (), [], and {}, respectively called the *vinculum*, the *parenthesis*, the *brackets*, and the *brace*; they are used when it is desired to indicate that all the quantities included by them

are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $5+8\times 7$, $(5+8)\times 7$, $[5+8]\times 7$, $\{5+8\}\times 7$. The vinculum is placed above the quantities which are to be treated as one quantity and subjected to the same operations.

While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{5+8}$, $\sqrt[3]{[5+8]}$, or $\sqrt[3]{\{5+8\}}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed (except, of course, when a root is to be indicated), the parenthesis is always used. Hence, $(5+8)\times 7$ would be the usual way of expressing the product of 5 plus 8 and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20-5)\div 3]\times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20-5)\div 3]\times 9-21\}\div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9; and that 21 is to be subtracted from the product thus obtained, and the result divided by 8.

Should it be necessary to use all four signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(20-5)\div 3]\times 9-21\}\div 8\}\times 12$. The reason for using the brace in this last instance will be explained, as it is not generally understood.

When several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication

must always be performed first. Thus, $2 + 3 \times 4$ equals 14, 3 being multiplied by 4 before adding to 2. Similarly, $10 \div 2 \times 5$ equals 1, since 2×5 equals 10, and $10 \div 10$ equals 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{2}$; whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in its order of importance. For example, $5 - 9 \div 3$ equals 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be performed; it is ($=$), and is called the sign of *equality*; it means that all on one side of it is exactly equal to all on the other side. For example, $2 = 2$, $5 - 3 = 2$, $5 \times (14 - 9) = 25$.

Having described the signs used in formulas, the formulas themselves will now be explained. First consider the well-known rule for finding the horsepower of a steam engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (*viz.*, the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \frac{\text{mean effective pressure (in pounds per square inch)} \times \text{stroke (in feet)} \times \text{area of piston (in square inches)} \times \text{number of strokes (per minute)}}{33,000}.$$

This expression could be shortened by representing each quantity by a single letter, thus: representing horsepower by the letter "*H*," the mean effective pressure in pounds per square inch by "*P*," the length of the stroke in feet by "*L*," the area of the piston in square inches by "*A*," the number of strokes per minute by "*N*," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P \times L \times A \times N}{33,000},$$

a much simpler and shorter expression. This last expression is called a *formula*.

The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{PLAN}{33,000}.$$

The sign of multiplication, evidently, cannot be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers that are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus, 2·3 means the same as 2×3 ; 542·749·1,006 indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3(b+c)$, $(b+c)a$, etc. The sign is not omitted when several quantities are included by a vinculum, and it is desired to indicate that the quantities

so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{\overline{b+c}} \times a$, etc., are always written as here printed.

Before proceeding further, we will explain one other device that is used by formula makers, and which is apt to puzzle one who encounters it for the first time. It is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names of the quantities. For example, they represent volume by *v*, pressure by *p*, height by *h*, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity that it represents. Some authors carry the practice a little further and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned when two quantities have the same name, but represent different things. Thus, consider the word *pressure* as applied to steam at different stages between the boiler and the condenser. First, there is *absolute pressure*, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by *p*, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter, and the gauge pressure by a small (lower-case) letter. Doing so, *P* represents absolute pressure, and *p* gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways, and still retain the letter *p* or *P* to represent the word pressure: First, by the use of the prime mark; thus, *p'* or *P'* (read *p prime* and *p major prime*) may be considered to represent the initial gauge pressure or the initial absolute pressure.

Second, by the use of sub. figures; thus, p_1 or P_1 (read p sub. one and p major sub. one). Third, by the use of sub. letters. thus, p_i or P_i (read p sub. i and P major sub. i). Likewise, p'' (read p second), p_2 , or p_r might be used to represent the gauge pressure at release, etc. Sub. letters have the advantage of still further identifying the quantity represented; in many instances, however, it is not convenient to use them, in which case primes and subs. are used instead. The prime notation may be continued as follows: p''' , p^{iv} , p^v , etc.; it is inadvisable to use superior figures, for example, p^1 , p^2 , p^3 , p^4 , etc., as they are liable to be mistaken for exponents.

The main thing to be remembered by the reader is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those that differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1 , w_2 , and w_3 represent the weights of three different bodies; s_1 , s_2 , and s_3 their specific heats; and t_1 , t_2 , and t_3 their temperatures; while t represents the final temperature, after the bodies have been mixed together.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters, and then perform the indicated operations. Thus, suppose that the values of w_1 , s_1 , and t_1 are, respectively, 2 pounds, .0951, and 80° ; of w_2 , s_2 , and t_2 , 7.8 pounds, 1, and 80° , and of w_3 , s_3 , and t_3 , $3\frac{1}{4}$ pounds, .1138, and 780° ; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 1 \times 80 + 3\frac{1}{4} \times .1138 \times 780}{2 \times .0951 + 7.8 \times 1 + 3\frac{1}{4} \times .1138} = \frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97^\circ.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

The reader should now be able to apply any formula involving only algebraic expressions that he may meet with, not requiring the use of logarithms for their solution. We will, however, call his attention to one or two other facts which he may have forgotten.

Expressions similar to $\frac{160}{\frac{660}{25}}$ sometimes occur, the heavy line

indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result

were desired, the expression would be written $\frac{160}{\frac{660}{25}}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\frac{160}{7 + \frac{660}{25}}$, the heavy line

is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7 + \frac{660}{25} = \frac{175 + 660}{25}$, if we substitute $\frac{175 + 660}{25}$ for $7 + \frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7 + \frac{660}{25}} = \frac{160}{\frac{175 + 660}{25}} = \frac{160}{\frac{835}{25}}$$

Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc. of some quantity, as 37 by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc. these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the denominator is

always the index of the root. Hence, instead of expressing the cube root of the square of 37 as $\sqrt[3]{37^2}$, it may be expressed $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1+a^2b)^3}$ may also be written $(1+a^2b)^{\frac{3}{5}}$, a much simpler expression.

We will now give several examples showing how to apply some of the more difficult formulas that the reader may encounter.

The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula.

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h),$$

in which A = area of segment;

π = 3.1416;

r = radius;

E = angle obtained by drawing lines from the center to the extremities of arc of segment;

c = chord of segment;

h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 in. long, angle subtended by chord is 83.46° , radius is 7.5 in., and height of segment is 1.91 in.?

SOLUTION.—Applying the formula just given,

$$\begin{aligned} A &= \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5 - 1.91) \\ &= 40.968 - 27.95 = 13.018 \text{ sq. in., nearly.} \end{aligned}$$

The area of any triangle may be found by means of the following formula, in which A = the area, and a , b , and c represent the lengths of the sides:

$$A = \frac{b}{2} \sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b} \right)^2}.$$

EXAMPLE.—What is the area of a triangle whose sides are 21 ft., 46 ft., and 50 ft. long?

SOLUTION.—In order to apply the formula, suppose we let a represent the side that is 21 ft. long; b , the side that is 50 ft. long; and c , the side that is 46 ft. long. Then, substituting in the formula.

$$\begin{aligned}
 A &= \frac{b}{2} \sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b} \right)^2} = \frac{50}{2} \sqrt{21^2 - \left(\frac{21^2 + 50^2 - 46^2}{2 \times 50} \right)^2} \\
 &= \frac{50}{2} \sqrt{441 - \left(\frac{441 + 2,500 - 2,116}{100} \right)^2} = 25 \sqrt{441 - \left(\frac{825}{100} \right)^2} \\
 &= 25 \sqrt{441 - 8.25^2} = 25 \sqrt{441 - 68.0625} = 25 \sqrt{372.9375} \\
 &= 25 \times 19.312 = 482.8 \text{ sq. ft., nearly.}
 \end{aligned}$$

The above operations have been extended much further than was necessary; this was done in order to show the reader every step of the process.

The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P = \frac{SA}{1 + q \frac{l^2}{G^2}},$$

in which P = load (pressure) in lb.; S = ultimate strength (in lb. per sq. in.) of material composing column; A = area of cross-section of column in sq. in.; q = a factor (multiplier) whose value depends on the shape of the ends of the column and on the material composing the column; l = length of the column in in.; G = least radius of gyration of cross-section of column.

The values of S , q , and G^2 are all given in printed tables on pages 151, 153, and 156.

EXAMPLE.—What is the least load that will break a hollow wrought-iron column whose outside diameter is 14 in., inside diameter 11 in., length 20 ft., and whose ends are flat?

SOLUTION.—For steel, $S = 150,000$, and $q = \frac{1}{25,000}$ for flat-ended steel columns; A , the area of the cross-section, = $.7854(d_1^2 - d_2^2)$, d_1 and d_2 being the outside and inside diameters, respectively; $l = 20 \times 12 = 240$ in.; and $G^2 = \frac{d_1^2 + d_2^2}{16}$. Substituting these values in the formula,

$$\begin{aligned}
 P &= \frac{SA}{1 + q \frac{l^2}{G^2}} = \frac{150,000 \times .7854(14^2 - 11^2)}{1 + \frac{1}{25,000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}} \\
 &= \frac{150,000 \times 58.905}{1 + .1163} = \frac{8,835,750}{1.1163} = 7,915,211 \text{ lb.}
 \end{aligned}$$

EVOLUTION.

METHOD OF EXTRACTING ROOTS.

The process of finding the root of a number is called *evolution*, and consists of finding one of the equal factors, which, when multiplied together, will give a product equal to the given number.

When the given number is resolved into *two* equal factors, one of them is called the *square root*; when resolved into *three* equal factors, one of them is called the *cube root*; when resolved into *four* equal factors, one of them is called the *fourth root*; etc. For example, the number 36 is equal to the product of the two equal factors 6 and 6; hence, 6 is the square root of 36. Again, the number 343 is equal to the product of the three equal factors, 7, 7, and 7; hence, 7 is the cube root of 343. Also, 32 is equal to the product of the five equal factors, 2, 2, 2, 2, and 2; hence, 2 is the fifth root of 32.

To indicate that some root of a number is to be extracted, a sign, called the *radical sign*, is placed before the number; and to indicate what root is to be extracted, a small figure, called the *index*, is placed within the sign. Thus, the square root of 36, the cube root of 243, and the fifth root of 32 would be respectively indicated by $\sqrt[2]{36}$, $\sqrt[3]{243}$, and $\sqrt[5]{32}$. It is usual, however, to combine the vinculum with the radical sign, in which case the above expressions would become $\sqrt{36}$, $\sqrt[3]{243}$, and $\sqrt[5]{32}$, respectively. Square root is always indicated by simply placing the radical sign before the number and omitting the index, thus, $\sqrt{36}$.

The roots most commonly required to be found are the square root and cube root, and, occasionally, the fifth root. The fourth root is very seldom required, and may always be found by extracting the square root, and then extracting the square root of the result; in other words, by extracting the square root twice. Hence, in the following pages we shall describe only the methods to be pursued in extracting square, cube, and fifth roots. Roots higher than the fifth are practically never required.

The following method of extracting the roots of numbers is the simplest of any of which we are aware, and for all those cases in which the root is an interminable decimal it is, perhaps, the easiest of all methods to apply.

SQUARE ROOT.

The greatest number that can be written with *one* figure is 9, and $9^2 = 81$; the greatest number that can be written with *two* figures is 99, and $99^2 = 9,801$; with *three* figures 999, and $999^2 = 998,001$; with *four* figures 9,999, and $9,999^2 = 99,980,001$, etc.

In each of the above it will be noticed that the square of the number contains just twice as many figures as the given number. Hence, in order to find the square root of a number, first find how many figures there will be in the root; this is done by pointing off the number into *periods* of *two* figures each, *beginning with the unit figure*. The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain four figures; since, pointing off the periods, we get 83'74'08'01, or four periods, consequently, there must be four figures in the root. In like manner, the square root of 50,625 must contain three figures, since there are three (5'06'25) periods.

The square of any number wholly decimal always contains twice as many figures as the number. For example, $.1^2 = .01$; $.13^2 = .0169$; $.751^2 = .564001$, etc. Hence, in pointing off a decimal, begin at the decimal point and proceed to the right. If the last period contains but one figure, complete it by annexing a cipher; thus, $.43'42'94'40$.

NOTE.—The order of a figure denotes the position that it occupies with respect to the decimal point. Thus, a figure in unit's place is said to be a figure of the first order; a figure in hundred's place, a figure of the third order, etc. In the number 7,854, 7 is a digit of the fourth order; 8, a digit of the third order; 5, a digit of the second order; and 4, a digit of the first order.

Hereafter, when referring to any digit of a number, the digit occupying the highest order in the number will be called the first digit or figure; that occupying the next highest order will be called the second digit or figure; etc.

So, also, when a number is divided into periods, the extreme left-hand period will be called the first period; the one next to it, the second period, etc.

The process of extracting the square root of a number is best illustrated by means of an example.

EXAMPLE.—Extract the square root of 1,971.14.

SOLUTION.—First point off the number into periods of two figures each, beginning at the decimal point; this gives 19'71'.14. Consider the first two periods 19'71, and try to find two numbers that, multiplied together, will give a product equal to, or very nearly equal to, 1,971, the two numbers themselves being equal, or very nearly so. This may be done as follows: $1,971 = 2,000$, nearly $= 50 \times 40$, nearly. Now find the arithmetic mean* between these two

numbers; this is $\frac{50 + 40}{2} = 45$. Since the square of the arithmetic mean of two numbers is always greater than the product of the two numbers, we know that 45^2 is greater than 50×40 , and hence greater than 1,971. Consequently, we will try 44 for one of the factors of 1,971, and find the other by division. $1,971 \div 44 = 44.8$; that is, $44 \times 44.8 = 1,971$, nearly. The arithmetic mean of these two numbers is $\frac{44 + 44.8}{2} = 44.4$, and its square, though greater than 44×44.8 ,

differs from the product but slightly; in fact, the more nearly the two factors approach equality, the less will be the difference between their product and the square of their arithmetic mean.

Now using all the figures of the given number and dividing it by the result last obtained, we have $1,971.14 \div 44.4 = 44.395+$, and the arithmetic mean is $\frac{44.4 + 44.395}{2} = 44.3975$.

This last result is the square root of 1,971.14 to 6 figures, the exact root to 8 figures being 44.397522.

*The arithmetic mean of several quantities is the sum of the quantities divided by the number of quantities. Thus, the arithmetic mean of 4, 6, 10, and 14 is $\frac{4 + 6 + 10 + 14}{4} = 8\frac{1}{2}$.

If more than five or six figures are desired, repeat the process; thus, $1,971.14 \div 44.3975 = 44.397544906808$. The arithmetic mean is $\frac{44.3975 + 44.397544906808}{2} = 44.397522453404$.

The exact root to 14 figures is $44.397522453409+$.

EXAMPLE.— $\sqrt[3]{.00217} = ?$

SOLUTION.—Pointing off the number into periods of two figures each, it becomes .00'21'70, annexing a cipher to complete the last period. Neglect the decimal point for the present, and assume that our number is 2,170. Then, $2,170 = 2,150$, nearly $= 50 \times 43$, nearly. The arithmetic mean is $\frac{50 + 43}{2} = 46.5$. Trying 46.5 for one factor (since 2,150 is less than 2,170, 46.5² will be quite near 2,170) we have for the other $2,170 \div 46.5 = 46.6666+$, and the arithmetic mean is $\frac{46.5 + 46.6666}{2} = 46.5833$. The exact root of 2,170 to 8 figures

is 46.583259; hence, the result obtained is correct to 6 figures.

We locate the decimal point as follows: Since the number is wholly decimal, the root must also be wholly decimal; and since two ciphers follow the decimal point, there must be *one cipher after the decimal point in the root*; for there must be as many ciphers following the decimal point in the root as there are entirely cipher periods between the decimal point and the first digit of the given number. Hence, $\sqrt[3]{.00217} = .046583$.

In order to determine how many figures of any given arithmetic mean are correct as expressing the true root, apply the principle that *any arithmetic mean expresses the root correctly to twice as many figures (or 1 less) as the number of figures in it agrees with the next preceding arithmetic mean*. Thus, in the example just given, the two arithmetic means are 46.5 and 46.5833, and since they agree to 3 figures, 46.5833 is the root correct to 2×3 figures = 6 figures. In the first example, the last arithmetic mean is 44.397522453404, and the one preceding is 44.3975; since they agree to 6 figures, the former must be correct to at least 12 figures, which is the case.

When there are ciphers between the decimal point and the first digit of a number that is wholly decimal, and it is required to ascertain the position of the decimal point in the root, count

the ciphers between the decimal point and the first digit and divide the result by 2. The quotient in each case, neglecting any remainder that may be obtained, will denote the number of ciphers between the decimal point and the first digit of the root. Thus, the square root of .0000027 will have $6 \div 2 = 3$ ciphers between the decimal point and the first digit of the root; likewise, the square root of .0000027 will have $5 \div 2 = 2$ ciphers between the decimal point and the first figure of the root.

CUBE ROOT.

The first step in finding the cube root of a number is to find how many figures there will be in the root. Proceeding as in square root, we have $9^3 = 729$; $99^3 = 970,299$; $999^3 = 997,002,999$. It appears that in each of the above cases the cube of the number contains three times as many figures as the number. Therefore, if a number is pointed off in periods of three figures each, beginning at the right, the cube root of the number will contain as many figures as there are periods. Thus, the cube root of 1,767,172,329 must contain four figures, since the number contains four periods.

The cube of any number wholly decimal contains three times as many figures as the number cubed. Thus, $.2^3 = .008$, $.02^3 = .000008$, $.007^3 = .000000343$, $.317^3 = .031855013$. The first step in finding the cube root of a decimal is to separate it into periods of three figures each, beginning at the decimal point. The location of the decimal point in the root is readily determined from the number of entirely cipher periods between the decimal point and the first digit. The cube root of .000076, neglecting the decimal point, is 42,358. To locate the decimal point, we observe that there is one entirely cipher period between the decimal point and the first digit in the given number. Therefore, there will be one cipher following the decimal point in the root; or $\sqrt[3]{.000076} = .042358$.

The process of extracting the cube root of a number is very similar to that just shown for square root, and may be shown by an example.

EXAMPLE.— $\sqrt[3]{2,571.14} = ?$

SOLUTION.—Pointing off into periods, we have 2'571'.140.

Call the number 2,500 and divide it into three factors that will be as nearly equal as we can determine directly. Thus, $2,500 = 25 \times 10 \times 10$; but, since 25 is considerably larger than the other two factors, it will be well to make them more nearly equal. This may be done in several ways, one of which is given here.

$2,500 = 10 \times 10 \times 25 = 10 \times 250 = 10 \times 16 \times 15.6$, nearly $= 160 \times 15.6$, nearly $= 13 \times 12.3 \times 15.6$, nearly. Since these factors do not differ greatly, we will take the arithmetic mean between the greatest and least, obtaining $\frac{15.6 + 12.3}{2}$

$= 14$, nearly. Now call two of the equal factors 14, and find the other by dividing the given number by the product of the two equal factors. Thus, $14 \times 14 = 196$, and $2,571.14 \div 196 = 13.12$; that is, $2,571.14 = 14 \times 14 \times 13.14$, nearly. The arithmetic mean of these three factors is $\frac{14 + 14 + 13.1}{3} = 13.7$;

in other words, $2,571.14 = 13.7 \times 13.7 \times 13.7 = 13.7^3$, nearly. Repeating this last process, we have $13.7 \times 13.7 = 187.69$, and $2,571.14 \div 187.69 = 13.698865$; the arithmetic mean is $\frac{13.7 + 13.7 + 13.698865}{3} = 13.699622$, the cube root of 2,571.14,

correct to 8 figures.

It was not necessary to take so much pains to equalize the factors as was done in the foregoing example, but it saves figures and time to do so. For instance, we might have proceeded as follows: $2,500 = 10 \times 10 \times 25$; the arithmetic mean

$= \frac{10 + 10 + 25}{3} = 15$; $15 \times 15 = 225$; $2,571.14 \div 225 = 11.4$,

nearly; the arithmetic mean of these three factors is $\frac{15 + 15 + 11.4}{3} = 13.8$, and so on.

We will give one more example in cube root.

EXAMPLE.— $\sqrt[3]{.1756} = ?$

SOLUTION.—Pointing off into periods of three figures each, we have $.175'600$, annexing two ciphers to complete the last period, since the given number is a decimal. Considering the first period only, and neglecting the decimal point for the present, $175 = 5 \times 35 = 5 \times 6 \times 6$, nearly $= 5.5 \times 5.5 \times 5.8$,

nearly. The arithmetic mean is $\frac{5.5 + 5.5 + 5.8}{3} = 5.6$, since we discarded the decimal point before the first period, in order to avoid unnecessary trouble in looking after the decimal point; in other words, we consider the number .1756 as being 175.6, for the present.

$175.6 \div 5.6 = 31.35714$; $31.35714 \div 5.6 = 5.59949$. The arithmetic mean = $\frac{5.6 + 5.6 + 5.59949}{3} = 5.59983$. Since this number is *very* nearly equal to one of the equal factors 5.6, it is extremely probable that all 6 figures are correct; in fact, the exact root to 11 figures is 5.5998299259. Hence, $\sqrt[3]{.1756} = .559983$.

OTHER ROOTS.

The process is very similar for other roots. First point off the number into periods that shall contain as many figures as there are units in the index of the root; then divide the number into as many factors (making them as nearly equal as possible) as there are units in the index of the root, and find their arithmetic mean, as in the case of square and cube root. We will illustrate by giving one example in fifth root.

EXAMPLE.— $\sqrt[5]{2,571} = ?$

SOLUTION.—There is only one period. Consider the number to be 2,500. Then, $2,500 = 5 \times 5 \times 10 \times 10 = 5 \times 5 \times 100 = 5 \times 5 \times 4 \times 4 \times 6$, nearly. The arithmetic mean is $\frac{5 + 5 + 4 + 4 + 6}{5} = 4.8$. Now, dividing 2,571 by 4.8 four times, to find the unequal factor, we have: $2,571 \div 4.8 = 535.625$; $535.625 \div 4.8 = 111.5885$; $111.5885 \div 4.8 = 23.2476$; $23.2476 \div 4.8 = 4.843$. The arithmetic mean is

$$\frac{4.8 + 4.8 + 4.8 + 4.8 + 4.843}{5} = 4.8086.$$

Since the first three figures, 4.80, of the first and second arithmetic means are the same, it is extremely probable that all 5 figures of the result just obtained are correct. (The principle stated on page 106 is perfectly general, and applies to all roots.) In fact, the exact root to 8 figures is 4.8086195. Hence, $\sqrt[5]{2,571} = 4.8086$.

The fifth power of any number wholly decimal contains five times as many figures as the first power of the number. Thus, $.01^5 = .00000'00001$; $.9^5 = .59049$, etc.

To locate the decimal point in the root, count the number of periods in the integral part of the given number; the result is the number of figures in the integral part of the root. Thus, in the fifth root of 27,892,797.436, there will be two figures in the integral part of the root, since there are two periods (278'92797.436) in the integral part of the given number. If the number is wholly decimal, there will be as many ciphers between the decimal point and the first digit of the root as there are entirely cipher periods. Thus,
 $\sqrt[5]{.00032} = .2$; $\sqrt[5]{.00000'00032} = .02$, etc.

The first three (usually, the first four) figures of the square, cube, or fifth root of a number may be obtained by using the accompanying table (page 112), which gives the squares and cubes of numbers from 1 to 10, varying by tenths, and the fifth powers of the same numbers to five figures.

To use the table, proceed as follows: First point off the given number into periods corresponding to the root to be extracted; then consider the first period as a whole number and the remaining figures as a decimal, regardless of where the decimal point is in the given number. For example: for square root, consider the number 67,265.7 as 6.72657; for cube root, as 67.2657; and for fifth root, as 67,265.7. To extract the square root of 67,265.7, consider it as 6.72657 and find between what two numbers in the columns headed "square" it occurs in the table. We find that it falls between 6.25, the square of 2.5, and 6.76, the square of 2.6; hence, the square root of 6.72657 is greater than 2.5 and less than 2.6; that is, the first two figures are 2.5. To find the next figure, find the difference between the two numbers in the table between which the given number falls and the difference between the lower number in the table and the given number; divide the second difference by the first, obtaining two figures of the quotient. The quotient will usually be the next two figures of the root. Thus, $6.76 - 6.25 = .51$; $6.72657 - 6.25 = .47657$; $.47657 \div .51 = .93$. Hence, $\sqrt{6.72657} = 2.693$, and $\sqrt{67,265.7} = 269.3$ to four figures.

The cube root and the fifth root are found in the same manner. Thus, the first two figures of $\sqrt[3]{67.2657}$ are 4.0; the first difference is $68.921 - 64.000 = 4.921$; the second difference is $67.2657 - 64.000 = 3.2657$; the quotient is $3.2657 \div 4.921 = .66$. Hence, $\sqrt[3]{67.2657} = 4.066$ and $\sqrt[3]{67,265.7} = 40.66$ to four figures.

The first two figures of $\sqrt[5]{67,265.7}$ are 9.2; the first difference is $69,569 - 65,908 = 3,661$; the second difference is $67,265.7 - 65,908 = 1,357.7$; the quotient is $1,357.7 \div 3,661 = .37$. Hence, $\sqrt[5]{67,265.7} = 9.237$ to four figures.

The fourth figure cannot always be depended upon; consequently, it is better to take the first three figures (increasing the third figure by 1 if the fourth figure is 5 or greater) and then use the rule previously given for finding more figures of the root.

GENERAL RULE.

I. *Divide the given number into periods containing as many figures as there are units in the index of the root. Only the left-hand period of the integral part of a number can have a less number of figures.*

II. *Divide the given numbers into as many factors (making them as nearly equal as possible) as there are units in the index of the root, and find the arithmetic mean of the factors; the result will be the first approximate value of the root, correct, usually, to 3 figures, if the factors selected are nearly equal.*

III. *Divide the given number by the arithmetic mean obtained in II; divide the resulting quotient by the arithmetic mean; and so continue until a quotient is obtained that is nearly equal to the divisor. There will be as many divisions as there are units in the index of the root, less one; carry all quotients to 6 figures. The arithmetic mean of all the divisors and the last quotient will be the second approximate value of the root, which should be correct to at least 5 figures.*

IV. *If more than 5 figures of the root are desired, repeat the process described in III, carrying all results to 10 figures, or as many less as are desired.*

SQUARES, CUBES, AND FIFTH POWERS.

No.	Square.	Cube.	Fifth Power.	No.	Square.	Cube.	Fifth Power.
1.0	1.00	1.000	1.0000	5.5	30.25	166.375	5,032.8
1.1	1.21	1.331	1.6105	5.6	31.36	175.616	5,507.3
1.2	1.44	1.728	2.4883	5.7	32.49	185.193	6,016.9
1.3	1.69	2.197	3.7129	5.8	33.64	195.112	6,563.6
1.4	1.96	2.744	5.3782	5.9	34.81	205.379	7,149.2
1.5	2.25	3.375	7.5938	6.0	36.00	216.000	7,776.0
1.6	2.56	4.096	10.486	6.1	37.21	226.981	8,446.0
1.7	2.89	4.913	14.199	6.2	38.44	238.328	9,161.3
1.8	3.24	5.832	18.896	6.3	39.69	250.047	9,924.4
1.9	3.61	6.859	24.761	6.4	40.96	262.144	10,737
2.0	4.00	8.000	32.000	6.5	42.25	274.625	11,603
2.1	4.41	9.261	40.841	6.6	43.56	287.496	12,523
2.2	4.84	10.648	51.536	6.7	44.89	300.763	13,501
2.3	5.29	12.167	64.363	6.8	46.24	314.432	14,539
2.4	5.76	13.824	79.626	6.9	47.61	328.509	15,640
2.5	6.25	15.625	97.656	7.0	49.00	343.000	16,807
2.6	6.76	17.576	118.81	7.1	50.41	357.911	18,042
2.7	7.29	19.683	143.49	7.2	51.84	373.248	19,349
2.8	7.84	21.952	172.10	7.3	53.29	389.017	20,731
2.9	8.41	24.389	205.11	7.4	54.76	405.224	22,190
3.0	9.00	27.000	243.00	7.5	56.25	421.875	23,730
3.1	9.61	27.791	286.29	7.6	57.76	438.976	25,355
3.2	10.24	32.768	335.54	7.7	59.29	456.533	27,068
3.3	10.89	35.937	391.35	7.8	60.84	474.552	28,872
3.4	11.56	39.304	454.35	7.9	62.41	493.039	30,771
3.5	12.25	42.875	525.22	8.0	64.00	512.000	32,768
3.6	12.96	46.656	604.66	8.1	65.61	531.441	34,868
3.7	13.69	50.653	693.44	8.2	67.24	551.368	37,074
3.8	14.44	54.872	792.35	8.3	68.89	571.787	39,390
3.9	15.21	59.319	902.24	8.4	70.56	592.704	41,821
4.0	16.00	64.000	1,024.0	8.5	72.25	614.125	44,371
4.1	16.81	68.921	1,158.6	8.6	73.96	636.056	47,043
4.2	17.64	74.088	1,306.9	8.7	75.69	658.503	49,842
4.3	18.49	79.507	1,470.1	8.8	77.44	681.472	52,773
4.4	19.36	85.184	1,649.2	8.9	79.21	704.969	55,841
4.5	20.25	91.125	1,845.3	9.0	81.00	729.000	59,041
4.6	21.16	97.336	2,059.6	9.1	82.81	753.571	62,408
4.7	22.09	103.823	2,293.5	9.2	84.64	778.688	65,908
4.8	23.04	110.592	2,548.0	9.3	86.49	804.357	69,569
4.9	24.01	117.649	2,824.8	9.4	88.36	830.584	73,390
5.0	25.00	125.000	3,125.0	9.5	90.25	857.375	77,378
5.1	26.01	132.651	3,450.3	9.6	92.16	884.736	81,537
5.2	27.04	140.608	3,802.1	9.7	95.09	912.673	85,873
5.3	28.09	148.877	4,182.0	9.8	96.04	941.192	90,392
5.4	29.16	157.464	4,591.7	9.9	98.01	970.299	95,099

MENSURATION.

In the following formulas, the letters have the meanings here given, unless otherwise stated.

D = larger diameter;

d = smaller diameter;

R = radius corresponding to D ;

r = radius corresponding to d ;

p = perimeter or circumference;

C = area of convex surface = area of flat surface which can be rolled into the shape shown;

S = area of entire surface = C + area of the end or ends;

A = area of plane figure;

π = 3.1416, nearly = ratio of any circumference to its diameter;

V = volume of solid.

The other letters used will be found on the cuts.

CIRCLE.

$$p = \pi d = 3.1416 d.$$

$$p = 2 \pi r = 6.2832 r.$$

$$p = 2 \sqrt{\pi A} = 3.5449 \sqrt{A}.$$

$$p = \frac{2 A}{r} = \frac{4 A}{d}.$$

$$d = \frac{p}{\pi} = \frac{p}{3.1416} = .3183 p.$$

$$d = 2 \sqrt{\frac{A}{\pi}} = 1.1284 \sqrt{A}.$$

$$r = \frac{p}{2 \pi} = \frac{p}{6.2832} = .1592 p.$$

$$r = \sqrt{\frac{A}{\pi}} = .5642 \sqrt{A}.$$

$$A = \frac{\pi d^2}{4} = .7854 d^2.$$

$$A = \pi r^2 = 3.1416 r^2.$$

$$A = \frac{pr}{2} = \frac{pd}{4}.$$

TRIANGLES.



$$D = B + C.$$

$$E + B + C = 180.$$

$$B = D - C.$$

$$E' + B + C = 180^\circ.$$

$$E' = E.$$

$$B' = B.$$

The above letters refer to angles.

For a right-angled triangle, c being the hypotenuse,

$$c = \sqrt{a^2 + b^2}.$$

$$a = \sqrt{c^2 - b^2}.$$

$$b = \sqrt{c^2 - a^2}.$$

c = length of side opposite an acute angle of an oblique triangle.

$$c = \sqrt{a^2 + b^2 - 2be}.$$

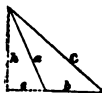
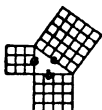
$$h = \sqrt{a^2 - e^2}.$$



c = length of side opposite an obtuse angle of an oblique-angled triangle.

$$c = \sqrt{a^2 + b^2 + 2be}.$$

$$h = \sqrt{a^2 - e^2}.$$



For a triangle inscribed in a semicircle; i.e., any right-angled triangle,

$$c : b :: a : h.$$

$$h = \frac{ab}{c} = \frac{ce}{a}.$$

$$a : b + e = c : a = h : c.$$



For any triangle,

$$A = \frac{bh}{2} = \frac{1}{2}bh.$$

$$A = \frac{b}{2} \sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b} \right)^2}.$$



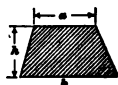
RECTANGLE AND PARALLELOGRAM.

$$A = ab.$$

$$A = b \sqrt{c^2 - b^2}.$$

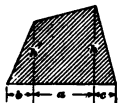
TRAPEZOID.

$$A = \frac{1}{2} h(a + b).$$



TRAPEZIUM.

Divide into two triangles and a trapezoid.



$$A = \frac{1}{2} b h' + \frac{1}{2} a(h' + h) + \frac{1}{2} c h,$$

or, $A = \frac{1}{2} [b h' + c h + a(h' + h)].$

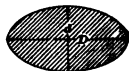
Or, divide into two triangles by drawing a diagonal. Consider the diagonal as the base of both triangles, call its length l ; call the altitudes of the triangles h_1 and h_2 ; then

$$A = \frac{1}{2} l (h_1 + h_2).$$

ELLIPSE.

$$p^* = \pi \sqrt{\frac{D^2 + d^2}{2} - \frac{(D - d)^2}{8.8}}.$$

$$A = \frac{\pi}{4} D d = .7854 D d.$$



SECTOR.

$$A = \frac{1}{2} l r.$$

$$A = \frac{\pi r^2 E}{360} = .008727 r^2 E.$$

l = length of arc.

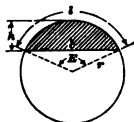
SEGMENT.

$$A = \frac{1}{2} [l r - c(r - h)].$$

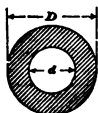
$$A = \frac{\pi r^2 E}{360} - \frac{c}{2} (r - h).$$

$$l = \frac{\pi r E}{180} = .0175 r E.$$

$$E = \frac{180 l}{\pi r} = 57.2956 \frac{l}{r}.$$



* The perimeter of an ellipse cannot be exactly determined without a very elaborate calculation, and this formula is merely an approximation giving fairly close results.

**RING.**

$$A = \frac{\pi}{4} (D^2 - d^2).$$

CHORD.

c = length of chord.

$$r = \frac{c^2 + 4h^2}{8h} = \frac{e^2}{2h}$$

$$c = 2\sqrt{2hr - h^2}.$$

$$l = \frac{8e - c}{3}, \text{ approximately.}$$

**HELIX.**

To construct a helix.

l = length of helix;

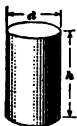
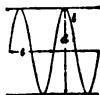
n = number of turns;

t = pitch.

$$l = \sqrt{\frac{l^2}{n^2} - \pi^2 d^2}.$$

$$l = n \sqrt{\pi^2 d^2 + t^2}.$$

$$n = \frac{l}{\sqrt{\pi^2 d^2 + t^2}}.$$

**CYLINDER.**

$$C = \pi d h.$$

$$S = 2\pi r h + 2\pi r^2.$$

$$= \pi d h + \frac{\pi}{2} d^2.$$

$$V = \pi r^2 h = \frac{\pi}{4} d^2 h.$$

$$V = \frac{p^2 h}{4\pi} = .0796 p^2 h.$$

FRUSTUM OF CYLINDER.

h = $\frac{1}{2}$ sum of greatest and least heights.

$$C = p h = \pi d h.$$

$$S = \pi d h + \frac{\pi}{4} d^2 + \text{area of elliptical top.}$$

$$V = A h = \frac{\pi}{4} d^2 h.$$



CONE.



$$C = \frac{1}{2} \pi d l = \pi r l.$$

$$S = \pi r l + \pi r^2 = \pi r \sqrt{r^2 + h^2} + \pi r^2.$$

$$V = \frac{\pi d^3}{4} \times \frac{h}{3} = \frac{.7854 d^3 h}{3} = \frac{p^3 h}{12 \pi}.$$

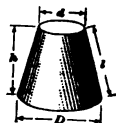
FRUSTUM OF CONE.

$$C = \frac{1}{2} l (P + p) = \frac{\pi}{2} l (D + d).$$

$$S = \frac{\pi}{2} [l (D + d) + \frac{1}{3} (D^2 + d^2)].$$

$$V = \frac{\pi}{4} (D^2 + D d + d^2) \times \frac{1}{3} h$$

$$= .2618 h (D^2 + D d + d^2).$$



SPHERE.

$$S = \pi d^2 = 4 \pi r^2 = 12.5664 r^2.$$

$$V = \frac{1}{6} \pi d^3 = \frac{4}{3} \pi r^3 = .5236 d^3 = 4.1888 r^3.$$

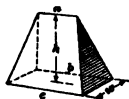
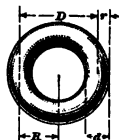
CIRCULAR RING.

D = mean diameter;

R = mean radius.

$$S = 4 \pi^2 R r = 9.8696 D d.$$

$$V = 2 \pi^2 R r^2 = 2.4674 D d^2.$$



WEDGE.

$$V = \frac{1}{6} w h (a + b + c).$$

PRISMOID.

A prismoid is a solid having two parallel plane ends, the edges of which are connected by plane triangular or quadrilateral surfaces.



A = area one end;
 a = area of other end;
 m = area of section midway between ends;
 l = perpendicular distance between ends.

$$V = \frac{1}{3} l (A + a + 4m).$$

The area m is not in general a mean between the areas of the two ends, but its sides are means between the corresponding lengths of the ends.

Approximately,
$$V = \frac{A + a}{2} l.$$

REGULAR PYRAMID.

P = perimeter of base;

A = area of base.

$$C = \frac{1}{2} Pl.$$

$$S = \frac{1}{2} Pl + A.$$

$$V = \frac{Ah}{3}.$$



To obtain area of base, divide it into triangles, and find their sum.

The formula for V applies to any pyramid whose base is A and altitude h .

FRUSTUM OF REGULAR PYRAMID.



a = area of upper base;

A = area of lower base;

p = perimeter of upper base;

P = perimeter of lower base.

$$C = \frac{1}{2} l (P + p).$$

$$S = \frac{1}{2} l (P + p) + A + a.$$

$$V = \frac{1}{3} h (A + a + \sqrt{Aa}).$$

The formula for V applies to the frustum of any pyramid.

LENGTH OF SPIRAL.

$$l = \pi n \left(\frac{D + d}{2} \right).$$

$$l = \frac{\pi}{t} (R^2 - r^2).$$

n = number of coil;

l = length of spiral;

t = pitch.



PRISM OR PARALLELOPIPED.

$$C = Ph.$$

$$S = Ph + 2A.$$

$$V = Ah.$$



For prisms with regular polygon as bases, P = length of one side $\times$ number of sides.

To obtain area of base, if it is a polygon, divide it into triangles, and find sum of partial areas.

FRUSTUM OF PRISM.

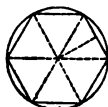


If a section perpendicular to the edges is a triangle, square, parallelogram, or *regular* polygon,

$$V = \frac{\text{sum of lengths of edges}}{\text{number of edges}} \times \text{area of right section.}$$

REGULAR POLYGONS.

Divide the polygon into equal triangles and find the sum of the partial areas. Otherwise, square the length of one side and multiply by proper number from the following table:



Name.	No. Sides.	Multiplier.
Triangle	3	.433
Square	4	1.000
Pentagon	5	1.720
Hexagon	6	2.598
Heptagon	7	3.634
Octagon	8	4.828
Nonagon	9	6.182
Decagon	10	7.694

IRREGULAR AREAS.

Divide the area into trapezoids, triangles, parts of circles, etc., and find the sum of the partial areas.

If the figure is very irregular, the approximate area may be found as follows: Divide the figure into trapezoids by equidistant parallel lines b, c, d , etc. The lengths of these lines being measured, then, calling a the first and n the last length, and y the width of strips,



$$\text{Area} = y \left(\frac{a+n}{2} + b + c + \text{etc.} + m \right).$$

MECHANICS.

FALLING BODIES.

Let $g = 32.16 =$ constant acceleration due to the attraction of the earth;

$t =$ number of seconds that the body falls;

$v =$ velocity in feet per second at the end of the time t ;

$h =$ distance that the body falls during the time t .

$$\text{Then, } v = g t = \frac{2 h}{t} = \sqrt{2 g h} = 8.02 \sqrt{h}.$$

$$h = \frac{v t}{2} = \frac{g t^2}{2} = \frac{v^2}{2 g} = .015547 v^2.$$

$$t = \frac{v}{g} = \frac{2 h}{v} = \sqrt{\frac{2 h}{g}} = .24938 \sqrt{h}.$$

PROJECTILES.

The formulas under this and the preceding heading are rigidly true only for bodies moving in a vacuum or in space (as the stars and planets); they are approximately true for bodies moving in air, provided they are dense and the velocity is not very great. Fairly good results may be obtained by applying the formulas for projectiles in calculating the range of a jet of water issuing from a small orifice in the side of a vessel.

Let $g = 32.16 =$ acceleration due to gravity;

$v =$ initial velocity in feet per second;

$r =$ range;

$y =$ vertical height of starting point above ground;

$A =$ elevation in degrees = angle that the direction of the projectile at the start makes with the horizontal.

Then the range, or distance from the starting point to the point where the projectile crosses a horizontal line through the starting point, is

$$r = \frac{v^2}{g} \sin 2 A.$$

If the body is projected in a horizontal direction, the range is the distance from the starting point to the point where the projectile strikes the ground, and

$$r = v \sqrt{\frac{2y}{g}} = .24938 v \sqrt{y}.$$

The range of a projectile fired in a horizontal direction, 30 ft. above the ground, with a velocity of 800 ft. per second, equals $r = .24938 \times 800 \times \sqrt{30} = 409.77$ ft.

CENTRIFUGAL FORCE.

F = centrifugal force in pounds;

W = weight of revolving body in pounds;

r = distance from the axis of motion to the center of gravity of the body in feet;

N = number of revolutions per minute;

v = velocity in feet per second.

$$F = \frac{W v^2}{g r} = .00034 W r N^2.$$

In calculating the centrifugal force of flywheels, it is customary to neglect the arms and take r equal to the mean radius of the rim; in such cases W is taken as one-half the weight of the rim. The result thus obtained, divided by π , is approximately the force tending to burst the flywheel rim.

EXAMPLE.—What is the force tending to burst a flywheel rim weighing 7 tons, making 150 rev. per min., and having a mean radius of 5 ft.?

SOLUTION.—

$$F = \frac{.00034 \times (\frac{1}{2} \times 7 \times 2,000) 5 \times 150^2}{3.1416} = 85,227 \text{ lb.}$$

CENTER OF GRAVITY.

The center of gravity of a body, or of a system of bodies, is that point from which, if the body or system were suspended, it would be in equilibrium.

If a line or a surface has two axes, or a solid has three axes of symmetry, the center of gravity lies at their point of intersection, and corresponds with the geometrical center of the figure.

An axis of symmetry is any line so drawn that, if part of the figure on one side of the line is folded on this line, it will coincide exactly with the other part, point for point and line for line. Thus, in Fig. 1, if the part ab is folded on the line AB , the upper half will coincide exactly with the lower half; also, if bc is folded on the line CD , the right-hand half will coincide exactly with the left-hand half. Hence, the point O where AB and CD intersect is the center of gravity of the rectangle $abcd$. If the figure has one axis of symmetry, the center of gravity may be found as follows: Let

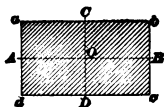


FIG. 1.

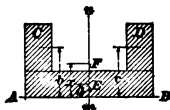


FIG. 2.

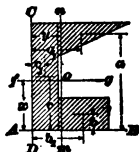


FIG. 3.

mn be an axis of symmetry of the area in Fig. 2. The center of gravity will lie somewhere on this line. Draw any line AB perpendicular to mn . Divide the area into squares, rectangles, triangles, parallelograms, circles, etc., whose centers of gravity are easily found, and measure the perpendicular distances of these centers of gravity from the line AB . Add the sum of the products obtained by multiplying each area by the distance of its center of gravity from the line AB , and divide by the area of the entire figure; the result is the distance x of the center of gravity from AB measured on mn , or the point F .

If the figure has no axis of symmetry, as in Fig. 3, draw any line, as AB , and find the distance x of the center of gravity from AB , and through x draw fg parallel to AB . Choose any other line, CD , and find the distance y of the center of gravity from CD by the same method, and through y draw mn parallel to CD . The point of intersection o of fg and mn is the center of gravity.

Thus, suppose that the area of the triangle, Fig. 3, is Δ sq. in., and the distance of its center of gravity from AB is

a in., and from CD , a_1 in.; that the area of the small rectangle is B sq. in., and the distance of its center of gravity from AB is b in., and from CD is b_1 in.; that the area of the large rectangle is C sq. in., and the distance of its center of gravity from AB is c in., and from CD is c_1 in.; then,

$$x = \frac{(A \times a) + (B \times b) + (C \times c)}{A + B + C},$$

and
$$y = \frac{(A \times a_1) + (B \times b_1) + (C \times c_1)}{A + B + C}.$$

To find the center of gravity mechanically, suspend the object from a point near its edge and mark on it the direction of a plumb-line from that point; then suspend it from another point and again mark the direction of a plumb-line. The intersection of these two lines will be directly over the center of gravity.

The center of gravity of a body having parallel sides may be found by drawing the outline of one of the sides upon heavy paper, and cutting out the exact shape of the figure. Then suspend the paper from the two points and find the center of gravity, as in the last case.

The center of gravity of a triangle lies on a line drawn from a vertex to the middle point of the opposite side, and at a distance from that side equal to one-third of the length of the line. Or, draw a line from another vertex to the middle point of the side opposite, and the intersection of the two lines will be the center of gravity.

For a parallelogram, the center of gravity is at the intersection of the two diagonals.

For an irregular four-sided figure, draw a diagonal, dividing it into two triangles. Draw a line joining these centers of gravity. Draw the other diagonal, dividing the figure into two other triangles, and join the centers of gravity by a straight line. The intersection of these lines is the center of gravity of the figure.

For a figure having more than four sides, find the center of gravity by the general method explained in connection with Fig. 3.

For an arc of a circle, the center of gravity lies on the radius drawn to the middle point of the arc (an axis of

symmetry) and at a distance from the center equal to the length of the chord multiplied by the radius and divided by the length of the arc.

For a semicircle, the distance from the center $= \frac{2r}{\pi}$
 $= .6366 r$, when r = the radius.

For the area included in a half circle, the distance of the center of gravity from the center $= \frac{4r}{3\pi} = .4244 r$.

For circular sector, the distance of the center of gravity from the center equals two-thirds of the length of the chord multiplied by the radius and divided by the length of the arc.

For a circular segment, let A be its area and C the length of its chord; then the distance of the center of gravity from the center of the circle is equal to $\frac{C^3}{12A}$.

For a solid having three axes of symmetry, all perpendicular to each other, like a sphere, cube, right parallelepiped, etc., their point of intersection is the center of gravity.

For a cone or pyramid, draw a line from the apex to the center of gravity of the base; the required center of gravity is one-fourth the length of this line from the base, measured on the line.

For two bodies, the larger weighing W lb., and the smaller P lb., the center of gravity will lie on the line joining the centers of gravity of the two bodies and at a distance from the larger body equal to $\frac{Pa}{P+W}$, where a is the distance between the centers of gravity of the two bodies.

For any number of bodies, first find the center of gravity of two of them as above, and consider them as one weight whose center of gravity is at the point just found. Find the center of gravity of this combined weight and a third body. So continue for the rest of the bodies, and the last center of gravity will be the center of gravity of the whole system of bodies.

MOMENT OF INERTIA.

The *moment of inertia* of a body or section is a **mathematical expression** that is much used in computations relating to rotating bodies and to the strength of materials.

It may be defined as follows:

The moment of inertia of a body, rotating about a given axis, is the sum of the products obtained by multiplying the weights of the elementary particles of which it is composed by the square of their distances from the axis.

It is often desirable to use the moment of inertia for a plane section; but as a plane surface has no weight, it is apparent that the above definition does not correctly apply. The following definition applies to plane surfaces:

The moment of inertia of a plane surface about a given axis is the sum of the products obtained by multiplying each elementary areas into which the surface may be conceived to be divided by the square of its distance from the axis.

The axis about which the body or surface rotates, or is assumed to rotate, i. e., the axis from which the distance to each area or particle is measured, is called the *axis of rotation*. The least moment of inertia is that value of the moment of inertia of a body or section when the axis of rotation passes through the center of gravity, since its value is less for that position of the axis than for any other.

To find the moment of inertia of a body about a given axis:

Divide the body or section into many small parts and multiply the weight or area of each part by the square of the distance from its center of gravity to the axis of rotation; the sum of these products will be the moment of inertia.

NOTE.—The results obtained by the above rules are really only approximate; for practically it is impossible to divide a body or surface into parts sufficiently small for absolute accuracy. The smaller the parts the more accurate will be the result; but the results obtained by these rules will always be *slightly too small*.

The moment of inertia is usually designated by the letter *I*.

Formulas for the values of *I* about an axis of rotation passing through the center of gravity of the section are given for various forms of sections in Table V, page 153.

The moment of inertia about an axis of rotation not passing through the center of gravity is equal to the moment of inertia about a parallel axis through the center of gravity plus the product of the entire weight of the body (or area of the section) multiplied by the square of the distance between the two axes.

EXAMPLE.—It is desired to find the moment of inertia of a 6" I-beam of the dimensions shown in Fig. 1 about an axis xy perpendicular to the web of the beam at the center.

SOLUTION.—Since the axis about which the moment of inertia is to be found is an axis of symmetry of the beam, it is necessary to make the computations only for the half section of the beam lying at one side of the axis, and multiply the result by 2. As stated before, the smaller the parts into which the area is divided, the more accurate will be the result.

It will be sufficiently accurate for present purposes to divide the section in the manner shown in Fig. 2.

The operations are given at the side of the figure, and will be readily understood. The sum of the products is the approximate value of the moment of inertia of this half of the section about the axis xy , and when multiplied by 2 is the approximate value of I for the entire section. It is found to equal 23.444.

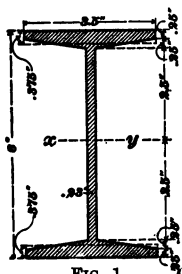


FIG. 1.

Area.	Square of Distance.
$3.50 \times .25 = .875$	$.875 \times 2.875^2 = 7.232$
$3.27 \times .125 = .409$	$.409 \times 2.667^2 = 2.907$
$.23 \times .50 = .115$	$.115 \times 2.50^2 = .719$
$.23 \times .50 = .115$	$.115 \times 2.00^2 = .460$
$.23 \times .50 = .115$	$.115 \times 1.50^2 = .259$
$.23 \times .50 = .115$	$.115 \times 1.00^2 = .115$
$.23 \times .50 = .115$	$.115 \times 0.50^2 = .029$
$.23 \times .25 = .058$	$.058 \times 0.125^2 = .001$
1.917	11.722
2	2
$A = 3.834$	$I = 23.444$

If the web of the beam is divided into areas $\frac{1}{2}$ in. in height (instead of $\frac{1}{4}$ in.), the value of I obtained will be 23.46 in. If the section is considered to be of the form indicated by the dotted lines in Fig. 1, and to have the same area as the original section, then, by the formula for the moment of inertia of an I-beam given in Table V, page 153, the value of

$$I = \frac{3.50 \times 6^3 - 3.27 \times 5.25^3}{12} = 23.57.$$

The true value is almost exactly 23.48 in. Any one of these values would be sufficiently correct for most practical purposes.

If it is desired to find the moment of inertia of a body about a given axis with reference to the *weight* of the body, the process is substantially the same as in the example given for the plane section, except that the *weight* of each small part of the body is taken instead of the *area* of each small part of the section.

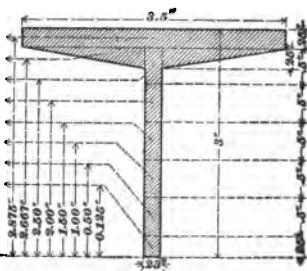


FIG. 2.

CENTER OF OSCILLATION.

The *center of oscillation* of a pendulum or other body vibrating or rotating about a fixed axis or center is that point at which, if the entire weight of the body were concentrated, the body would continue to vibrate in the same intervals of time.

When a pendulum, or other suspended body, is oscillating backward and forward, it is plain that those particles that are farther from the point of suspension travel through greater distances, and therefore move with greater velocities than those particles that are nearer the point of suspension.

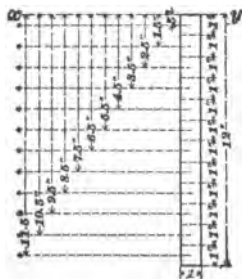
But there is evidently some point on the pendulum that travels through the same distance and has the same velocity as the average distance and average velocity of all the particles. This point is called the *center of oscillation*; it is not situated at the center of gravity. It always exists in the ball of a revolving governor or other rotating body. The axis or center around which the body rotates (corresponding to the point of suspension in pendulum) is the *axis of rotation*.

The distance from the axis, or center of rotation, to the center of oscillation is sometimes called the *true length of the pendulum*; it is also called the *radius of oscillation*; the latter name is preferable. To find the radius of oscillation:

Divide the moment of inertia of the body about the given axis of rotation by the product of the total weight of the body, multiplied by the distance from the given axis to the center of gravity of the body.

The centers of oscillation and of rotation (point of suspension) are *interchangeable*. If the position of a pendulum is reversed, and suspended from its center of oscillation, the pendulum will vibrate in the same intervals of time.

EXAMPLE.—It is desired to find the position of the center of oscillation of a wrought-iron bar 1 in. square and 12 in. long, axis of rotation perpendicular to the bar at one end:



<i>Weight of Each Cu. In.</i>	<i>Sq. of Dist.</i>	
.281	$\times 0.5^2 =$	0.070
.281	$\times 1.5^2 =$	0.632
.281	$\times 2.5^2 =$	1.756
.281	$\times 3.5^2 =$	3.442
.281	$\times 4.5^2 =$	5.690
.281	$\times 5.5^2 =$	8.500
.281	$\times 6.5^2 =$	11.872
.281	$\times 7.5^2 =$	15.806
.281	$\times 8.5^2 =$	20.302
.281	$\times 9.5^2 =$	25.360
.281	$\times 10.5^2 =$	30.980
.281	$\times 11.5^2 =$	37.162
3.372		161.572 = <i>I</i>

SOLUTION.—For the purposes of the example it will be sufficiently accurate to find the moment of inertia by considering the bar to be divided into 12 equal cubes, each containing 1 cu. in. of metal, as indicated in the figure, and the weight of each cube to be concentrated at its center of gravity.

The weight of 1 cu. in. of wrought iron is .281 lb., and of a bar 1 in. square and 1 ft. long it is $.281 \times 12 = 3.372$ lb. Hence, $I = .281 \times .5^2 + .281 \times 1.5^2 + \text{etc.} = 161.572$. (See page 128.) The exact value of I is 161.856; this shows that the approximate method is very close.

According to the rule previously given, if the moment of inertia is divided by the product of the weight of the body, by the distance from the axis of rotation to the center of gravity, the quotient will be the radius of oscillation.

Therefore, the distance from the exact center of oscillation of a wrought-iron bar, 1 in. square and 12 in. long, to an axis of rotation perpendicular to the end of the bar, is

$$\frac{161.856}{3.372 \times 6} = 8 \text{ in.,}$$

or two-thirds of the length of the bar.

The value of I for a bar of any cross-section, provided it is uniform throughout its length, revolving about an axis perpendicular to it and passing through its end, is

$$\frac{W l^2}{3},$$

in which W is the weight of the bar. and l is its length.

$$\text{Hence, } I = \frac{W l^2}{3} = \frac{3.372 \times 12^2}{3} = 161.856.$$

If the axis passes through the center of gravity of the bar,

$$I = \frac{W l^2}{12}.$$

CENTER OF PERCUSSION.

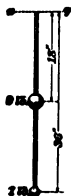
The *center of percussion* with respect to a given axis of rotation may be defined as the point of application of the resultant of the forces that cause the body to rotate. It is that point at which if a force is applied, the force will have no effect at the axis of rotation.

Strike anything solid, as an anvil, with a stick. If the end of the stick hits the anvil, the opposite end will sting your hand and will jerk in the direction in which the blow is struck; if the center of the stick hits the anvil it will again sting your hand, but you will jerk it in a direction opposite to the movement of the blow. But somewhere between the end and the center of the stick will be a point where it may hit the anvil and not sting your hand at all. This point is the center of percussion.

Level off the surface of some wet sand and lay a strip of board upon it (say 18 in. long and 3 in. wide). Strike or press the board near the center and the entire length of the board will be imprinted in the sand; but press it near one end and the opposite end will be raised up from the sand and will make no imprint. Between the center and the end of the board is a point that if pressed upon will cause no movement in the opposite end, i. e., the end of the board will neither press into the sand nor be lifted from it, but the imprint in the sand will diminish to zero at the end of the board. The point pressed or struck will be the center of percussion. If the board is of uniform width, the center of percussion will be at one-third of the distance from one end of the board.

Similarly in the preceding illustration, if the stick is of uniform size and weight, and your hand grasps it at one end, the point at which it can strike the anvil without affecting your hand will be at one-third the distance from the opposite end.

In all cases the center of percussion is identical with the center of oscillation, and its position is found in the same manner.



EXAMPLE.—It is desired to find the position of the center of oscillation or percussion of two balls fastened upon a rod. The first, weighing 2 lb., is at a distance of 18 in. from the axis of rotation, and the second, weighing 1 lb., is at a distance of 36 in. from the axis. (See figure.)

SOLUTION.—For simplicity, the rod will be assumed to have no weight. Consider the weight of each ball to be concentrated at its center of gravity.

The moment of inertia is found as follows:

<i>Wt.</i>	<i>Sq. of Dist.</i>	
2	$\times 18^2$	= 648
1	$\times 36^2$	= 1,296
		1,944 = <i>I</i> .

The center of gravity of the two balls is found to be at a distance of 6 in. from the larger, or 24 in. from the axis of rotation (see page 124), and the combined weight of the two balls is $2 + 1 = 3$ lb. Therefore, the center of percussion is found to be at a distance of $\frac{1,944}{3 \times 24} = 27$ in. from the axis of rotation.

But, in an actual case, the rod would have weight, and its moment of inertia must be considered as well as the moment of inertia of the balls.

If we assume that the rod is of steel, $\frac{3}{8}$ in. in diameter and 36 in. long, it will weigh $\left(\frac{3}{8}\right)^2 \times .7854 \times 36 \times .283 = 1.125$ lb. .283 lb. is the weight of 1 cu. in. of steel.

Using the formula given on page 129,

$$I = \frac{WT^2}{3} = \frac{1.125 \times 36^2}{3} = 486.$$

Adding this result to the former, $1,944 + 486 = 2,430$ = moment of inertia of rods and balls. The center of gravity of the combination is found by the formula (see page 124)

$\frac{Pa}{P+W}$. Substituting, $\frac{1.125 \times 6}{1.125 + 3} = 1\frac{1}{3}$. $24 - 1\frac{1}{3} = 22\frac{2}{3}$ in. = distance from end of rod to center of gravity.

Applying the rule given for finding the center of oscillation, the distance of the center of percussion from the end of the bar is $\frac{2,430}{(1 + 2 + 1.125) \times 22\frac{2}{3}} = 26.34$ in., very nearly.

RADIUS OF GYRATION.

The *center of gyration* is that point in a revolving body at which, if the entire mass of the body were concentrated, the moment of inertia with respect to a given axis would be the same as in the body.

An ounce of cork occupies about 94 times as much space as

an ounce of platinum; but the ounce of platinum can have the same moment of inertia as the ounce of cork, if its center of gyration has the same position with respect to the axis of rotation.

The center of gyration is not at the center of gravity, nor at the center of oscillation, but at some point in a straight line between those centers.

The radius of gyration is the distance from the axis of rotation to the center of gyration.

The square of the radius of gyration is the average of the squares of the distances from the axis of rotation to each elementary particle of the body, or to each elementary area of the section, as the case may be. But the sum of these squares of distances, multiplied by the weight or area of each elementary part, equals the moment of inertia; therefore, the moment of inertia divided by the weight of the body or area of the section equals the square of the radius of gyration; the square root of this quotient is the radius of gyration.

But, according to the rule for finding the radius of oscillation, the quotient obtained by dividing the moment of inertia by the weight or area equals the product of the distance from the axis of rotation to the center of gravity, multiplied by the radius of oscillation; and, therefore, *the radius of gyration is a mean proportional between these distances.*

If the distance from the axis of rotation to the center of gravity is known, and the radius of oscillation is known, the radius of gyration may be found by multiplying these two known distances together and extracting the square root of the product.

In the example of the I-beam, Fig. 2, page 126, the sum of the areas of the half section of the beam is 1.917, and the area of the entire section is 3.834 sq. in. Therefore, the radius of gyration of this beam about an axis through the center of gravity perpendicular to the web = $\sqrt{\frac{23.44}{3.834}} = 2.47$ in.

In the example of the iron bar 12 in. long (see figure, page 128), the distance from the axis of rotation to the center of gravity is 6 in., and the radius of oscillation was found to equal 8 in. Therefore, the radius of gyration about an

axis perpendicular to the bar at one end $= \sqrt{6 \times 8} = 6.93$ in. Or, the moment of inertia of the bar $= 161.586$, and the weight of the bar $= 3.372$ lb. Therefore, the radius of gyration $= \sqrt{\frac{161.586}{3.372}} = 6.93$ in., very nearly.

The radius of gyration is used in determining the strength of columns. The axis must be taken in such a direction that the result will be the *least* radius of gyration of the column; this condition is usually obtained when the axis is perpendicular to the least diameter or side of the column.

The various relations between these quantities may be concisely expressed by the following formulas, in which A = area of section (or weight of body if the weight is used); g = distance from axis of rotation to center of gravity; G = radius of gyration; r_o = radius of oscillation; I = moment of inertia.

Then,

$$\begin{array}{l|l|l} I = A G^2. & I = A g r_o. & G^2 = g r_o. \\ G = \sqrt{\frac{I}{A}}. & g = \frac{I}{A r_o}. & r_o = \frac{I}{A g}. \\ G = \sqrt{g r_o}. & g = \frac{G^2}{r_o}. & r_o = \frac{G^2}{g}. \end{array}$$

$$g : G = G : r_o.$$

To find the radius of oscillation, radius of gyration, and moment of inertia, experimentally.

The connecting-rod of an engine is represented in the

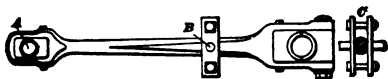


figure. It is desired to find the moment of inertia of the rod about an axis of rotation through the center of the crosshead pin A .

This may be accomplished, experimentally, as follows: Suspend the rod from the crosshead pin in such a manner

that it will swing freely; cause it to swing, or oscillate, and note the exact time of the vibrations. Remove the crosshead pin and reverse the rod, but, instead of suspending it by the crankpin, suspend it by a movable pin *B*, that can be clamped at any desired point upon the rod. *C* is another view of this pin. There will be a point on the rod from which it may be suspended by means of the movable pin, so that it will vibrate in exactly the same intervals of time as when suspended from the crosshead pin. This point is the *center of oscillation*, for the center of oscillation and the center of rotation are interchangeable; the point will be found at about one-third the length of the rod from the crankpin. Find this center of oscillation, experimentally, and carefully measure the distance from the center of the movable pin to the center of the crosshead-pin hole. This distance is the *radius of oscillation* = r_o . Next remove the movable pin, and find the center of gravity (lengthwise) of the rod by balancing it across a knife edge, and measure the distance from the center of gravity thus found to the center of the crosshead-pin hole; this distance = g . Finally, weigh the rod.

The product of the weight (= A), the radius of oscillation (= r_o), and the distance from the center of crosshead pin (axis of rotation) to the center of gravity (= g) will be the moment of inertia. For, by the formula, $I = A g r_o$. The radius of gyration G may be found by the formula

$$G = \sqrt{\frac{I}{A}}, \text{ or } G = \sqrt{g r_o}.$$

MOMENT OF RESISTANCE.

If the moment of inertia of the cross-section of a beam is divided by the distance from the neutral axis (see definition on next page) to the extreme fiber, i. e., the fiber that is farthest from the axis, the quotient will be the quantity known as the *moment of resistance*.

It is evident that, if a beam is strained by a vertical load, the greatest stress will be in the extreme upper and lower fibers of the beam.

The intensity of the stress that can be borne by the extreme fibers is the limit of the strength of the beam.

The upper fibers are compressed and the lower fibers are stretched, but somewhere along or near the center of a vertical section of the beam, the fibers are neither extended nor compressed; the position of these fibers is called the *neutral surface*, and the line where this neutral surface intersects a right section of the beam is the *neutral axis* of the section.

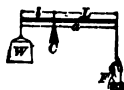
The neutral axis passes through the center of gravity of the section.

If the moment of resistance is multiplied by the amount of stress that may be allowed per square inch upon the extreme fiber, the product will represent the efficiency of the beam to resist bending moment.

EXAMPLE.—Referring to the 6" I-beam, Figs. 1 and 2, pages 126 and 127, for which the moment of inertia of the section has been found, it is desired to ascertain the load that a wrought-iron beam of the same dimensions as Fig. 1 will carry at the center of a span 8 ft. between supports.

SOLUTION.—The moment of resistance for the section = $\frac{23.48}{8} = 7.83$. In Table II, page 151, the ultimate strength or fiber stress for wrought iron is given as 50,000 lb. per sq. in., and in Table I, page 151, the factor of safety given for wrought iron under a steady stress is 4; therefore, the safe fiber stress for wrought iron = $\frac{S}{f} = \frac{50,000}{4} = 12,500$ lb. per sq. in., and the moment of resistance multiplied by the safe fiber stress, or $\frac{SR}{f} = 7.83 \times 12,500 = 97,875$ in.-lb. But $l = 8$ ft., or 96 in.; equating the bending moment for a load at the center of a beam $\left(= \frac{Wl}{4} \right)$ with the moment of resistance, or putting $M = \frac{SR}{f} = \frac{Wl}{4}$; then $\frac{96 W}{4} = 97,875$; therefore, $W = 4,078$ lb., the load that can be safely supported at the center of the beam.

MECHANICAL POWERS.



$$F : W = l : L. \quad FL = Wl$$

$$F = \frac{Wl}{L}. \quad W = \frac{FL}{l}.$$

$$l = \frac{Fa}{W + F}. \quad L = \frac{Wa}{W + F}$$



$$F : W = l : L. \quad FL = Wl$$

$$F = \frac{Wl}{L}. \quad W = \frac{FL}{l}.$$

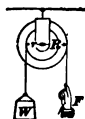
$$L = \frac{Wa}{W - F}. \quad l = \frac{Fa}{W - F}$$



$$F : W = l : L. \quad FL = Wl$$

$$F = \frac{Wl}{L}. \quad W = \frac{FL}{l}.$$

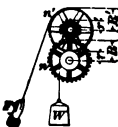
$$L = \frac{Wa}{F - W}. \quad l = \frac{Fa}{F - W}.$$



$$F : W = r : R. \quad FR = Wr.$$

$$F = \frac{Wr}{R}. \quad R = \frac{Wr}{F}.$$

$$W = \frac{RF}{r}. \quad r = \frac{RF}{W}.$$



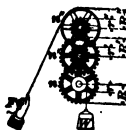
$$F = \frac{Wr r'}{R R'}. \quad W = \frac{F R R'}{r r'}.$$

n = number of revolutions of large gear.

$$n : n' = r' : R.$$

$$v : v' = r r' : R R'.$$

v = velocity of W ; v' = velocity of F .



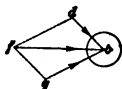
$$F = \frac{W r r' r''}{R R' R''}. \quad W = \frac{F R R' R''}{r r' r''}.$$

$$n : n'' = r' r'' : R R'.$$

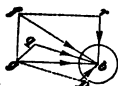
$$v : v' = r r' r'' : R R' R''.$$

$r, r', r'',$ etc. = radii of the pinions;
 $R, R', R'',$ etc. = radii of the wheels.

Let db and qb represent the magnitudes and directions of two forces that act to move the body b . By completing the parallelogram there will be obtained a diagonal force fb , whose magnitude and direction are equal to the effect produced by db and qb . fb is called the resultant of db and qb .



If three or more forces act in different directions to move a body b , find the resultant of any two of them, and consider it as a single force. Between this and the next force find a second resultant. Thus, pb , qb , and rb are magnitudes and directions of the forces. $pb + qb + rb = fb$, the magnitude and direction of the three forces, pb , qb , and rb .

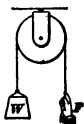


A SINGLE FIXED PULLEY.

$$F = W.$$

$$v = v'.$$

v = velocity of W ; v' = velocity of F .



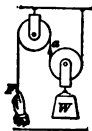
A SINGLE MOVABLE PULLEY.

$$F : W = 1 : 2, \text{ or } F = \frac{1}{2} W.$$

If the force F be applied at a and act upwards, the result will be the same.

$$v' = 2v.$$

v = velocity of W ; v' = velocity of F .



A DOUBLE MOVABLE PULLEY.

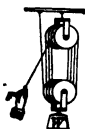
$$F : W = 1 : 4, \text{ or } F = \frac{1}{4} W.$$

Let u = number of parts of rope, not counting the free end.

$$F = W \div u. \quad v : v' = 1 : u.$$

v = velocity of W ; v' = velocity of F .



**QUADRUPLE MOVABLE PULLEY.**

$$F = \frac{1}{4} W. \quad F : W = 1 : 4.$$

Let u = number of parts of rope, not counting the free end; then,

$$F = W \div u. \quad v : v' = 1 : u.$$

v = velocity of W ; v' = velocity of F .

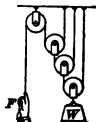
COMPOUND PULLEY.

u = number of movable pulleys.

$$F = \frac{W}{2^u}. \quad W = 2^u F.$$

$$v : v' = 1 : 2^u.$$

v = velocity of W ; v' = velocity of F .

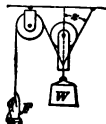
**DIFFERENTIAL PULLEY.**

$$W = \frac{2PR}{R-r}.$$

AN OBLIQUE FIXED PULLEY.

$$F : W = 1 : 2 \cos z.$$

$$W = 2 F \cos z. \quad F = \frac{W}{2 \cos z}.$$

**INCLINED PLANE.**

$$F = \frac{Wh}{l} = W \sin a.$$

$$W = \frac{Fl}{h} = \frac{F}{\sin a}.$$

WEDGE.

F = force required to drive the wedge;

R = resistance.

$$F = \frac{Ra}{l}. \quad R = \frac{Fl}{a}.$$



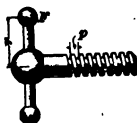
SCREW.

P = pitch of the screw;

r = radius on which the force F acts.

$$F : W :: P : 2\pi r.$$

$$F = \frac{WP}{2\pi r} \quad W = \frac{2\pi r F}{P}.$$



WORK.

Work is the overcoming of resistance through a distance. The unit of work is the *foot-pound*; that is, it equals 1 pound raised vertically 1 foot. The amount of work done is equal to the resistance in pounds multiplied by the distance in feet through which it is overcome. If a body is lifted, the resistance is the weight or the overcoming of the attraction of gravity, the work done being the weight in pounds multiplied by the height of the lift in feet. If a body moves in a horizontal direction, the work done is the friction overcome, or the force needed to move a resistant body or combination of bodies, multiplied by the distance moved. In order to compare the different amounts of work done by different systems of forces, time is also considered.

One *horsepower* is 550 ft.-lb. of work in 1 second, or 33,000 ft.-lb. in 1 minute, or 1,980,000 ft.-lb. in 1 hour.

The work necessary to be done in raising a body weighing W lb. through a height of h ft. equals Wh ft.-lb. The total work that any moving body is capable of doing in being brought to rest equals its kinetic energy, or $\frac{Wv^2}{2g}$, when v is the velocity in feet per second.

Thus, the work that a cannon ball weighing 800 lb. and traveling with a velocity of 1,200 ft. per sec. could do, is $\frac{800 \times 1,200^2}{2 \times 32.16} = 17,910,447$ ft.-lb.

If stopped in 1 min., the horsepower would be $17,910,447 \div 33,000 = 542.8$, nearly.

FORCE OF A BLOW.

In order to determine the force of a blow, the velocity of the object at the instant of striking must be known, and also the time required to bring the body to rest. It is a very difficult matter to determine the exact time, but a close approximation to the striking force may be obtained by dividing the kinetic energy of the body at the instant of striking by the average amount of penetration or compression produced by the striking body.

Let F = striking force in pounds;

W = weight of striking body in pounds;

v = velocity of striking body in feet per second;

R = distance penetrated or amount of compression = the distance through which the resistance acts, in feet;

t = time required to bring the body to rest;

h = height in feet which would produce the velocity v .

$$\text{Then, } F = \frac{Wv}{gt}, \text{ or } F = \frac{Wv^2}{2gR} = \frac{Wh}{R}.$$

EXAMPLE.—A steam hammer weighing 1,000 lb. (with its piston) falls from a height of 8 ft., and compresses a piece of iron $\frac{1}{4}$ in.; what is its striking force?

SOLUTION.—If gravity be considered as the only force acting, the steam on top of the piston being used to prevent a rebound of the hammer,

$$F = \frac{Wh}{R} = \frac{1,000 \times 8}{(\frac{1}{4} \div 12)} = 1,000 \times 8 \times 8 \times 12 = 768,000 \text{ lb.}$$

Divide $\frac{1}{4}$ in. by 12, to obtain the amount of compression in feet or parts of a foot.

BELTING.

D = diameter of larger pulley in inches;

d = diameter of smaller pulley in inches;

N = revolutions per minute of larger pulley;

n = revolutions per minute of smaller pulley;

W = width of double belt in inches;

w = width of single belt in inches;

H = horsepower that can be transmitted by the belt.

Then,

$$H = \frac{D N w}{2,750} \text{ for single belts.}$$

$$H = \frac{D N W}{1,925} \text{ for double belts.}$$

$$w = \frac{2,750 H}{D N} = \frac{2,750 H}{d n}.$$

$$W = \frac{1,925 H}{D N} = \frac{1,925 H}{d n}.$$

$$D = \frac{2,750 H}{w N} \text{ for single belt.}$$

$$D = \frac{1,925 H}{W N} \text{ for double belt.}$$

$$N = \frac{2,750 H}{w D} \text{ for single belt.}$$

$$N = \frac{1,925 H}{W D} \text{ for double belt.}$$

The above rules are for open belts and pulleys having the same diameter, the arc of contact being, in this case, half the circumference, or 180° . For open belts and pulleys of different diameters, the arc of contact is less than 180° on the smaller pulley, and a different constant, to be taken from the following table, must be substituted in the formulas. To find the arc of contact, let l be the distance in inches between the centers of the pulleys. Then, $\frac{D-d}{2l} = \text{cosine of half the angle}$. Find this half angle from a table of natural cosines, and

Degrees.	Fraction of Circumference.	Single Belt Constant.	Double Belt Constant.
90	$\frac{1}{4} = .25$	6,080	4,250
$112\frac{1}{2}$	$\frac{5}{8} = .3125$	4,730	3,310
120	$\frac{1}{3} = .3333$	4,400	3,080
135	$\frac{3}{8} = .375$	3,850	2,700
150	$\frac{1}{2} = .4167$	3,410	2,390
$157\frac{1}{2}$	$\frac{5}{8} = .4375$	3,220	2,250
180 to 270	$\frac{1}{2} \text{ to } \frac{3}{4} = .5 \text{ to } .75$	2,750	1,925

multiply by 2. The result is the arc of contact in degrees. Find the number in the first column of the table, which is nearest to this result, and use the constant corresponding to

that number. If a table of natural cosines is not at hand, measure the length of the arc of contact on the smaller pulley and divide it by the circumference of the pulley. Find the fraction in the second column that corresponds nearest to this result, and opposite this its corresponding constant.

EXAMPLE.—What must be the width of a single belt to transmit 12 horsepower, when the diameter of the larger pulley is 42 in., of the smaller pulley 20 in., distance between their centers 14 ft. = 168 in., and R. P. M. of smaller pulley 150?

SOLUTION.— $\frac{42 - 20}{2 \times 168} = .06548 = \text{cosine of half the arc of contact, which thus} = 86^\circ 15', \text{ nearly; } 86^\circ 15' \times 2 = 172^\circ = \text{arc of contact; the nearest number in the table is } 180^\circ, \text{ and the corresponding constant is } 2,750; \text{ hence, } w = \frac{2,750 \times 12}{20 \times 150} = 11 \text{ in.}$

Oak-tanned leather makes the best belts. When belts are run with the hair side over the pulley, they have greater adhesion.

The ordinary thickness of leather belts is $\frac{1}{4}$ in., and their weight is about 60 lb. per cu. ft.

Ordinarily, four-ply cotton belting is considered equivalent to single-leather belting.

RULES FOR CALCULATING THE SPEED OF GEARS OR PULLEYS.

In calculating for gears, multiply or divide by the diameter or the number of teeth, as may be required. In calculating for pulleys, multiply or divide by their diameters in inches.

The driving wheel is called the *driver*, and the driven wheel the *driven* or *follower*.

PROBLEM I.

The revolutions of driver and driven, and the diameter of the driven, being given, required the diameter of the driver.

Rule.—*Multiply the diameter of the driven by its number of revolutions, and divide by the number of revolutions of the driver.*

PROBLEM II.

The diameter and revolutions of the driver being given, required the diameter of the driven to make a given number of revolutions in the same time.

Rule.—*Multiply the diameter of the driver by its number of revolutions, and divide the product by the required number of revolutions.*

PROBLEM III.

The diameter or number of teeth, and number of revolutions of the driver, with the diameter or number of teeth of the driven, being given, required the revolutions of the driven.

Rule.—*Multiply the diameter or number of teeth of the driver by its number of revolutions, and divide by the diameter or number of teeth of the driven.*

PROBLEM IV.

The diameter of driver and driven, and the number of revolutions of the driven, being given, required the number of revolutions of the driver.

Rule.—*Multiply the diameter of the driven by its number of revolutions, and divide by the diameter of the driver.*

PUMPS.

In all pumps, whether lifting, force, steam, single-acting, double-acting, or centrifugal, the number of foot-pounds of work performed by the pump is equal to the weight of the water discharged in pounds, multiplied by the vertical distance in feet between the level of the water in the well or source and the point of discharge, plus the work done in overcoming the friction and other resistances. (It is assumed that the water is delivered with practically no velocity.)

To find the discharge of a pump in gallons per minute:

Let T = piston travel in feet per minute;

d = diameter of cylinder in inches;

G = number of gallons discharged per minute.

Then,

$$G = .03264 T d^2.$$

To find the horsepower of a pump, use the following formula, in which T and d are the same as above, and h is the vertical distance in feet between the level of the water at the source and the point of discharge:

$$H. P. = .00083724 G h = .00001238 T d^2 h.$$

In both the above formulas, allowance has been made for friction, leakage, etc.

DUTY.

The duty of a pump is the number of foot-pounds of work actually done for 100 lb. of coal burned.

$$\text{Duty} = 835.53 \frac{G h}{W},$$

where W = weight of coal burned, in pounds.

HYDROMECHANICS.**HYDROSTATICS.**

Hydrostatics treats of liquids at rest under the action of forces. If a liquid is acted on by a pressure, the pressure per unit of area exerted anywhere on the mass of liquid is transmitted undiminished in all directions, and acts with the same force on all surfaces, in a direction at right angles to those surfaces.

General Law for the Downward Pressure on the Bottom of Any Vessel.—The pressure on the bottom of a vessel containing a liquid is independent of the shape of the vessel, and is equal to the weight of a prism of the liquid whose base is the same as the bottom of the vessel, and whose altitude is the distance between the bottom and the upper surface of the liquid, plus the pressure per unit of area upon the upper surface of the liquid multiplied by the area of the bottom of the vessel.

General Law for Upward Pressure.—The upward pressure on any submerged horizontal surface equals the weight of a prism of the liquid whose base has an area equal to the area of the submerged surface, and whose altitude is the distance between the submerged surface and the upper surface of the liquid, plus the pressure per unit of area on the upper surface of the liquid multiplied by the area of the submerged surface.

General Law for Lateral Pressure.—The pressure on any vertical surface due to the weight of the liquid is equal to the weight of a prism of the liquid whose base has the same area as the vertical surface, and whose altitude is the depth of the center of gravity of the vertical surface below the level of the liquid. Any additional pressure is to be added, as in the previous cases.

Pressure on Oblique Surfaces.—The pressure exerted by a liquid in any direction on a plane surface is equal to the weight of a prism of the liquid whose base is the projection of the surface at right angles to the given direction, and whose height is the depth of the center of gravity of the surface below the level of the liquid.

If a cylinder is filled with water, and a pressure applied, the total pressure on any half section of the cylinder is equal to the projected area of the half cylinder (or the diameter multiplied by the length of the cylinder) multiplied by the depth of the center of gravity of the half cylinder, multiplied by the weight of a cubic inch of water, plus the diameter of the shell, multiplied by the pressure per square inch, multiplied by the length of the cylinder.

If d = the diameter, and l = the length of the cylinder, the pressure due to the weight of the water when the cylinder is vertical upon the half cylinder = $d \times l \times \frac{l}{2} \times$ the weight of a cubic inch of water = $d \times \frac{l^2}{2} \times$ the weight of a cubic inch of water; d and l are to be measured in inches.

The pressure in pounds per square inch due to a head of water is equal to the head in feet multiplied by .434.

The head equals the pressure in pounds per square inch multiplied by 2.304.

EXAMPLE.—(a) What is the pressure per square inch corresponding to a head of water of 175 ft. ? (b) If the pressure had been 90 lb. per sq. in., what would the head have been ?

SOLUTION.—(a) $175 \times .434 = 75.95$ lb. per sq. in.

(b) $90 \times 2.304 = 207.36$ ft.

HYDROKINETICS.

Hydrokinetics, also called hydrodynamics and hydraulics, treats of water in motion. When water flows in a pipe, conduit, or channel of any kind, the velocity is not the same at all points of the flow, unless all cross-sections of the pipe or channel are equal. That velocity which, being multiplied by the area of the cross-section of the stream, will equal the total quantity discharged, is called the *mean velocity*.

Let Q = quantity that passes any section in 1 second;

A = area of the section;

v = mean velocity in feet per second.

Then, $Q = Av$, and $v = \frac{Q}{A}$.

The vertical distance between the level surface of the water and the center of the aperture through which it flows, is called the *head*.

Let V = mean velocity of efflux through a small aperture;

h = head in feet at the center of the aperture;

w = weight of water flowing through the aperture per second.

Then, $V = \sqrt{2gh}$; that is, the velocity of efflux is the same as if the water had fallen through a height equal to the head.

Let Q = theoretical number of cubic feet discharged per second;

V_m = mean velocity through orifice in feet per second;

A = area of orifice;

h = theoretical head necessary to give a mean velocity V_m ;

Q_a = actual quantity discharged in cubic feet per second.

Then, for an orifice in a thin plate, or a square-edged orifice (the hole itself may be of any shape, triangular, square, circular, etc., but the edges must not be rounded), the actual quantity discharged is

$$Q_a = .615 Q = .615 A V_m.$$

The *weir* is a device used for measuring the discharge of water. It is a rectangular orifice through which the water flows.

If d = the depth of the opening in feet, and b its breadth in feet, the area of the opening is $A = d \times b$, and the theoretical discharge is $Q = d \times b \times V_m = db \times \frac{1}{2} \sqrt{2gd}$, the head for this case being taken as d .

The actual discharge when the top of the weir lies at the surface of the water is

$$Q_a = .615 Q = .615 \times db \times \frac{1}{2} \sqrt{2gd} = .615 \times \frac{1}{2} b \sqrt{2gd^3} = 3.288 b \sqrt{d^3}.$$

If h_1 is the depth in feet of the top of a weir below the surface of the water, and h is the depth in feet of the bottom of the weir below the surface of the water, the actual discharge Q_a , in cubic feet per second, is

$$Q_a = .615 \times \frac{1}{2} b \sqrt{2g} (\sqrt{h^3} - \sqrt{h_1^3}) = 3.288 b (\sqrt{h^3} - \sqrt{h_1^3}).$$

FLOW OF WATER IN PIPES.

- Let V_m = mean velocity of discharge in feet per second;
 h = total head in feet = vertical distance between the level of water in reservoir and the point of discharge;
 l = length of pipe in feet;
 d = diameter of pipe in inches;
 f = coefficient of friction.

Then, for straight cylindrical pipes of uniform diameter, the mean velocity of efflux may be calculated by the formula,

$$V_m = 2.315 \sqrt{\frac{hd}{fl + 125d}}. \quad (a)$$

NOTE.—The head is always taken as the vertical distance between the point of discharge and the level of the water at the source, or point from which it is taken, and is always measured in feet. It matters not how long the pipe is—whether vertical or inclined, whether straight or curved, nor whether any part of the pipe goes below the level of the point of discharge or not—the head is always measured as stated above.

EXAMPLE.—What is the mean velocity of efflux from a 6" pipe, 5,780 ft. long, if the head is 170 ft.? Take $f = .021$.

SOLUTION.—

$$V_m = 2.315 \sqrt{\frac{hd}{f^2 + .125d}} = 2.315 \sqrt{\frac{170 \times 6}{.021 \times 5,780 + (.125 \times 6)}} = 6.69 \text{ ft. per sec.}$$

When the pipe is very long compared with the diameter, as in the above example, the following formula may be used:

$$V_m = 2.315 \sqrt{\frac{hd}{f}}, \quad (b)$$

in which the letters have the same meaning as in the preceding formula. This formula may be used when the length of the pipe exceeds 10,000 times its diameter.

The actual head necessary to produce a certain velocity V_m may be calculated by the formula

$$h = \frac{f l V_m^2}{5.36 d} + .0233 V_m^2. \quad (c)$$

If the head, the length of the pipe, and the diameter of the pipe are given, to find the discharge, use the formula

$$Q = .09445 d^2 \sqrt{\frac{h d}{f l + .125 d}}; \quad (d)$$

that is, the discharge in gallons per second equals .09445 times the square of the diameter of the pipe in inches, multiplied by the square root of the head in feet, multiplied by the diameter of the pipe in inches, divided by the coefficient of friction times the length of the pipe in feet, plus .125 times the diameter of the pipe in inches.

To find the value of f , calculate V_m by formula (b) assuming that $f = .025$, and get the final value of f from the following table:

V_m	f	V_m	f	V_m	f
.1	.0686	.7	.0349	2	.0265
.2	.0527	.8	.0336	3	.0243
.3	.0457	.9	.0325	4	.0230
.4	.0415	1	.0315	6	.0214
.5	.0387	$1\frac{1}{4}$	.0297	8	.0205
.6	.0365	$1\frac{1}{2}$	.0284	12	.0193

EXAMPLE.—The length of a pipe is 6,270 ft., its diameter is 8 in., and the total head at the point of discharge is 215 ft. How many gallons are discharged per minute?

SOLUTION.—

$$V_m = 2.315 \sqrt{\frac{215 \times 8}{.025 \times 6,270}} = 7.67 \text{ ft. per sec., nearly.}$$

Using the value of $f = .0205$ for $V_m = 8$ (see table), $Q =$

$$.09455 \times 8^2 \sqrt{\frac{215 \times 8}{.0205 \times 6,270 + (.125 \times 8)}} = 22.03 \text{ gal. per sec.} = 22.03 \times 60 = 1,321.8 \text{ gal. per min.}$$

If it is desired to find the head necessary to give a discharge of a certain number of gallons per second through a pipe

whose length and diameter are known, calculate the mean velocity of efflux by using the formula

$$V_m = \frac{24.51 Q}{d^2}; \quad (e)$$

find the value of f from the table, corresponding to this value of V_m , and substitute these values of f and V_m in the formula for the head.

EXAMPLE.—A 4" pipe, 2,000 ft. long, is to discharge 24,000 gal. of water per hr.; what head is necessary?

SOLUTION.— $\frac{24,000}{60 \times 60} = 6\frac{2}{3}$ gal. per sec. $V_m = \frac{24.51 \times 6\frac{2}{3}}{4^2} = 10.2$ ft. per sec.

From the table, $f = .0205$ for $V_m = 8$, and $.0193$ for $V_m = 12$; assume that $f = .02$ for $V_m = 10.2$.

$$\text{Then, } h = \frac{.02 \times 2,000 \times 10.2^2}{5.36 \times 4} + .0233 \times 10.2^2 = 196.53 \text{ ft.}$$

To find the diameter of a pipe that will give any required discharge in gallons per second, the total length of the pipe and the head being known, find the value of d by formula (f); substitute this value in formula (e), and find the value of V_m . Then find from the table the value of f corresponding to this value of V_m . Substitute the values of d and f just found in the right-hand member of formula (g) and solve for d ; the result will be the diameter of the pipe, accurate enough for all practical purposes.

$$d = 1.229 \sqrt[5]{\frac{l Q^2}{h}}. \quad (f) \quad d = 2.57 \sqrt[5]{\frac{(f l + \frac{1}{8} d) Q^2}{h}}. \quad (g)$$

EXAMPLE.—A pipe 2,000 ft. long is required to discharge 24,000 gal. of water per hr. The head being 195 ft., what should be the diameter of the pipe?

SOLUTION.— $Q = \frac{24,000}{60 \times 60} = 6\frac{2}{3}$ gal. per sec. Substituting in formula (f), $d = 1.229 \sqrt[5]{\frac{2,000 \times (6\frac{2}{3})^2}{195}} = 4.18 + \text{in.}$

Substituting this value in formula (e), $V_m = \frac{24.51 \times 6\frac{2}{3}}{4.18^2} = 9.352$ ft. per sec. From the table, the value of f for $V_m = 9.352$ is $.0201$. Substituting this value of f and the value of d , found above, in formula (g),

$$d = 2.57 \sqrt[5]{\frac{(.0201 \times 2,000 + \frac{1}{8} \times 4.18) \times (6\frac{2}{3})^2}{195}} = 4.01 +; \text{ say, } 4 \text{ in.}$$

STRENGTH OF MATERIALS.

The ultimate strengths of different materials vary greatly from the average values given in the following tables. In actual practice, the safest procedure would be to make a test of the material for its ultimate strength and coefficient of elasticity, or else specify in the contract that it shall not fall below certain prescribed limits. In the following formulas,

A = area of cross-section of material in square inches;

E = coefficient of elasticity in pounds per square inch;

G^2 = square of least radius of gyration;

I = moment of inertia about an axis passing through the center of gravity of the cross-section;

M = maximum bending moment in inch-pounds;

P = total stress in pounds;

R = moment of resistance;

S = ultimate stress in lb. per sq. in. of area of section;

W = weight placed on a beam in pounds;

b = breadth of cross-section of beam in inches;

d = depth of beam (in.) = diam. of circ. section = altitude of triangular section = length of vertical side;

e = amount of elongation or shortening in inches;

f = factor of safety;

l = length in inches;

p = pressure in pounds per square inch;

π = ratio of circumference to diameter = 3.1416, nearly;

q = a constant used in formula for columns;

r = radius of a circular section;

s = elastic set or deflection in inches of a beam under a transverse (bending) stress;

t = thickness of a shell or hollow section.

For tension, compression (where the piece does not exceed 10 times its least diameter), and shear,

$$P = \frac{AS}{f}. \quad (1)$$

To find the breaking stress (P), make $f = 1$. For safe load, take f from Table I, and S from Table II, according to the nature and character of stress.

TABLE I.
FACTORS OF SAFETY (*f*).

Name of Material.	Steady Stress.	Varying Stress.	Shocks (Machines).
Cast iron	6	15	20
Wrought iron	4	6	10
Steel	5	7	15
Wood	8	10	15
Brick and stone	15	25	80

TABLE II.
ULTIMATE STRENGTHS (*S*).

Name of Material.	Tension.	Compression.	Shear.	Flexure.
Cast iron	20,000	90,000	20,000	86,000
Wrought iron	50,000	50,000	47,000	50,000
Steel	100,000	150,000	70,000	120,000
Wood	10,000	8,000	600 to 3,000	9,000
Stone		6,000		2,000
Brick	200	2,500		

EXAMPLE.—A square cast-iron pillar 18 in. long is required to sustain a steady load of 75,000 lb.; what must be the length of a side?

SOLUTION.—From the table, $f = 6$, and $S = 90,000$. By formula (1),

$$P = \frac{AS}{f}, \text{ or } A = \frac{Pf}{S} = \frac{75,000 \times 6}{90,000} = 5 \text{ sq. in.}$$

Length of side = $\sqrt{5} = 2.236$ in., say 2½ in.

The amount of elongation or of shortening of a piece under a stress is given by the formula

$$e = \frac{Pl}{AE} \quad (2)$$

The coefficient of elasticity (E) must be taken from the following table:

TABLE III.

Name of Material.	Coefficient of Elasticity.	Elastic Limit for Tension.
Cast iron	15,000,000	6,000
Wrought iron	25,000,000	25,000
Steel	30,000,000	50,000
Wood	1,500,000	3,000

A wrought-iron bar 24 ft. long, $1\frac{1}{4}$ in. in diameter, would elongate, under a tensile stress of 15 tons,

$$e = \frac{(15 \times 2,000) \times (24 \times 12)}{\frac{1}{4} \pi (1\frac{1}{4})^2 \times 25,000,000} = .196 \text{ in.}$$

To find the breaking strength of a beam, use the formula

$$M = SR. \quad (3)$$

Obtain M and R from the two following tables, according to the kind of beam and nature of cross-section. A simple beam is one merely supported at its ends. In the expression for R , d is always understood to be the vertical side or depth; hence, that beam is the stronger which always has its greatest depth or longest side vertical. The moment of inertia I is taken about an axis perpendicular to d , and lying in the same plane.

TABLE IV.

Kind of Beam and Manner of Loading.	Bending Moment. M	Deflection. δ
Cantilever, load at end	Wl	$\frac{1}{8} \frac{Wl^3}{EI}$
Cantilever, uniformly loaded	$\frac{1}{2} Wl$	$\frac{1}{8} \frac{Wl^3}{EI}$
Simple beam, load at middle	$\frac{1}{4} Wl$	$\frac{1}{16} \frac{Wl^3}{EI}$
Simple beam, uniformly loaded	$\frac{1}{8} Wl$	$\frac{1}{16} \frac{Wl^3}{EI}$
Beam fixed at both ends, load at middle	$\frac{1}{8} Wl$	$\frac{1}{32} \frac{Wl^3}{EI}$
Beam fixed at both ends, uniformly loaded	$\frac{1}{16} Wl$	$\frac{1}{32} \frac{Wl^3}{EI}$

TABLE V.

Name of Section.		I	R	G^2
Solid circular		$\frac{\pi d^4}{64}$	$\frac{\pi d^3}{32}$	$\frac{d^2}{16}$
Hollow circular		$\frac{\pi (d^4 - d_1^4)}{64}$	$\frac{\pi (d^4 - d_1^4)}{32d}$	$\frac{d^2 + d_1^2}{16}$
Solid square		$\frac{d^4}{12}$	$\frac{d^3}{6}$	$\frac{d^2}{12}$
Hollow square		$\frac{d^4 - d_1^4}{12}$	$\frac{d^4 - d_1^4}{6d}$	$\frac{d^2 + d_1^2}{12}$
Solid rectangular		$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	$\frac{b^2}{12}$
Hollow rectangular		$\frac{bd^3 - b_1d_1^3}{12}$	$\frac{bd^3 - b_1d_1^3}{6d}$	$\frac{b^2d - b_1^2d_1}{12(bd - b_1d_1)}$
Solid triangular		$\frac{bd^3}{36}$	$\frac{bd^2}{24}$	$\frac{d^2}{18}$
Solid elliptical		$\frac{\pi bd^3}{64}$	$\frac{\pi bd^2}{32}$	$\frac{b^2}{16}$
Hollow elliptical		$\frac{\pi}{64} (bd^3 - b_1d_1^3)$	$\frac{\pi (bd^3 - b_1d_1^3)}{32d}$	$\frac{b^2d - b_1^2d_1}{16(bd - b_1d_1)}$
I-beam		$\frac{bd^3 - b_1d_1^3}{12}$	$\frac{bd^3 - b_1d_1^3}{6d}$	$\frac{b^2d - b_1^2d_1}{12(bd - b_1d_1)}$
Cross with equal arms (approximate)				$\frac{d^2}{22.5}$
Angle with equal arms (approximate)				$\frac{d^2}{25}$

Thus, the breaking strength of a cast-iron simple beam uniformly loaded and 20 ft. long between the supports, having a hollow rectangular cross-section 8 in. by 6 in. outside and 6 in. by 4 in. inside, is given by the formula

$$M = SR, \text{ or } \frac{1}{2}Wl = 36,000 \times \frac{bd^3 - b_1d_1^3}{6d};$$

whence,
$$W = \frac{36,000 \times 8 \times (6 \times 8^3 - 4 \times 6^3)}{(20 \times 12) \times (6 \times 8)} = 55,200.$$

Using a factor of safety of 6, the beam should support

$$\frac{55,200}{6} = 9,200 \text{ lb.}$$

with perfect safety. The value of S for beams should be taken from the flexure column of Table II.

To find the amount of deflection in a beam due to a load, substitute the values of W , l , E , and I in the different expressions for the deflection s in Table IV.

The value of I is to be taken from Table V.

EXAMPLE.—What is the deflection of a wrought-iron beam fixed at both ends, 7 ft. long between the supports, having a solid rectangular cross-section 6 in. wide and 2½ in. deep, carrying a load of 21,000 lb. in the middle?

SOLUTION.—From the table,

$$s = \frac{Wl^3}{192EI} = \frac{Wl^3}{192E \times \frac{bd^3}{12}} = \frac{21,000 \times (7 \times 12)^3 \times 12}{192 \times 25,000,000 \times 6 \times (2\frac{1}{2})^3} = .249''.$$

EXAMPLE.—It is desired to calculate the depth (d) of a cast-iron cantilever 36 in. in length ($= l$) that will sustain at its end a weight of 4,000 lb. ($= W$), the lever to be of rectangular section and 2 in. in width.

SOLUTION.—The ultimate stress per square inch for cast iron in flexure is given in Table II as 36,000 lb. ($= S$). The weight will be a steady load, and therefore, according to Table I, a factor of safety of 6 should be used. By formula (3), $M = SR$. For a cantilever beam carrying a load at the end, $M = Wl$ (Table IV); and for a rectangular section, $R = \frac{bd^2}{6}$ (Table V).

Then, as $W = 4,000$, $l = 36$, $b = 2$, $f = 6$, we have

$$\frac{SR}{f} = M, \text{ or } \frac{Sbd^2}{6f} = Wl.$$

The value of d is found by substituting in this equation the known values of S , b , W , l , and f , as follows:

$$\frac{36,000 \times 2 \times d^2}{6 \times 6} = 4,000 \times 36; \text{ whence, } d = 8.49 \text{ in.}$$

At the point where the beam is supported, the required depth is found to be 8.49, or, practically, 8½ in. At a point 6 in. from the support, the depth may again be calculated by substituting in the equation the value of l (the overhanging length beyond this point); $l = 30$, and the equation becomes

$$\frac{36,000 \times 2 \times d^2}{6 \times 6} = 4,000 \times 30.$$

$$d = 7.75 \text{ in.}$$

At a point 12 in. from the support, $l = 24$, and

$$\frac{36,000 \times 2 \times d^2}{6 \times 6} = 4,000 \times 24; \text{ whence, } d = 6.93 \text{ in.}$$

At a point 18 in. from the support, $l = 18$; and from the equation, $d = 6$ in.; at 24 in. from the support, $l = 12$ and $d = 4.9$ in.; at 30 in. from the support, $l = 6$ and $d = 3.46$ in.; at 36 in. from the support, or at the end of the beam, $l = 0$ and $d = 0$.

The depths required to be given to the lever or beam at the point of support and at intervals of 6 inches along its



length, are found to be 8.49, 7.75, 6.93, 6, 4.90, and 3.46 inches, respectively.

The lever is shown in the figure; theoretically, it would taper to nothing at the end, as indicated by dotted lines, but practically sufficient metal must be added at that point to provide means of attaching the weight.

NOTE.—In the preceding examples the weight of the beam has been neglected. If, however, this weight is large in comparison with the weight or weights carried by the beam, it should be taken into account, considering it (when the cross-section of the beam is the same throughout) as a load uniformly distributed over the whole length of the beam.

COLUMNS.

To find the breaking strength of a column, use the following formula:

$$P = \frac{SA}{1 + q \frac{l^2}{G^2}}. \quad (4)$$

S is taken from Table II, in the column for compression, G^2 from Table V, and q from the following table, according to the character of the ends.

TABLE VI.

Material.	Both Ends Flat or Fixed.	One End Round.	Both Ends Round.
Cast iron	$\frac{1}{5,000}$	$\frac{1.78}{5,000}$	$\frac{4}{5,000}$
Wrought iron.....	$\frac{1}{36,000}$	$\frac{1.78}{36,000}$	$\frac{4}{36,000}$
Steel	$\frac{1}{25,000}$	$\frac{1.78}{25,000}$	$\frac{4}{25,000}$
Wood.....	$\frac{1}{3,000}$	$\frac{1.78}{3,000}$	$\frac{4}{3,000}$

The breaking load of an elliptical wooden column 18 ft. long, having rounded ends, the diameters of the cross-section being 12 in. and 8 in., is

$$P = \frac{SA}{1 + q \frac{l^2}{G^2}} = \frac{8,000 \times (\frac{1}{4} \pi \times 12 \times 8)}{1 + \frac{4}{3,000} \times \frac{(18 \times 12)^2}{\frac{8^3}{16}}} = 36,442 \text{ lb.}$$

Using a factor of safety of 8, the column should support $\frac{36,442}{8} = 4,555$ lb. with perfect safety.

SHAFTING.

The diameter of a shaft may be found by the following formulas. The first is used when great stiffness is required, and the shafts are very long; the second when strength only is required to be considered.

d = diameter of shaft in inches;

H = horsepower transmitted;

N = number of revolutions per minute;

c = constant in formula (5);

k = constant in formula (6).

$$d = c \sqrt[4]{\frac{H}{N}}. \quad (5) \qquad d = k \sqrt[4]{\frac{H}{N}}. \quad (6)$$

c = 5.26 for cast iron; 4.75 for wrought iron; 3.96 for steel;

k = 4.02 for cast iron; 3.63 for wrought iron; 3.03 for steel.

NOTE.—To extract the fourth root, extract the square root twice.

PIPES AND CYLINDERS.

p = pressure in pounds per square inch;

d = diameter of pipe or cylinder in inches;

t = thickness in inches;

S = ultimate tensile strength taken from Table II;

r = inside radius in inches;

f = factor of safety, usually taken as 6 for wrought iron and 12 for cast iron.

For thin pipes, $p d f = 2 t S.$ (7)

For thick pipes or cylinders,

$$p f = \frac{S t}{r + t}. \quad (8)$$

ROPES AND CHAINS.

D = diameter of the rope in inches = diameter of iron from which the link in chain is made;

W = safe load in tons of 2,000 lb.

For common hemp rope, $W = \frac{1}{4} D^2.$

For iron-wire rope, $W = \frac{3}{8} D^2.$

For steel-wire rope, $W = \frac{1}{2} D^2.$

For close-link wrought-iron chain, $W = 6 D^2.$

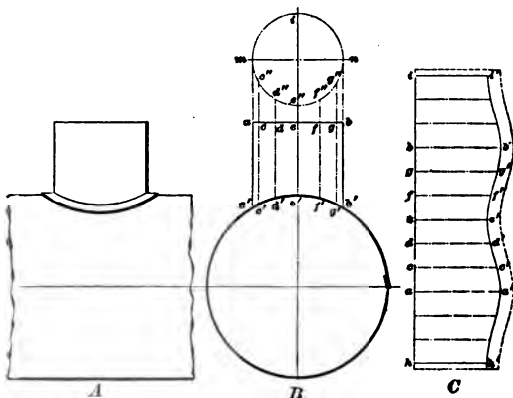
For stud-link wrought-iron chain, $W = 9 D^2.$

BOILERS.

BOILER DESIGN.

TO DEVELOP THE DOME OF A BOILER.

A side view of the dome, together with a section of the boiler, is shown in Fig. A. Draw Fig. B, the end view of the dome and of the boiler. Above the dome draw a circle $n e' m$ of the same diameter as the dome. Divide the lower



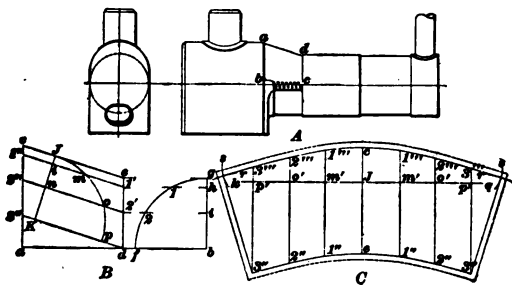
half of this circle, as $n e' m$, into any number of equal parts, as $m c'$, $c' d'$, $d' e'$, $e' f'$, and $f' g'$. The greater the number of these divisions, the more accurate will be the results. From the points of division c' , d' , e' , f' , and g' , draw lines parallel to the vertical center line of the boiler, as $c' c'$, $d' d'$, $f' f'$, and $g' g'$.

We are now ready to draw the templet of the dome, as shown in Fig. C. Draw a straight line of indefinite length, and on it lay off a distance $h i$ equal to the circumference of

the dome. (The circumference of the dome is found by multiplying the diameter ab of the dome by 3.1416.) Divide the distance hi into twice the number of equal parts that the semicircle above the dome in Fig. *B* has. In the figure it has been divided into 6 equal parts; therefore, divide this line into $2 \times 6 = 12$ equal parts, as bg , gf , fe , ed , etc., and through these points of division draw lines at right angles to the line hi , as shown; make the length of each of these lines the same as the length of the line that corresponds to it in Fig. *B*. Thus, ee' is equal to ee' in Fig. *B*, dd' is equal to dd' in Fig. *B*, aa' is equal to aa' in Fig. *B*, etc. After having laid off the lengths of these lines, draw the curved line $i'e'h'$. This being done, we have the templet of the dome on the seam. The lap for riveting must be allowed, as shown by the dotted lines around the templet.

TO DEVELOP THE SLOPE SHEET $abcd$ OF A BOILER, SHOWN AT *A* IN THE FIGURE BELOW.

Draw a straight line ab , as shown in Fig. *B*, and on it lay off the distance ad , equal to bc , Fig. *A*. At a and d , erect



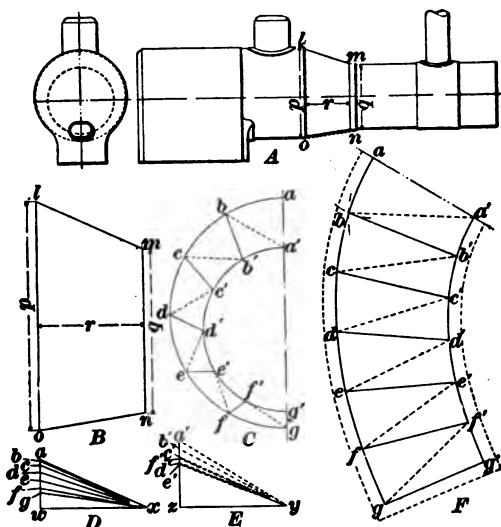
perpendiculars ac and de , respectively, making ac equal to ba , and de equal to cd , of Fig. *A*. With a point b on ab as a center, and a radius de , describe the quadrant fg . Divide this quadrant into any number of parts; the greater the number,

the more accurate will be the results. Here it is divided into three, as $g-1$, $1-2$, and $2-f$. Through the points g , 1 , and 2 , draw lines parallel to ab , intersecting the perpendicular de in e , $1'$, and $2'$, and the perpendicular bg in h and i . Through the points $1'$, $2'$, and d , draw lines parallel to ce . Through any point, as J , on the line ce , draw JK perpendicular to ce , cutting the lines $1''-1'$, $2''-2'$, and $3''-d$ in the points i , n , and K , respectively. From the line JK lay off the distances im , no , and Kp , equal to the distances hi , $i2$, and bf , respectively, and pass the dotted curve $Jmop$ through the points. Now draw Fig. C. Draw the straight line kq , and through the point J draw ec perpendicular to it. Lay off on the line kq , on each side of the line ce , points m' and m'' at distances from it equal to the length of Jm in Fig. B. Lay off, also, points o' and o'' at distances from m' and m'' equal to mo in Fig. B; also, points p' and p'' at distances from o' and o'' equal to op of Fig. B. Through the points thus laid off, draw lines parallel to ce . Lay off the distances Jc and Je from J , in Fig. C, equal to Jc and Je , respectively, in Fig. B; the distances $m'1'''$ and $m''1''$ from m' equal to $i1'''$ and $i1''$ in Fig. B; $o'2'''$ and $o''2''$ from o' equal to $n2'''$ and $n2''$; and $p'3'''$ and $p''3''$ from p' equal to $K3'''$ and Kd of Fig. B. Through the points thus laid off draw the curved lines $3'''c3'''$ and $3''e3''$. With the points $3'''$ as centers and a radius ad , Fig. B, describe the arcs r and r . With the points $3''$ as centers and a radius $3''a$, Fig. B, describe the arcs s and s . From the points of intersection of these arcs, draw lines to the points $3'''$ and $3''$. This being done, we have the templet of the slope sheet on the seams. The laps for riveting must be allowed as shown by the dotted lines around the templet.

**TO DEVELOP THE SLOPE SHEET $lmno$ OF A BOILER,
SHOWN AT A IN THE FIGURE ON THE
FOLLOWING PAGE.**

Draw the two views of the sheet as shown in Figs. B and C. Suppose the seam to be at on , Fig. A, and the sheet to be made in one piece. Divide the semicircles adg and $a'd'g'$, Fig. C, into any number of equal parts; the greater the number

of these divisions, the more accurate will be the results. Join the points b and b' , c and c' , d and d' , e and e' , and f and f' by full lines, and join the points b and a' , c and b' , d and c' , e and d' , f and e' , and g and f' by dotted lines, as shown. Then draw Figs. D and E . Draw at right angles to one another the lines wa and wx , also the lines za' and zy . Make the length of the line wx equal to r , Fig. B , and the



length of the line wa equal to aa' , Fig. C . From w lay off on the line wa , Fig. D , distances wb , wc , wd , we , wf , and wg , respectively, equal to the lengths of the full lines bb' , cc' , etc. of Fig. C , and draw the lines ax , bx , cx , dx , ex , fx , and gx , as shown. Make the length of the line zy , Fig. E , the same as that of wx , Fig. D . From z lay off on the line zy ,

Fig. *E*, distances za' , zb' , zc' , zd' , ze' , and zf' , respectively, equal to the lengths of the dotted lines ba' , cb' , etc., in Fig. *C*, and draw the lines $a'y$, $b'y$, $c'y$, $f'y$, $d'y$, and $e'y$.

We are now ready to draw the templet of the slope sheet. Instead of drawing the whole templet, we will draw only one-half of it, as is shown in Fig. *F*, since the other half is exactly the same. Draw the line aa' , and make it equal in length to the distance ax , Fig. *D*. With a' as a center, and a radius ya' , Fig. *E*, describe an arc at b . With a as a center and a radius = arc ab , Fig. *C*, describe another arc intersecting the first arc in b . With a' as a center, and a radius = arc $a'b'$, Fig. *C*, describe an arc at b' . With b as a center, and a radius xb , Fig. *D*, describe an arc, intersecting the arc already drawn, at b' ; draw the full line bb' and dotted line ba' . With b' as a center, and a radius $y b'$, Fig. *E*, describe an arc at c . With b as a center, and a radius = arc cb , Fig. *C*, describe an arc cutting the last arc at c . With b' as a center, and a radius = arc $c'b'$, Fig. *C*, describe an arc at c' . With c as a center, and a radius xc , Fig. *D*, describe an arc cutting the last arc at c' ; draw the full line cc' and dotted line cb' .

Continue to construct the remaining portion of the half templet in a similar manner, taking the distances for the full lines from Fig. *D*, and those for the dotted lines from Fig. *E*. Through the points a , b , c , d , e , f , and g , and through the points a' , b' , c' , d' , e' , f' , and g' , draw the curved lines shown. Since this is the development of the slope sheet at the seam, the laps for riveting must be allowed; they are shown by the dotted lines around the templet in Fig. *F*.

CARE AND INSPECTION OF BOILERS.

POINTS TO BE OBSERVED.

Preliminary to a boiler inspection, the boiler, flues, mud-drum, ash-pit, and all connections should be thoroughly cleaned, to facilitate a careful examination. Blisters may occur in the best iron or steel, and their presence, and also that of thin places, is ascertained by going over all parts of the boiler with a hammer. When blisters are discovered, the plates should be repaired or replaced. Repairing a blister

consists in cutting out the blistered space and riveting a "hard patch" over the hole on the inside of the boiler, if possible, to avoid forming a pocket for sediment. All seams, heads, and tube ends should be examined for leaks, cracks, corrosions, pitting, and grooving, detection of the latter possibly requiring the use of a magnifying glass. Uniform corrosion is a wasting away of the plates, and its depth can be determined only by drilling through the plate and measuring the thickness, afterwards plugging the hole. Pitting is due to a local chemical action, and is readily perceived. Grooving is usually due to buckling of the plates when under pressure, and frequently to the careless use of the sharp calking tool. Seam leaks are generally caused by overheating, and demand careful examination, as there may be cracks under the rivet heads. If such cracks are discovered, the seam should be cut out, and a patch riveted on. Loose rivets should be carefully looked for, and should be cut out and replaced, if found. Pockets, or bulging, and burns should be looked for in the firebox. The former are not necessarily dangerous, but if there are indications of their increasing, they should be heated and forced back into place or cut out and a patch put on. Burns are due to low water, the presence of scales, or to the continuous action of flames formed on account of air leaking through the brickwork. The burned spots should be cut out and patched as previously described. The conditions of all stays, braces, and their fastenings should be examined and defective ones replaced. The shell of the boiler should be thoroughly examined externally for evidences of corrosion, which is liable to set in on account of dampness, exposure to weather, leakage, etc., and may be serious. The boiler should be so set that joints and seams are accessible for inspection, and should have as little brickwork in contact with it as possible. The brickwork should be in good condition, and not have air holes in it, since they decrease the efficiency of the boiler and are liable to cause injury to the plates by burning, as above explained, and also by unevenly heating and distorting them. The mud-drum and its connections are liable to corrosion, pitting, and grooving, and should be examined as carefully as the boiler.

All valves about a boiler should be easy of access, and should be kept clean and working freely. Each boiler should have at least three gauge-cocks, properly located, and it is of the utmost importance that they be kept clean and in order, and the same may be said of the glass water gauge. The middle gauge-cock should be at the water level of the boiler, and the other two should be placed one above and one below it, at a distance of about 6 in.

The condition of the pumps or injectors should be looked into to make sure that they are in the best working order. The steam gauge should be tested to ascertain that it indicates correctly, and if it does not, it should be corrected. If the hydraulic test is to be used, the boiler should be tested to a pressure of 50% higher than that at which the safety valve will be set.

External Inspection When Boiler is Under Steam.—The gauge-cocks, and also the gauge glass, should be tried, to make sure that they are not choked. The steam gauge should be taken down, if permissible, and tested, and corrected if necessary. The gauge pointer should move freely. Blowing out the gauge connection will show whether it is clear or not. The boiler connections should be examined for leaks. The safety valve should be lifted from its seat, to make sure that it does not stick from any cause, and it should be seen that the weight is in the right place. Observe from the steam gauge if the valve blows off at the pressure it is set for. See that all pumps and feed-apparatus are working properly, and that the blow-off and check-valves are in order. Blisters and bagging may sometimes be detected in the furnace. The condition of the brickwork is of considerable importance, since the existence of air holes is a source of trouble, as already explained.

Incorustation.—One of the chief sources of trouble to the boiler user is that of incorustation. All water is more or less impure; and as the water in the boiler is continuously evaporated, the impurities are left behind as powder or sediment. This collects on the plates, forming a scaly deposit, varying in nature from a spongy, friable texture to a hard, stony one. This deposit impedes the transmission of heat from the plates

to the water and often causes overheating and injury to the plates. It is probable that $\frac{1}{8}$ in. of scale necessitates the consumption of 12% to 20% more fuel. The various impurities in the water may be either in suspension or solution. If the former, the water can be purified by filtration before going into the boiler. If the latter, the substances must first be precipitated and then filtered. Many impurities (sulphate and carbonate of lime, etc.) may be removed by heating the water before feeding it into the boiler.

The first thing to do, when dealing with a water supply, is to have an analysis of it made by a competent chemist. The fact that a water contains a certain amount of solid matter is no criterion as to its unfitness for boiler use. The presence of certain salts, as carbonate or chloride of sodium, even in large quantities (say 40 to 50 gr. per gal.), would not be serious if due attention were given to the blowing off. On the other hand, salts of lime in the above proportion would be very objectionable, requiring greatly increased attention in the matter of purification and blowing off or else cleaning out.

The various methods of dealing with impure water may be classed as follows:

1. *Filtration*.—Where the matter (sand, mud, etc.) is held in suspension, it can be removed, before the water enters the boiler, by the aid of settling tanks or by filtering, or by forcing the water up through layers of sand, broken brick, etc., or by using filtering cloths in a proper machine.

2. *Chemical Treatment*.—Clark's process, combined with a subsequent filtration (the joint process being known as the *Atkins* system), has been successfully applied on both small and large scales in the chalk districts of England. Lime water is mixed with the water to be purified, the amount used depending on the composition of the water, as determined by a careful analysis. The lime is thus precipitated, and the water is then filtered in a machine containing traveling cotton cloths. Not only is the carbonate of lime entirely removed, but it has been proved that any sulphate of lime that may be present is also prevented from incrusting. This is important, as the latter impurity forms, perhaps, the worst scale one has to contend with.

Various chemical compounds are in use for boilers. Carbonate of soda is perhaps the best general remedy. It forms the basis, in fact, of nearly all boiler compounds, whatever their name or appearance. This soda deals efficaciously both with the carbonate and the sulphate of lime. The precipitates thus thrown down do not form a hard crust; they can be washed out in the form of sludge or mud.

Carbonate of soda is also useful where condensers are employed, as it counteracts the effect of the grease, which is brought over with the exhaust steam. If used in too large quantities, it will cause priming. The best way to use it is to make a solution of it and connect with the feed, fixing a cock so as to regulate the amount fed in. Soda ash is cheaper, but more of it is required, and, besides, it is generally impure. Caustic soda removes lime scale quicker than ordinary soda does, but it is much stronger and liable to attack the plates. It should be used in smaller quantities than the ordinary kind.

Barks, molasses, vinegar, etc. develop acids that attack the plates. Animal and vegetable oils do the same, and also harden the deposits and make their removal more difficult. It is a good rule to keep all animal and vegetable matter out of boilers altogether.

Feed-Water Heaters.—Carbonates and sulphates of lime are precipitated by high temperatures. The heaters should be arranged so that the deposit forms chiefly on a series of plates that can be easily removed for cleaning. If the deposit gathers in pipes, however, it is simply transferring the evil from one vessel to another. A double advantage is gained by these heaters, for the feedwater is put into the boiler already heated, and so fuel is saved.

Mechanical Aids.—Deposits take place chiefly in sluggish places. Various devices to aid circulation have been brought out. With good attention and a not too impure water, they give satisfactory results.

Potatoes, linseed oil, molasses, etc. are sometimes put into the boiler with the idea of lessening scale formation, by forming a kind of coating round the particles of solid matter and so preventing their adhering together. This certainly takes place, but the substances are injurious, as already pointed out.

Whenever a boiler has been cleaned out, we may with advantage give the inside a thin coating of oil, or tallow and black lead; this arrests the incrustation to a great extent.

Sand, sawdust, etc. are often used, the idea being that their grains act as centers for the gathering together of the solid matter in the water, the resulting small masses not readily collecting together themselves and therefore being easily washed out. This may be so, but the cocks, valves, etc. are liable to suffer from the practice.

Kerosene is strongly recommended by some boiler users. There is no doubt that in many cases its use has given good results. It prevents incrustation, by coating the particles of matter with a thin covering of oil, the deposit thus formed being easily blown out. The oil also seems to act on the scale already formed, breaking it up and thus facilitating its removal. As already remarked, it is a good plan, when the boiler is empty, to give the inside a good coating of this oil, afterwards putting it in with the feed, the supply being regulated automatically. As to the quantity required, this will be found to vary in different cases, according to the nature of the water; an average of 1 qt. per day for every 100 horsepower will give good results in most cases.

In marine boilers, strips of zinc are often suspended; the deposit largely settles on them instead of on the boiler plates. Also, any scale that may be formed on the latter is less hard and compact and more easily broken up. Further, any acids formed by the oil and grease brought over from the condenser attack this zinc instead of the boiler plates.

Miscellaneous.—Acids are often introduced into boilers to dissolve the scale already formed, the solid matter then being washed out. This treatment should be adopted with great care, if at all, as the plates are likely to be affected.

Scale is often loosened and broken up by deliberately inducing sudden expansion or contraction in the boiler. In the former case, the expansion is brought about by blowing off the boiler, and then, when it is quite cooled down, turning on steam at as high a temperature as obtainable, thus causing the scale to expand more quickly than the plates and thus become loose.

In the second method, the boiler is blown off when the steam (and therefore the temperature) is at its highest and a stream of cold water then turned in. The fires are then drawn and the fire-hole doors, dampers, etc. opened, letting in a rush of cold air. All this cools the plates and, by the contraction thus brought about, loosens the scale. These two practices should be guarded against.

Foaming or priming is usually due either to forcing a boiler beyond its capacity for furnishing dry steam, or to the presence of foreign matter. It is dangerous if occurring to any great extent, since water may be carried along with steam into the engine, and a cylinder head knocked out. Foaming, when it cannot be checked by the use of the surface blow-out apparatus, may necessitate the emptying of the boiler, which must then be filled with fresh water; this rids the boiler of the impurities that have collected during the operation of the boiler.

HORSEPOWER OF BOILERS.

In actual practice, the result of a great many tests has shown that an evaporation of 30 lb. of water per hr. from a feedwater temperature of 100° F. into steam at 70 lb. gauge pressure is the equivalent of 1 horsepower, or that this steam, in a properly designed engine, will do the equivalent of $33,000 \times 60 = 1,980,000$ ft.-lb. of work per hr. In order, however, to have a more ready standard of comparison, the above evaporation has been reduced to another standard, and is found to be equal to the evaporation of 34.5 lb. of water from and at a temperature of 212° F. under atmospheric pressure, and it is on this latter quantity that the calculations of the horsepower of boilers are usually based.

In making an approximation of the horsepower of a given boiler, the square feet of water-heating surface of the boiler should first be determined, and in doing this the area of all the surfaces exposed to the fire and hot gases, which, on their opposite sides come in contact with the water in the boiler, should be taken into account.

EXAMPLE.—An externally-fired flue boiler, having a shell 88 in. in diameter, and containing two flue pipes 10 in. in

diameter, is 22 ft. long without the smokebox. If the greatest depth of the water in the boiler is $\frac{1}{4} \times 38 = 25.33$ in., what is the total water-heating area of the boiler?

SOLUTION.—Six feet of the circumference of the boiler shell lies below the water-line, as could be found by actual measurement, and the circumference of the two flues is equal to

$$\left(\frac{10 \times 3.1416}{12} \right) \times 2 = 5.24 \text{ ft.}$$

Therefore, the water-heating surface of the shell is $6 \times 22 = 132$ sq. ft., and that of the flues is $5.24 \times 22 = 115.28$ sq. ft. The water-heating surface of the heads of the shell (that is, the area below the water-line, minus the area of the flues, which could be obtained by direct measurement) is $4.5 \times 2 = 9$ sq. ft. Therefore, the total water-heating surface of the boiler is the sum of all these, or 256.28 sq. ft.

Having determined the water-heating surface of a boiler, to approximate its horsepower:

Rule.—*Divide the total water-heating surface in square feet by the number of square feet of heating area, as given in the table below, required to produce an evaporation equivalent to 1 horsepower in boilers of the given type.*

EXAMPLE.—The total water-heating surface of the above externally-fired flue boiler is 256.28 sq. ft. What is the horsepower of the boiler?

SOLUTION.—By referring to the table, we find that it takes about 10 sq. ft. of heating surface to produce 1 horsepower; therefore, the above boiler would be rated at about

$$\frac{256.28}{10} = 25.63 \text{ H. P.}$$

Type of Boiler.	Water-Heating Surface for 1 Horsepower. Square Feet.	Ratio of Water-Heating Area to Grate Area Required.
Cylindrical	9	From 12 to 15 : 1
Flue	10	From 20 to 25 : 1
Firebox tubular	12	From 25 to 35 : 1
Return tubular	15	From 25 to 35 : 1
Vertical	15	From 25 to 30 : 1
Water tube	11	From 35 to 40 : 1

The above rule must not be taken as furnishing anything but an approximate method, since the same boiler will give a different horsepower whenever the conditions under which it is operated are changed; or, in other words, the horsepower developed depends largely on the amount of coal burned per square foot of grate area per hour, the velocity and character of the furnace draft, and the quality of the coal used. In ordinary practice, however, we may expect an evaporation of from 8 to 11 lb. of water from and at 212° F. for each pound of good coal burned, where from 11 to 13 lb. of coal are consumed per sq. ft. of grate surface per hr., or about from 3 to 4 lb. per H. P. per hr.

CHIMNEYS.

The *chimney* serves the double purpose of creating a draft and carrying away obnoxious gases. The production of the draft depends on the fact that the furnace gases (the products of combustion) passing up the chimney have a high temperature, and are, consequently, lighter than an equal volume of outside air at the ordinary temperature; that is, the pressure within the chimney is slightly less than the pressure of the outside air. Consequently, the air will flow from the place of higher pressure to the place of lower pressure, that is, into the chimney through the furnace.

Suppose, for example, the average temperature of the gases in a chimney 150 ft. high is 500° F. A pound of the gases at 62° F. has a volume of 12.5 cu. ft.; its volume at 500° is, then,

$$\frac{12.5 \times (500 + 460)}{62 + 460} = 23 \text{ cu. ft.}$$

Therefore, a column of the

gases 1 ft. square and 150 ft. long would weigh $\frac{150}{23} = 6.52$ lb.

A similar column of air at 62° F. would weigh $\frac{150}{13.14} = 11.42$ lb., nearly. Hence, the pressure of the draft is $11.42 - 6.52 = 4.9$ lb. per sq. ft. = .941 in. of water. It is evident that the pressure of the draft depends on the temperature of the furnace gases and the height of the chimney. The higher the chimney, the lower may be the temperature of the gases to produce

the same draft, and the greater will be the economy of the furnace. In general, chimneys are not built much less than 100 ft. in height.

The relation between the height of the chimney and the pressure of the draft in inches of water is given by the following formula:

$$p = H \left(\frac{7.6}{T_a} - \frac{7.9}{T_g} \right),$$

where p = draft in inches of water;

H = height of chimney in feet;

T_a = absolute temperature of outside air;

T_g = absolute temperature of chimney gases.

Absolute temperatures are found by adding 460° F. to the ordinary temperatures.

EXAMPLE.—What draft pressure will be produced by a chimney 120 ft. high, the temperature of the chimney gases being 600° F. and the external air 60° F.?

SOLUTION.—By the formula we find

$$p = H \left(\frac{7.6}{T_a} - \frac{7.9}{T_g} \right) = 120 \left(\frac{7.6}{460 + 60} - \frac{7.9}{460 + 600} \right) = .86 \text{ in. of water.}$$

The draft pressures ordinarily produced by chimneys vary from 0 to 2 in. of water. A water-gauge pressure of 1 in. is equivalent to .03617 lb. per sq. in. Wood requires least draft, and the small sizes of anthracite coal the greatest draft. To successfully burn anthracite, slack, or culm, a draft of 1½ in. is necessary.

To find the height of chimney to give a specified draft pressure, the formula may be transformed:

$$H = \frac{p}{\frac{7.6}{T_a} - \frac{7.9}{T_g}}$$

EXAMPLE.—Required the height of the chimney to produce a draft of 1½ in. of water, the temperature of the gases and of the external air being, respectively, 550° and 62° F.

SOLUTION.—By the formula we find

$$H = \frac{p}{\frac{7.6}{T_a} - \frac{7.9}{T_g}} = \frac{1.125}{\frac{7.6}{522} - \frac{7.9}{1,010}} = 167 \text{ ft.}$$

The sizes of chimneys for boilers of various horsepowers are given in the following table

SIZES OF CHIMNEYS AND HORSEPOWERS OF BOILERS.

Height of Chimney in Feet.											Actual Area in Sq. Ft.	Side of Sq. in In.	Diameter in In.
50	60	70	80	90	100	110	125	150	175	200			
Commercial Horsepower.													
23	25	27									1.77	16	18
35	38	41									2.41	19	21
49	54	58	62								3.14	22	24
65	72	78	83								3.98	24	27
84	92	100	107	113							4.91	27	30
	115	125	133	141							5.94	30	33
	141	152	163	173	182						7.07	32	36
		183	196	208	219						8.30	35	39
		216	231	245	258	271					9.62	38	42
			311	330	348	365	389				12.57	43	48
			363	427	449	472	503	551			15.90	48	54
			505	539	565	593	632	692	748		19.64	54	60
				658	694	728	776	849	918	981	23.76	59	66
				792	835	876	934	1,023	1,105	1,181	28.27	64	72
					995	1,038	1,107	1,212	1,310	1,400	33.18	70	78
					1,163	1,214	1,294	1,418	1,531	1,637	38.48	75	84
					1,344	1,415	1,496	1,639	1,770	1,893	44.18	80	90
					1,537	1,616	1,720	1,876	2,027	2,167	50.27	86	96

EXAMPLE.—A round chimney 100 ft. high is to be used for a battery of boilers of 550 H. P. What should be the internal diameter?

SOLUTION.—Looking under column 100 in "Height of Chimney in Feet" the nearest horsepower is 565, and the diameter corresponding is 60 in., which should be the internal diameter of the chimney.

Chimneys are usually built of brick, though in some cases iron stacks are preferred. The external diameter of the base should be $\frac{1}{6}$ of the height, in order to provide stability. The taper of a chimney is from $\frac{1}{8}$ to $\frac{1}{4}$ in. to the foot on each side. The thickness of brickwork is usually 1 brick (8 or 9 in.) for 25 ft. from the top, increasing $\frac{1}{4}$ brick for each 25 ft. from the top downward. If the inside diameter is greater than 5 ft., the top length should be $1\frac{1}{4}$ bricks, and if under 3 ft., it may be

$\frac{1}{2}$ brick in thickness for the first 10 ft. A round chimney is better than a square one, and a straight flue better than a tapering one. If the flue is tapering the area for calculation is measured at the top.

The flue through which the gases pass from the furnaces to the chimney should have an area equal to, or a little larger than, the area of the chimney. Abrupt turns in the flue or contractions of its area should be carefully avoided, as they greatly retard the flow of the gases. Where one chimney serves several boilers, the branch flue from each furnace to the main flue must be somewhat larger than its proportionate part of the area of the main flue.

SAFETY VALVES.

Balance the valve and lever over a sharp, knife-like edge, and measure the distance from the point of suspension to the fulcrum (center of pin on which the lever turns).

Let a = distance thus measured in inches;

b = distance from center of valve to fulcrum in inches;

x = distance of weight from fulcrum in inches;

W = weight in pounds hung on lever;

Q = weight of lever and valve in pounds;

A = area of safety valve in square inches;

p = pressure per square inch in the boiler.

$$\text{Then, } x = \frac{A p b - Q a}{W}; \quad W = \frac{A p b - Q a}{x}; \quad p = \frac{W x + Q a}{A b}.$$

EXHAUST HEATING.

Exhaust steam from non-condensing engines usually contains from 20% to 25% of water and oil, the latter being employed to lubricate the engine cylinders. Before exhaust steam is allowed to enter a heating system, the water and oil should be separated from it.

The effect of turning exhaust steam into a heating system is to form a back pressure on engine, which must be avoided as far as possible by using large steam-distributing pipes.

A direct connection to the steam boilers through a pressure-reducing valve must be employed, to automatically furnish

steam to the heating system when the exhaust fails. A relief valve, also, should be placed upon the system, so that surplus exhaust steam may escape to the atmosphere.

To proportion an exhaust-heating system, it is necessary to know about how many square feet of radiating surface we should employ to properly condense the exhaust steam from the non-condensing engines. To do this we must first know the weight of steam that would be discharged from the engine.

Class of Non-Condensing Engine.	Water Used per Hour for Indicated Horsepower.
Compound automatic	25 lb.
Simple Corliss	30 lb.
Simple automatic	35 lb.
Simple throttling.....	40 lb.

From this must be deducted about 10% for condensation in the cylinders, etc., in order to obtain the real available weight of steam for heating purposes.

APPROXIMATE RATIO BETWEEN CUBIC CONTENTS AND RADIATOR SURFACE FOR EXHAUST HEATING.

Class of Building.	Direct Radiation.	Indirect Radiation.	Blower System.
	sq.ft. cu.ft.	sq.ft. cu.ft.	sq.ft. cu.ft.
Dwellings	1 to 50	1 to 40	1 to 300
Offices	1 to 70	1 to 60	1 to 365
Stores and shops.....	1 to 100	1 to 80	1 to 500
Churches, etc.....	1 to 200	1 to 150	1 to 900

The figures in the foregoing tables simply form a reasonable average, and allowance must be made for exposure, etc.

Each square foot of direct radiating surface gives off to the air around it about $1\frac{1}{4}$ thermal units per hour per degree of difference between the temperature of the steam and that of the surrounding air. This is equivalent to about $\frac{1}{4}$ lb. of steam per hr., or, in other words, about 4 to $4\frac{1}{4}$ sq. ft. of surface to each pound of steam to be condensed.

MACHINE DESIGN.

BLUEPRINTS.

Blueprint paper for copying tracings of plans and other drawings may be prepared as follows: Dissolve 1 oz., avoirdupois, of ammonia citrate of iron in 6 oz. of water, and in a separate bottle dissolve the same quantity of potassium ferricyanide in 6 oz. of water. Keep these solutions separate, and in a dark place, or in opaque bottles.

To prepare the paper, mix equal quantities of the two solutions, and with a sponge spread it evenly over the surface. Let the paper remain in a horizontal position until the chemical has set on the surface, which will take but a few minutes; then hang the paper up to dry. In preparing the paper darken the room by pulling down the shades, as direct rays of light affect sensitized surfaces. The prepared paper should be kept in a closed drawer, well covered with heavy paper, so that no light can come in contact with the sensitized surface; otherwise it will lose much of its value.

To make a blueprint from a tracing, lay the tracing with ink side down against the glass of the printing frame, then take the prepared paper, and place the sensitized surface down on the tracing. On the top of the paper place the felt cushion, on top of which place the hinged back of the printing frame, after which expose to the sunlight. The exposure will vary in sunlight from about 3 to 10 minutes. After the exposure, wash the paper thoroughly in a trough of cold water for about 10 minutes, and hang it up to dry.

The print after washing should be of a deep-blue color, with clear white lines. If the color is a pale blue, this indicates that the print has not had sufficient exposure, and if the lines of the drawing are not perfectly clear and white, that the exposure has been too long.

Corrections may be made on the print with an ordinary writing or ruling pen and a solution of washing soda, caustic potash, strong ammonia, or any other alkali. When any of these are mixed with carmine ink, the marks on the print will be red, thus making the corrections clear.

MACHINE TOOLS.

SPEED OF EMERY WHEELS.

The speed most strongly recommended by their manufacturers is a peripheral velocity of 5,500 ft. per min. for all sizes. All things being considered, it is stated that no advantage is gained by exceeding this speed. If run much slower than this, the wear on the wheels is much greater in proportion to the work accomplished, and if run much faster, the wheel is likely to burst.

SPEED OF GRINDSTONES.

Grindstones used for grinding machinists' tools are usually run so as to have a peripheral speed of about 900 ft. per min., and those used for grinding carpenters' tools at about 600 ft. per min. With regard to safety, it may be stated in general that with any size of grindstone having a compact and strong grain, a peripheral velocity of 2,800 ft. per min. should not be exceeded.

SPEED OF POLISHING WHEELS.

Polishing wheels are run at about the following peripheral speeds:

Leather-covered wooden wheels.....	7,000 ft. per min.
Walrus-hide wheels.....	8,000 ft. per min.
Rag wheels.....	7,000 ft. per min.

SPEED OF CUTS FOR MACHINE TOOLS.

Brass: Use high speeds, about the same as for wood.

Bronze: 6 to 18 ft. per min., according to alloy used.

Cast or wrought iron: 20 ft. per min. is a good average for all machines, except millers. 30 is about the maximum.

Machinery steel: 15 ft. on shapers, planers, and slotters. 20 to 45 on turret lathes, according to cut.

Tool steel: 8 to 10 ft.

Milling Cutters.—*Gun metal*, 80 ft. per min.; *cast iron*, 30; *wrought iron*, 35 to 40; *machinery steel*, 30. These are good speeds to adopt, with a view to economy, time required for grinding, etc.

Twist Drills.—The best results are obtained when the rates of speed of twist drills are as given in the following table:

Diameter of Drills.	Revolutions of Drills per Minute.		
	Steel.	Iron.	Brass.
$\frac{1}{16}$	940	1,280	1,560
$\frac{1}{8}$	460	660	785
$\frac{3}{16}$	310	420	540
$\frac{1}{4}$	230	320	400
$\frac{5}{16}$	190	260	320
$\frac{3}{8}$	150	220	260
$\frac{7}{16}$	130	185	230
$\frac{1}{2}$	115	160	200
$\frac{9}{16}$	100	140	180
$\frac{5}{8}$	95	130	160
$\frac{11}{16}$	85	115	145
$\frac{3}{4}$	75	105	130
$\frac{7}{8}$	70	100	120
$\frac{15}{16}$	65	90	115
1	62	85	110
$1\frac{1}{16}$	58	80	100
$1\frac{1}{8}$	54	75	95
$1\frac{1}{4}$	52	70	90
$1\frac{3}{8}$	49	66	85
$1\frac{1}{2}$	46	62	80
$1\frac{5}{8}$	44	60	75
$1\frac{3}{4}$	42	58	72
$1\frac{7}{8}$	40	56	69
$1\frac{9}{8}$	39	54	66
$1\frac{11}{8}$	37	51	63
$1\frac{5}{4}$	36	49	60
$1\frac{13}{8}$	34	47	58
$1\frac{3}{2}$	33	45	56
$1\frac{15}{8}$	32	43	54
$1\frac{7}{4}$	31	41	52
$1\frac{17}{8}$	30	40	51
2	29	39	49

The following are recommended as the best rates of feed for twist drills:

Diameter of drill in inches.....	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$ $\frac{1}{2}$	$\frac{3}{4}$ 1 $1\frac{1}{2}$
Number of revolutions per inch depth of hole.....	125	125	120 to 140	1 in. feed per min.

CHANGE GEARS REQUIRED FOR CUTTING SCREW THREADS.

The pitch of a single-threaded screw is the distance between two adjacent threads, measured on a line parallel to the axis of the screw; or, in *any* screw, whether single- or multiple-threaded, it is the distance the nut is moved by 1 revolution of the screw. Usually, a screw is spoken of as having a certain number of threads to the inch, and this is equal to the number of revolutions the screw must make in order to move the nut a distance of 1 inch; so, whether the screw is single- or multiple-threaded, the pitch is always equal to 1 divided by the number of revolutions that the screw must make in order to move the nut 1 inch.

The Simple-Geared Lathe.—In Fig. 1 is shown the usual arrangement of the change gears of a simple-geared screw-cutting lathe. By a simple-geared lathe is meant a lathe in

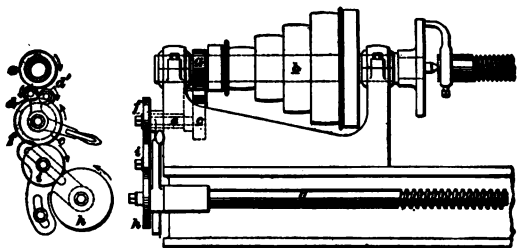


FIG. 1.

which the change gears are so arranged that the circumferential velocity of the change gear on the stud is the same as that of the change gear on the lead screw, which means that, when the change gear on the stud has rotated, say, 5 teeth, the change gear on the lead screw has also rotated 5 teeth, whatever the diameter of these gears, or of any intermediate gears between them, may be.

Referring to Fig. 1, the gear *a* is fastened to the spindle *b* and drives another gear *c* by means of either one of the

reversing gears d, d' . The gear c is keyed to one end of the spindle e ; this spindle is called the *stud*, and carries on its outer end a change gear f . The lead screw g carries a change gear h ; and these two change gears f and h are connected by means of the *idler* gear i , so that gear f drives gear h , and with it, the lead screw g .

In making calculations for the change gears of a simple-gear screw-cutting lathe, the idler gear i is ignored, as it is only introduced to connect gears f and h . The gears d and d' are also ignored, since they are only used to change the direction of rotation of the gear c , their duty being to facilitate the cutting of either right-hand or left-hand threads; when d meshes with gear a , as shown in Fig. 1, a right-hand thread is cut, and when d' meshes with gear a , a left-hand thread is cut.

The number of teeth in the gear a is not always the same as the number of teeth in the gear c ; it is so in some lathes, but in others it is not; hence, in calculating the change gears for any lathe, the number of teeth in the gears a and c must be taken into account.

By the following formulas and rules, the number of teeth required in each change gear in order to cut a given number of threads to the inch, or the number of threads to the inch that given change gears will produce may be found.

Let a = number of teeth in the spindle gear a ;

c = number of teeth in the gear c ;

f = number of teeth in the change gear on stud;

h = number of teeth in the change gear on lead screw;

g = number of threads to the inch in the lead screw;

n = number of threads to the inch to be cut.

$$\text{Then, } n = \frac{gch}{af}. \quad (1) \qquad h = \frac{naf}{gc}. \quad (3)$$

$$\frac{h}{f} = \frac{na}{gc}. \quad (2) \qquad f = \frac{gch}{na}. \quad (4)$$

Now, of the gears h, f, c, a , a and f are the *drivers*, and c and h being driven by a and f , are called the *driven* gears; remembering this, we deduce, from formula (1), the following rule for simple-gear screw-cutting lathes:

Rule.—*The number of threads to the inch to be cut is equal to the number of threads to the inch in the lead screw, multiplied by the product of the number of teeth in each driven gear, and divided by the product of the number of teeth in each driving gear.*

EXAMPLE.—If the lead screw g of a simple-gear lathe has 5 threads to the inch, and the gear a has 21 teeth, the gear c 42 teeth, the change gear f 60 teeth, and the change gear h 72 teeth, how many threads to the inch will be cut?

SOLUTION.—Using formula (1), we have

$$n = \frac{g c h}{a f} = \frac{5 \times 42 \times 72}{21 \times 60} = 12 \text{ teeth.}$$

From formula (2) we deduce the following rule for simple-gear screw-cutting lathes:

Rule.—*The number of teeth in the change gear on the lead screw, divided by the number of teeth in the change gear on the stud, is equal to the product of the number of threads to the inch to be cut and the number of teeth in the driving spindle gear, divided by the product of the number of threads to the inch in lead screw and the number of teeth in the fixed gear on the stud.*

EXAMPLE.—If the lead screw g of a simple-gear lathe has 8 threads to the inch, and the gear a has 16 teeth, and the gear c 32 teeth, how many teeth must there be in each of the gears f and h in order that the lathe may cut 10 threads to the inch?

SOLUTION.—Using formula (2),

$$\frac{h}{f} = \frac{n a}{g c} = \frac{10 \times 16}{8 \times 32} = \frac{5}{8},$$

and, if it were possible to have gears with 5 and 8 teeth, respectively, then a solution of the problem would be, $h = 5, f = 8$. It is evident that such gears are impracticable; but, as it does not change the value of a fraction to multiply both numerator and denominator by the same number, we may multiply 5 and 8, each by such a number that the resulting numbers of teeth in the gears are satisfactory. There is evidently, therefore, *more than one solution* to the problem—for if we multiply by 10 we, shall have $h = 50, f = 80$, which would give 12 threads to the inch; and if we multiply by 13, we shall have, as another solution, $h = 65, f = 104$, which would also give 12 threads to the inch, because $\frac{65}{104} = \frac{5}{8}$.

Having found that $\frac{h}{f} = \frac{1}{2}$, it is customary in practice to choose the change gears in the following manner: From the assortment of gears belonging to the lathe, choose one of convenient diameter, the number of whose teeth is divisible by either the numerator 5 or the denominator 8, and, after dividing by one of these numbers, multiply both numerator and denominator by the quotient.

EXAMPLE.—Given, $\frac{h}{f} = \frac{1}{2}$, to find the number of teeth in the two change gears h and f , respectively.

SOLUTION.—Choose a gear of convenient diameter, the number of whose teeth, say 60, is divisible by either 5 or 8; in this case by 5; divide 60 by 5, and the answer is 12. Then,

$$\frac{5 \times 12}{8 \times 12} = \frac{60}{96},$$

that is, h has 60 teeth, and f 96 teeth.

If one of the change gears is given, and it is desired to find the number of teeth in the other change gear in order to cut a given number of threads to the inch, use either formula (3) or formula (4) according as the number of teeth in gear h or in gear f is required. After the examples given, these formulas will not need explanation.

In a simple-gear screw-cutting lathe, it is often possible to cut a fractional number of threads to the inch, as is the case in the following example:

EXAMPLE.—If the lead screw g has 2 threads per inch, and the gear a has 20 teeth, and the gear c has 20 teeth, how many teeth must there be in each of the change gears f and h , in order to cut $5\frac{1}{2}$ threads to the inch?

SOLUTION.—Using formula (2),

$$\frac{h}{f} = \frac{na}{gc} = \frac{5\frac{1}{2} \times 20}{2 \times 20} = \frac{5\frac{1}{2}}{2}.$$

Then, choosing a gear whose number of teeth, say 32, is divisible by 2, divide 32 by 2 and the quotient is 16. Then, $\frac{64 \times 16}{2 \times 16} = \frac{84}{32}$; that is, h has 84 teeth, and f 32 teeth. In many cases, however, it is impossible, out of the assortment of gears supplied with a simple-gear screw-cutting lathe, to

find gears to cut a screw of the required number of threads to the inch. In such cases, it becomes necessary either to *make* suitable gears or to resort to a compound-gear lathe.

The Compound-Gear Lathe.—In Fig. 2 is shown the usual arrangement of the change gears of a compound-gear screw-cutting lathe. The difference between this and the simple-gear lathe lies in putting two change gears of different sizes on one spindle, in place of the idler between the gear on the stud and the gear on the lead screw. These two gears on one spindle are shown at *i* and *j* in Fig. 2, gear *j* meshing with gear *h* on the lead screw, and gear *i* meshing with gear *f* on the stud.

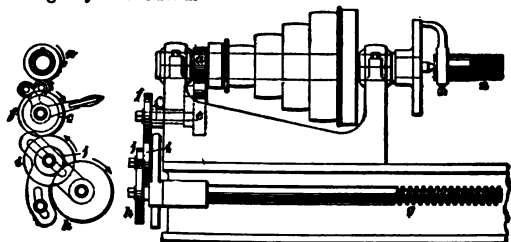


FIG. 2.

From the following formulas, the number of teeth in each change gear, or the number of threads per inch that can be cut with given change gears, can be found.

Let *a* = number of teeth in the spindle gear *a*;

c = number of teeth in the gear *c*;

f = number of teeth in the change gear *f*;

h = number of teeth in the change gear *h*;

i = number of teeth in the change gear *i*, which meshes with the change gear *f*;

j = number of teeth in the change gear *j*, which meshes with the change gear *h*;

g = number of threads to the inch in the lead screw;

n = number of threads to the inch to be cut.

Then,
$$n = \frac{g \times c h i}{a f j}. \quad (5)$$

Now, remembering that gears a , f , and j are the drivers, and gears c , h , and i are the driven gears, and also that the idlers are ignored in all calculations, we can, from formula (5), deduce the following rule for compound-gear screw-cutting lathes:

Rule.—*The number of threads to the inch to be cut is equal to the number of threads to the inch in the lead screw, multiplied by the product of the number of the teeth in each of the driven gears, and divided by the product of the number of teeth in each of the driving gears.*

EXAMPLE.—If the lead screw g of a compound-gear lathe has 2 threads to the inch, and the gear a has 20 teeth, gear c 40 teeth, change gear f 48 teeth, change gear i 72 teeth, change gear j 36 teeth, and change gear h 96 teeth, how many threads to the inch will be cut?

SOLUTION.—Using formula (5), we have

$$n = \frac{g \times c h i}{a f j} = \frac{2 \times 40 \times 96 \times 72}{20 \times 48 \times 36} = 16 \text{ threads to the inch.}$$

If it is desired to find what combination of change gears will enable us to cut a given number of threads to the inch, the following formula may be used:

$$\frac{i}{j} = \frac{n a f}{g c h} \quad (6)$$

From this formula the following rule is deduced:

Rule.—*Of the change gears of a lathe, any driven gear divided by any driver gear is equal to the product of the numbers of teeth in each of the other driver gears and the number of threads to the inch to be cut, divided by the product of the numbers of teeth in each of the other driven gears and the number of threads to the inch in the lead screw.*

EXAMPLE.—In a compound-gear lathe, in which the lead screw has 5 threads to the inch, gear a 20 teeth, gear c 40 teeth, and the number of threads per inch to be cut is $3\frac{1}{4}$, what must be the number of teeth in each of the change gears h , i , j , f ?

SOLUTION.—Using formula (6), we have

$$\frac{i}{j} = \frac{n a f}{g c h}$$

From the assortment of gears belonging to the lathe, choose, for the driven gear h , one whose number of teeth, say 28, can be divided by the number of threads per inch to be cut, in this case $8\frac{1}{2}$; 28 is a multiple of $8\frac{1}{2}$, because it is obtained by multiplying $8\frac{1}{2}$ by 8. Substitute this value in place of h ; then choose any gear of convenient size, say one having 40 teeth, and substitute 40 in place of f ; we shall then have,

$$\frac{i}{j} = \frac{n a \times 40}{g c \times 28},$$

or, substituting the given values of n , a , g , and c ,

$$\frac{i}{j} = \frac{3\frac{1}{2} \times 20 \times 40}{5 \times 40 \times 28} = \frac{1}{2}.$$

Choose, for j , a gear whose number of teeth, say 60, is divisible by 2; then, dividing the number of teeth in j by 2, we have $60 \div 2 = 30$. Now multiplying both terms of the fraction $\frac{1}{2}$ by 30,

$$\frac{i}{j} = \frac{1 \times 30}{2 \times 30} = \frac{30}{60},$$

that is, $i = 30$, and $j = 60$. Hence, one solution of the problem is, $h = 28$; $i = 30$; $j = 60$; $f = 40$.

HORSEPOWER OF ENGINES, BOILERS, AND PUMPS.

THEORETICAL HORSEPOWER.

The theoretical horsepower of any machine that uses a fluid (steam, gas, water, etc.) as a motive power, or that discharges a fluid (i. e., a pump or a fan), may be readily computed by the following formula, in which v is the volume of the fluid used or discharged in cubic feet per minute, and p is the average pressure in pounds per square inch:

$$\text{H. P.} = \frac{144 v p}{33,000}.$$

If, in the above formula, allowance for friction, etc. is made, the final result will be the actual horsepower.

EXAMPLE.—A ventilating fan delivers 5,000 cu. ft. of air per min. at a pressure of .56 lb. above the atmospheric pressure; what is the theoretical horsepower required to drive the fan?

SOLUTION.

$$\text{H. P.} = \frac{144 \, v \, p}{33,000} = \frac{144 \times 5,000 \times .56}{33,000} = 12.218.$$

If all hurtful resistances are taken in this case as 20% of the total horsepower, the actual horsepower will be

$$12.218 \div (1 - .20) = 12.218 \div .80 = 15.27 \text{ H. P.}$$

EXAMPLE.—The mean effective pressure computed from an indicator card taken from the air cylinder of an air compressor is 30.6 lb. per sq. in.; diameter of cylinder, 28 in.; stroke, 48 in.; number of strokes per minute, 108; what is the horsepower?

SOLUTION.—In this case

$$v = \frac{28^2 \times .7854 \times 48 \times 108}{1,728} \text{ cu. ft. per min.}$$

Hence,

$$\frac{144 \, v \, p}{33,000} = \frac{144 \times 28^2 \times .7854 \times 48 \times 108 \times 30.6}{1,728 \times 33,000} = 246.66 \text{ H. P.}$$

HORSEPOWER OF AN ENGINE.

Let P = mean effective pressure in pounds per square inch on the piston during one stroke;

L = length of stroke in feet;

A = area of piston in square inches;

N = number of strokes per minute;

D = diameter of piston in inches.

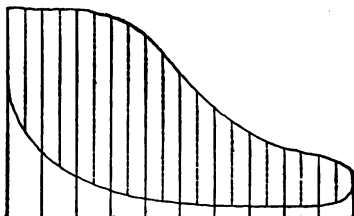
Then, to find the indicated horsepower,

$$\text{I. H. P.} = \frac{P L A N}{33,000} = \frac{238 \, P L D^2 N}{10,000,000}.$$

The actual horsepower may be taken as three-fourths of the indicated horsepower. The mean effective pressure may be found exactly by taking some indicator cards, finding the areas by means of a planimeter, and dividing the area by the length of the card. Multiply the result by the scale of the indicator spring, and the product will be the mean effective pressure, or M. E. P. If no planimeter is at hand, divide the card into 10 equal parts and measure each part in the middle, as shown by the dotted lines in the following figure.

Add all the dotted ordinates together, and divide by 10; this result, multiplied by the scale of the indicator spring, gives the M. E. P.

For example, a double-acting engine $26'' \times 30''$, making 80 rev. per min. (80 R. P. M.), gives an indicator card that, being divided up and measured as shown in the figure, gives, for the total length of the ordinates, 21.4 in. This divided by



10 = 2.14 in. for the length of the mean ordinate. If a No. 40 spring is used in the indicator, every inch measured vertically on the diagram = 40 lb. per sq. in., and $2.14 \times 40 = 85.6$ lb. per sq. in. for the M. E. P. on the piston. Then the indicated horsepower, or I. H. P., equals

$$\frac{PLAN}{33,000} = \frac{85.6 \times \frac{3}{4} \times (.7854 \times 26^2) \times (2 \times 80)}{33,000} = 550.88.$$

The calculation is rendered much easier by using the second formula. Thus,

$$\text{I. H. P.} = \frac{238 \times 85.6 \times \frac{3}{4} \times 26^2 \times (2 \times 80)}{10,000,000} = 550.88.$$

If an indicator card cannot be obtained, a fair approximation to the M. E. P. may be obtained by adding 14.7 to the gauge pressure, and multiplying the number opposite the fraction indicating the point of cut-off in the following table by the boiler pressure. Subtract 17 from the product, and multiply by .9. The result is the M. E. P. for good simple non-condensing engines. If the engine is a simple condensing engine, subtract the pressure in the condenser instead of 17. The fraction indicating the point of cut-off is obtained by dividing the distance that the piston has traveled when the steam is cut off by the whole length of the stroke. Thus, if the stroke is 30 in., and the steam is cut off when the piston

has traveled 20 in., the engine cuts off at $\frac{3}{8}$ = $\frac{1}{4}$ stroke. For a $\frac{1}{4}$ cut-off, and 92-lb. gauge pressure in the boiler, the M. E. P. is $[(92 + 14.7) \times .917 - 17] \times .9 = 72.76$ lb. per sq. in.

Cut-off.	Constant.	Cut-off.	Constant.	Cut-off.	Constant.
$\frac{1}{8}$	.566	$\frac{3}{8}$	.771	$\frac{2}{8}$	.917
$\frac{1}{4}$	.603	$\frac{4}{8}$	.789	$\frac{7}{8}$	.926
$\frac{3}{8}$	.659	$\frac{1}{2}$	.847	$\frac{5}{8}$	.937
$\frac{5}{8}$	.708	$\frac{6}{8}$	.895	$\frac{8}{8}$	.944
$\frac{7}{8}$	.743	$\frac{7}{8}$	.904	$\frac{7}{8}$	.951

THE SLIDE VALVE.

Figs. A, B, C, and D show sections of an ordinary D slide valve at different points of its travel. Fig. A shows the valve in its central position, with the center of the valve in line with the center line of the exhaust port. The names of the various parts are as follows: *p* and *p* are the *steam ports*; *e* is the *exhaust port*; *s, s* is the *valve seat*; the amount *o* by which the valve overlaps the outer edges of the steam ports is the *outside lap*; the amount *i* by which the valve overlaps the inside edges of the steam port is called the *inside lap*; the amount *l* (Fig. C) that the port is open when the piston is at the end of the stroke is called the *lead*. The valve travel is the total distance in one direction that the valve can be moved by the eccentric; it is the total distance between two extreme positions of the valve. The *displacement* of the valve is the distance that the valve has moved (in either direction) from its central position.

The line joining the center of the eccentric with the center of the crank-shaft is called the *eccentric radius*. When the eccentric radius makes a right angle with the center line of the crank, that is, when the eccentric radius is vertical (see *oe*, Fig. E), the valve is in its central position, provided the valve seat is horizontal, as is usually the case. When the crank is on a dead center, say *a*, Fig. E, the valve must be in the position shown in Fig. C; that is to say, the crank must

In Fig. *B*, the valve is shown in its extreme position at the right. The distance marked *m* is the *maximum port opening*. It matters not whether the outer edge of the valve travels beyond the inner edge of the port or falls short of it, as in the figure, the distance *m* between the edge of the valve and the edge of the port when the valve is in its extreme position is the maximum port opening. If, in Fig. *C*, the valve were shown moving to the left, a little farther movement would bring the left outer edge just even with the outer edge of the left steam port, and from here on to the end of the stroke no more steam could enter the left end of the cylinder; in other words, the valve *cuts off* at this point. A little farther movement of the valve to the left brings the valve to the position shown in Fig. *D*, with the right inner edge opposite the inner edge of the right steam port; it is at this point that compression begins.

When designing a valve for an engine, some of the above quantities are assumed and the remaining ones are required; these may be found by means of the diagram shown in Fig. *E*.

Let *ab*, Fig. *E*, drawn to any convenient scale, represent the stroke of the engine; then *adb* will represent the crankpin circle. About *o*, the center of the crankpin circle, describe a circle *a'eb'*, whose diameter *a'b'* is equal to the actual travel of the valve. Draw the line *gh* parallel to *ab* and at a distance from it equal to the lead of the valve. Then, with a radius *o'j* equal to the outside lap of the valve, describe a circle, called the *outside lap circle*, tangent to the line *gh*, and having its center *o'* on the circle *a'eb'*. Draw the line *oo'*, and produce it to *f*; then $\angle fob = \angle eoc = \text{angle of advance}$.

Now, draw any position of the crank center line, such as *ao*, and drop upon it, from the point *o'*, a perpendicular; the length of this perpendicular (marked *r* in Fig. *E*) is the displacement of the valve for that position of crank center line.

About the center *o'* with a radius equal to the inside lap of the valve, describe a circle; this is called the *inside lap circle*.

The radius *od*, drawn from the point *o* tangent to the outside lap circle, is the position of the center line of crank at the point of cut-off. Drop a perpendicular from point *d*,

meeting the line ab at k ; then ak is the distance moved by piston before cut-off, and the fraction of the stroke at which the valve cuts off is represented by the fraction $\frac{ak}{ab}$.

Draw the radius ol tangent to the upper side of the inside lap circle, and it will be the position of the center line of the crank when *compression* commences; if a perpendicular is dropped from point l , meeting the line ab at p , the fraction of the stroke of piston at which compression begins will be represented by the fraction $\frac{ap}{ab}$.

In like manner, the radius om , drawn tangent to the lower side of the inside lap circle, is the position of the center line of the crank at the moment of *release*; and $\frac{ay}{ab}$ is the fractional part of the stroke at which the expanding steam is released.

The maximum steam-port opening is equal to on , n being the point of intersection of the outside lap circle with the angle of advance line of .

The essential features of the valve diagram having been given, the following examples will make clear its application in practice:

EXAMPLE 1.—Given, the point of cut-off, the point of release, the lead, and the maximum port opening, to find the valve travel, the outside and inside lap, the angle of advance, and the point of compression.

SOLUTION.—Draw to a convenient scale the crankpin circle adb , Fig. *E*, having its center at o , and its diameter ab equal to the stroke of the piston.

From the point a , lay off, on the line ab , the distances ak and ay , so that $\frac{ak}{ab}$ and $\frac{ay}{ab}$ are equal, respectively, to the fractions of the stroke at which cut-off and release are to occur. At k and y draw perpendiculars to the line ab , intersecting the crankpin circle at d and m , respectively; the radii od and om will represent the positions of the crank at cut-off and release, respectively. Now draw gh parallel to ab , and at a distance above it equal to the lead; then, about o as

a center, and with a radius equal to the given maximum port opening, describe an arc. Find by trial a center o' , from which a circle can be drawn tangent to this arc, and also to the radius od , and to the line gh . The radius of this circle will be the required outside lap; and its center o' will be a point in the valve circle whose center is at o ; this circle can now be drawn, since the radius oo' is known.

The diameter $a'b'$ is equal to the required valve travel. Now, with o' as a center, draw a circle tangent to om , and the radius of this circle will be the required inside lap. Draw of through o' and the angle $fo b$ is the required angle of advance. Draw the radius ol tangent to the inside lap circle on its upper side, and lp perpendicular to ab .

Then, $\frac{ap}{ab}$ represents the fraction of the stroke at which compression begins.

EXAMPLE 2.—Given, the valve travel, the angle of advance, the cut-off, and the point of compression, to find the lead and the outside and inside lap.

SOLUTION.—Draw the crankpin circle, as before, and the valve circle $a'eb'$; construct the angle $fo b$ equal to the angle of advance. By the same method as employed in the last example, locate the radii od and ol , representing the positions of the crank at the points of cut-off and compression, respectively.

About the point o' , at which of intersects the valve circle, describe a circle tangent to od , and the radius $o'j$ of this circle will be the required outside lap. Now draw the line gh parallel to ab and tangent to the outside lap circle; then, the perpendicular distance between gh and ab is the required lead. The radius of a circle drawn from o' tangent to ol will be the inside lap.

EXAMPLE 3.—Given, the valve travel, outside lap, and the lead, to find the point of cut-off and angle of advance.

SOLUTION.—Draw the crankpin circle and the valve circle $a'eb'$ as before; draw a line parallel to ab , at a distance above it equal to the outside lap r plus the lead, intersecting the valve circle at the point o' . About o' as center, and with a radius equal to the given lap, describe a circle; draw od

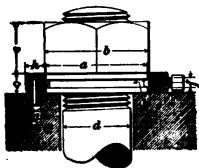
tangent to this circle, and drop a perpendicular from d , meeting line ab at a point k ; then the required cut-off is represented by the fraction $\frac{ak}{ab}$. Draw the radius of through the point o' and the angle $fo'b$ is the required angle of advance.

EXAMPLE 4.—Given, the outside lap, the lead, and the point of cut-off, to find the valve travel and the angle of advance.

SOLUTION.—Draw the crankpin circle as before, and by the same method as employed in Example 1 locate the radius od the position of the crank at the point of cut-off. Draw gh parallel to ab , and at a distance above it equal to the lead. At a distance above the line ab equal to the lap plus the lead, draw another line parallel to ab ; about a center o' on this line, and with a radius $o'j$ equal to the outside lap, describe a circle tangent to od and gh . Draw the radius of through o' , then $fo'b$ will be the required angle of advance. About o as a center, and with a radius oo' , describe the valve circle $a'e'b'$, and $a'b'$ will be the required valve travel.

LOCKNUTS.

A good method of locking a nut is shown in the figure.



The lower portion of the nut is turned down, and in the center of the circular portion a groove is cut. A collar is fastened by means of a pin to one of the pieces to be connected, and into this collar is fitted the circular part of the nut. The nut is then bound to the collar by a setscrew passing through the

latter, the point of the setscrew engaging into the groove turned in the nut. The following proportions have proved very satisfactory, in which d , the diameter of the bolt, is taken as the unit. All dimensions are in inches:

$$\begin{aligned} a &= 1\frac{1}{2}d - \frac{1}{16}''; & f &= \frac{1}{2}d + \frac{1}{16}''; \\ b &= 1\frac{1}{2}d + \frac{1}{16}''; & g &= \frac{1}{2}d + \frac{1}{16}''; \\ c &= \frac{1}{2}d + \frac{1}{16}''; & h &= \frac{1}{2}d + \frac{1}{16}''; \\ e &= \frac{1}{2}d; \end{aligned}$$

PROPORTION OF KEYS.

In common designing, the sizes of keys are determined by empirical formulas, which give an excess of strength. For an ordinary sunk key, these proportions may be adopted:

- t = thickness of key in inches;
- b = breadth of key in inches;
- d = diameter of shaft in inches;
- $b = \frac{1}{4} d$;
- $t = \frac{1}{3} b = \frac{1}{12} d$.

LINE SHAFTING.

The speed of a shaft is fixed largely by the speed of the driving belt or the diameters of the pulleys upon it. In general, machine-shop shafts run about 120 to 150 rev. per min.; shafts driving wood-working machinery, about 200 to 250 rev. per min.; in cotton mills, the practice is to make the shaft diameter smaller and run at a higher speed. Line shafts should generally not be less than $1\frac{1}{2}$ in. in diameter.

The distance between the bearings should not be great enough to permit a deflection of more than $\frac{1}{16}$ in. per foot of length; hence, the bearings must be closer when the shaft is heavily loaded with pulleys.

The maximum distances between bearings of different sizes of continuous shafts used for transmitting power are:

DISTANCES BETWEEN BEARINGS.

Diameter of Shaft. Inches.	Distance Between Bearings in Feet.	
	Wrought-Iron Shaft.	Steel Shaft.
2	11	11.50
3	13	13.75
4	15	15.75
5	17	18.25
6	19	20.00
7	21	22.25
8	23	24.00
9	25	26.00

Pulleys that give out a large amount of power should be placed as near a hanger as possible.

SHAFT COUPLINGS.

A box, or *muff*, coupling is shown in the figure. It consists of a cast-iron cylinder that fits over the ends of the shaft. The two ends are prevented from moving relatively to each other by the



sunk key. The keyway is cut half into the box and half into the shaft ends. Quite commonly the ends of the shafts are enlarged to allow the keyway to be cut without weakening the shaft.

The key may be proportioned by the formula already given. For the other dimensions, take

$$l = 2\frac{1}{2}d + 2''$$

$$t = .4d + .5''$$

EXAMPLE.—Find the dimensions of a muff coupling for a shaft $2\frac{1}{2}$ in. in diameter.

SOLUTION.—For the key we use the formula previously given,

$$b = \frac{1}{4}d = \frac{1}{4} \times 2\frac{1}{2} = \frac{5}{8}''$$

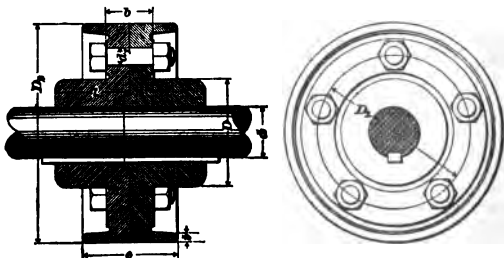
$$t = \frac{1}{8}d = \frac{1}{8} \times 2\frac{1}{2} = \frac{5}{16}''$$

For the muff,

$$l = 2\frac{1}{2}d + 2'' = 2\frac{1}{2} \times 2\frac{1}{2} + 2'' = 8\frac{1}{4}''$$

$$t = .4d + .5'' = .4 \times 2\frac{1}{2} + .5'' = 1\frac{1}{2}''$$

A *flange coupling* is shown in the following figure. Cast-



iron flanges are keyed to the ends of the shafts. To insure a

perfect joint the flange is usually faced in the lathe after being keyed to the shaft. The two flanges are then brought face to face and bolted together.

Sometimes the ends of the shafts are enlarged to allow for the keyway. To prevent the possibility of the shafts getting out of line, the end of one may enter the flange of the other.

The following proportions may be used for this form of flange coupling:

d = diameter of shaft; n = number of bolts.

$$D = 1\frac{1}{2}d + 1''$$

$$D_1 = 2\frac{1}{2}d + 2''$$

$$l = 1\frac{1}{2}d + 1''$$

$$n = 3 + \frac{d}{2}$$

(Take the nearest whole number for n .)

$$d_1 = \frac{d}{n} + \frac{1}{4}''$$

$$D_2 = 1.4 D_1$$

$$b = \frac{1}{2}d + \frac{1}{8}''$$

$$e = 2b$$

$$t = \frac{1}{8}d$$

The proportions for the key have already been given.

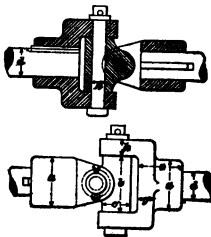
In the accompanying figure is shown a flexible coupling, or *universal joint*. These joints, when constructed of wrought iron, may have the following proportions in terms of the diameter d of the shaft:

$$a = 1.8d \quad g = .6d$$

$$b = 2d \quad h = .5d$$

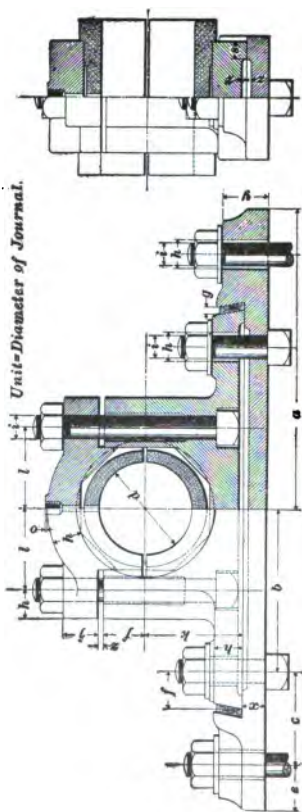
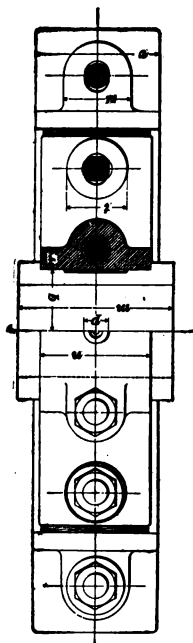
$$c = d \quad k = .6d$$

$$e = 1.6d$$



PEDESTALS.

The names *pedestal*, *pillow-block*, *bearing*, and *journal-box* are used indiscriminately. They are all a form of bearing, and indicate a support for a rotating piece.



$a = 2.25 d;$	$m = .25 d + .1875'';$
$b = 1.75 d;$	$n = .5 d;$
$c = d;$	$o = .625''$ (constant);
$e = .375 d;$	$p = 1.5 d;$
$f = .08 d + .0625'';$	$q = 1.333 d;$
$g = 1.75 d;$	$r = .08 d;$
$h = 2.45 d;$	$s = .125''$ (constant);
$i = .3 d;$	$t = .16 d;$
$j = .33 d;$	$u = 1.333 d;$
$k = .25 d + .125'';$	$v = .125 d.$
$l = .08 d;$	

In Fig. 2 is shown a common form of pedestal that is used for somewhat larger journals than the one shown in Fig. 1.

It consists of (1) a foundation plate that is bolted to the foundation on which the pedestal rests; the plate is essential when the pedestal rests on brickwork or masonry, but may be dispensed with when the pedestal rests on the frame of the machine; (2) the block that carries the seats and supports the journal; (3) the cap that is screwed down over the seats. The bolt holes in both foundation plate and block are oblong, so that the pedestal may be readily adjusted.

The following proportions may be used for this kind of pedestal, having journals from 2 in. to 6 in., inclusive. An oil cup having a $\frac{1}{4}$ in. pipe-tap shank may be used on pedestals for journals having diameters from 3 in. to 4 in., and $\frac{1}{2}$ in. pipe-tap shank for larger sizes up to 6 in. diameter.

NOTE.—The shanks of oil cups and grease cups bought in the market are made with a $\frac{1}{4}''$, $\frac{1}{2}''$, $\frac{3}{4}''$, or $1''$ pipe thread. The amount of oil or grease the cup holds when filled is usually expressed in ounces.

The diameter of journal d is the unit.

$a = 3.25 d;$	$j = .375 d;$	$r = .25 d;$
$b = 1.75 d;$	$k = 1.0625 d;$	$s = .1875 d;$
$c = d;$	$l = .875 d;$	$t = .65 d;$
$e = .5 d;$	$m = 1.75 d;$	$u = .75 d;$
$f = .4375 d;$	$n = 1.25 d;$	$v = 1.375 d;$
$g = .09 d;$	$o = .125''$ (constant);	$x = .25 d;$
$h = .3125 d;$	$p = .875''$ (constant);	$y = .5 d;$
$i = .25 d;$	$q = .625 d;$	$z = .0625 d.$

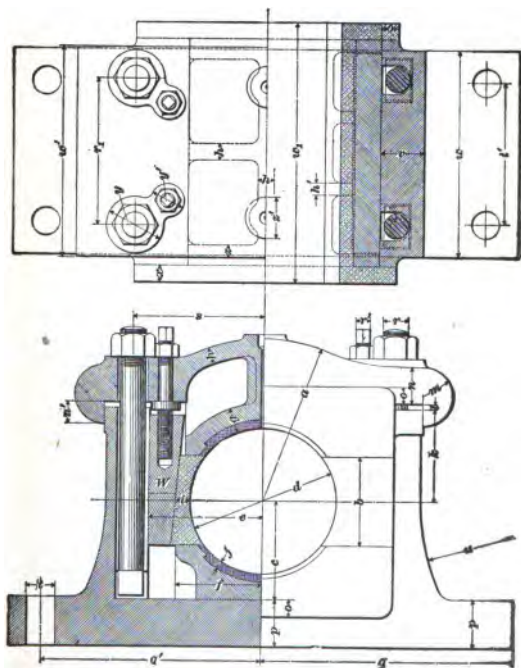


FIG. 3.

Fig. 3 shows a pedestal suitable for the crank-shaft of a horizontal engine with journals from 8 in. to 20 in. in diameter. The block may be complete in itself, as shown in the figure, but more often it forms part of the engine bed.

The seats are in three parts, and may be adjusted horizontally by means of the wedges *W*. The lower seat may be raised by placing packing pieces under it. To obtain its dimensions, use the following proportions, which are based on the unit d = the diameter of the crank-shaft journal.

$a = d + 1''$;	$q' = 1.5 d$;
$b = .5 d + 1''$;	$r = .15 d$;
$c = .66 d$;	$r' = .1 d$;
$e = .825 d - .25''$;	$r_1 = d$;
$f = .6 d$;	$s = .9 d$;
$g = .1 d + .5625''$;	$t = 15 d + .375''$;
$h = .1 d + .25''$;	$t' = .9 d$;
$h' = .08 d$;	$u = 1.5 d$;
$i = .11 d$;	$v = .25 d + .375''$;
$j = .625''$ (constant);	$w = 1.45 d$;
$k = .5 d + 1.25''$;	$w' = 1.47 d$;
$l = .375''$ (constant);	$w_1 = 1.75 d$;
$m = .175 d + .31.25''$;	$x = .1 d$;
$n = .25 d + 25''$;	$y = .3 d + .75''$;
$n' = .1 d + .375''$;	$y' = .2 d + .5''$;
$o = 1''$ (constant);	$z = .09 d$;
$p = .25 d + .625''$;	$z' = 2.5''$ (constant).
$q = 1.75 d$;	

Taper of adjusting wedge, 1 : 10.

Further details of the bottom seat and the cap are shown in Fig. 4, in which the unit is the same as in Fig. 3, and the proportions are as follows:

$a = 1''$ (constant);	$c = .08 d$;
$b = 1.65 d - .5''$;	$d = .1 d$.

The foundation casting, or the bed casting, is shown in Fig. 5, and has dimensions to suit the pedestal that is shown in Fig. 3. The proportions of the casting are given in connection with Fig. 5, on page 201. The diameter d of the crank-shaft journal is taken as the unit.

- $a = 2.45 d + 7.25''$;
 $b = 2.3 d + 5.25''$;
 $c = .5 d + 3.5''$;
 $e = 3.5 d + 2''$;
 $f = .25 d + .5''$;
 $g = .25 d + 1.75''$;
 $h = .25 d + 2.25''$;
 $i = .05 d + .5''$;
 $j = .05 d + 1.125''$;
 $k = .05 d + .75''$;
 $l = .25 d + .75''$;
 $m = .4 d$;
 $m' = .6 d$;
 $n = 1.55 d + 2.5''$;
 $o = .25 d + 2''$;
 $o' = .25 d + .5''$;
 $o'' = .5 d + 4.5''$;
 $p = .08 d$;
 $q = 1.5 d$;
 $r = .15 d + .375''$;
 $s = .15 d + .375''$;
 $t = .9 d$;
 $u = .15 d + .875''$;
 $v = .2 d$;
 $w = 1.5 d$;
 $x = 1.65 d$.

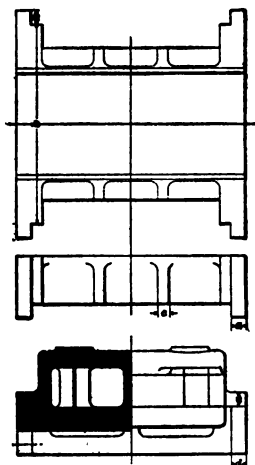


FIG. 4.

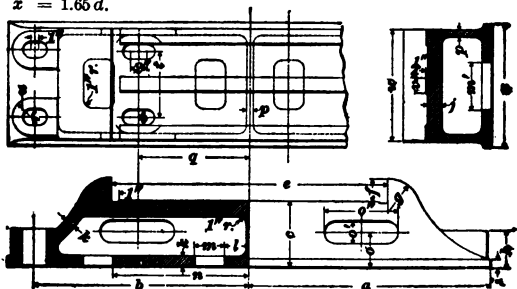


FIG. 5.

HANGERS.

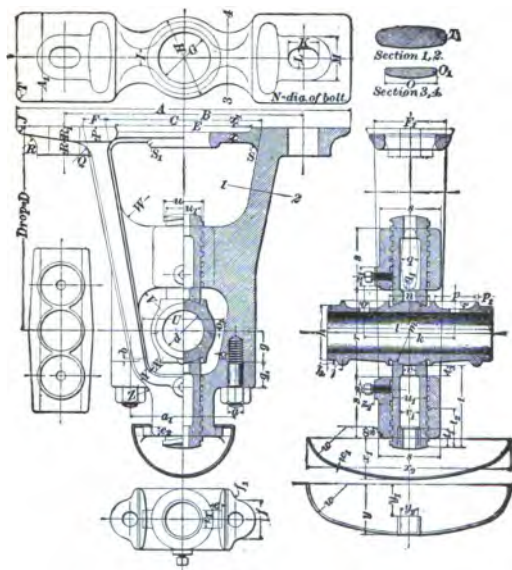
A hanger is used when a shaft bearing is to be suspended from the ceiling. The figure on page 203 shows a form of hanger made by a leading manufacturing company.

The frame of the hanger is divided and the parts are connected by bolts. With such a form, the shaft may be more easily removed than when the hanger frame is a solid piece.

The units for determining the leading dimensions of a shaft hanger are the diameter d of the shaft and the drop D of the hanger.

The following proportions are suitable for shafts ranging from $1\frac{1}{2}$ in. to $4\frac{1}{2}$ in. in diameter:

$A = 6d + .45D;$	$X = .375d;$
$A_1 = 2d + .03D;$	$Y = .25d + .125'';$
$B = 4d + .35D;$	$Z = .625d;$
$C = 2d + .3D;$	$a = .15d + .375'';$
$E = 2d + .25D;$	$a_1 = 2.4d + .3125'';$
$F = .5d + .01D;$	$b = .08d;$
$F_1 = 1.5d + .05D;$	$c = .125d + .0625'';$
$G = 1.25d;$	$e = .2d;$
$H = 2d;$	$e_1 = .4d;$
$I = .4d;$	$e_2 = .2d;$
$J = .125d + .01D;$	$f = .375d + 1'';$
$K = .5d + .5'';$	$f_1 = .09d + .25'';$
$L = .25d + .5'';$	$g = .75d;$
$M = .75d + .6875'';$	$g_1 = 1.3125d + .125'';$
$N = .25d + .375'';$	$h = 1.25d + .1875'';$
$O = 1.25d;$	$i = .1d;$
$O_1 = .094d + .002D;$	$j = .25d + .25'';$
$P = .375d + .008D;$	$j_1 = .125d + .0625'';$
$Q = .375d + .008D;$	$k = 2.2d;$
R and R_1 (see note);	$l = 4d;$
$S = .25d + .005D;$	$m = 1.4d + .375'';$
$S_1 = .125d + .003D;$	$n = d;$
$T = .125d + .01D;$	$o = .25d;$
T_1 = (see note);	$o_1 = .0625d;$
$U = 2d;$	$p = d;$
$V = .5d;$	$p_1 = .0625d;$
$W = .75d;$	$q = .4d;$



$$r_1 = .15 d;$$

$$r = 2.125 d;$$

$$s = 1.5 d;$$

$$s_1 = .125 d;$$

$$t = 2 d;$$

$$t_1 = .5 d;$$

$$t_2 = d;$$

$$t_3 = .25 d;$$

$$u = .95 d;$$

$$u_1 = .85 d;$$

$$v = .25 d + .125'';$$

$$v_1 = .5 d;$$

$$w = d;$$

$$w_1 = .125'' \text{ (constant);}$$

$$x = .25 d;$$

$$x_1 = d;$$

$$x_2 = 4 d + 2'';$$

$$y = 1.25 d;$$

$$y_1 = .75 d + .0625'';$$

$$y_2 = .4 d + .0625'';$$

$$z = .06 d + .75'';$$

$$z_1 = .12 d + .75'';$$

$$z_2 = .3125'' \text{ (constant).}$$

Thread of plugs, .5 in. pitch for all sizes.

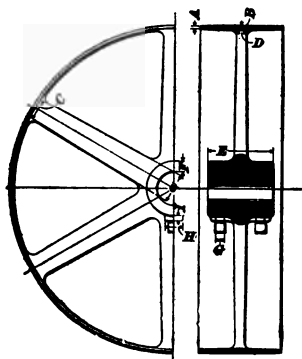
NOTE.—To find R_1 , draw the arc J ; also, draw the arc Q tangent to P ; then, draw a straight line tangent to these arcs, and R_1 will be the distance along the center line determined by B included between this tangent and the upper face of the hanger. Having found R_1 , make R equal to it.

The radius T_1 is made equal to three-eighths of the thickness at the middle.

The steps of the ball-and-socket bearings are of cast iron, and are bored to fit the journal without using either lining or brasses. The ball and the recesses in the ends of the plugs, into which the ball is fitted, should be faced. The screw threads on the plugs may be cast on the plugs or turned, the latter being preferable. It is customary to use 2 threads per inch for all sizes of plugs.

BELT PULLEYS.

The accompanying table gives the dimensions of a set of cast-iron belt pulleys ranging from 6 in. to 72 in. in diameter, as



made by a well-known manufacturing company. These pulleys are so designed that the number of patterns may be kept within reasonable limits, and at the same time have the dimensions correspond as nearly as possible with well-established rules.

The letters over the columns of dimensions given in the table correspond to the letters in the figure.

In all cases the number of arms is 6, and the arms increase in size toward the hub, the taper being $\frac{1}{4}$ in. per ft.

In order to prevent heavy stresses in shafts and bearings, pulleys that are to run at high speeds must be carefully

balanced. Perfect balance involves two conditions: (a) the center of gravity of the pulley must lie in the center line of the shaft, (b) the straight line joining the centers of gravity of any pair of opposite halves of the pulley must be perpendicular to the center line of the shaft.

The usual method of balancing a pulley is to rivet a weight to the light side and test the balance by putting the pulley on a mandrel that is placed on two carefully leveled ways on which it can roll with very little friction. If the center of gravity of the pulley lies in the center of the shaft, the pulley will stay in position when stopped with any point of its circumference over the mandrel; if, however, one side of the pulley is heavier, the mandrel will roll until the heavy side is at the lowest possible point.

While the above method does not determine whether or not the second condition of perfect balance is fulfilled, it is generally sufficient for pulleys running at ordinary limits of speed and reasonably well made.

In some cases, however, a failure to meet the requirements of the second condition of perfect balance may result in unsatisfactory running and severe stresses in the shaft and its bearings. Consider a pulley in which the center of gravity of one half is at the right of a line perpendicular to the center line of the shaft while the center of gravity of the opposite half is on the left of the perpendicular. This condition will not affect the balance of the pulley when tested by the mandrel rolling on the ways; when, however, the pulley revolves around the center line of the shaft, the centrifugal forces of the two halves act in opposite directions and along different lines. These forces thus form a couple that tends to bend the shaft. Since the centrifugal force is proportional to the square of the number of revolutions, it is apparent that, at high speeds, the bending effect may be considerable, even though the lack of symmetry is not very great.

It is usually considered unsafe to run a cast-iron pulley, gear-wheel, or flywheel at a higher rim speed than 100 ft. per sec. Since the centrifugal force increases in direct proportion to the cross-section of the rim, it is evident that it is useless to try to provide against it by putting more material in the rim.

PROPORTIONS OF PULLEYS.

Diam.	Face.	Rim.		Arm.		Hub.		Boss.		
		A	B	C	D	E	F	G	H	I
6"	4	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	3	$\frac{3}{8}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6			$\frac{3}{4}$	$\frac{7}{8}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	8			$\frac{3}{4}$	$\frac{7}{8}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	10			$\frac{3}{4}$	$\frac{7}{8}$	4	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	12			$\frac{3}{4}$	$\frac{7}{8}$	4	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
8	4	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{7}{8}$	3	$\frac{3}{8}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6					$3\frac{1}{2}$	$\frac{1}{2}$			
	8	$\frac{5}{16}$	$\frac{1}{4}$	$1\frac{1}{8}$	$\frac{7}{8}$	$4\frac{1}{2}$				
	10					$5\frac{1}{2}$				
	12									
10	4	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{7}{8}$	3	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6	$\frac{5}{16}$	$\frac{1}{4}$	$1\frac{1}{8}$		$3\frac{1}{2}$				
	8					$4\frac{1}{2}$				
	10			$1\frac{5}{8}$	$\frac{5}{8}$	$5\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	12									
12	4	$\frac{5}{16}$	$\frac{1}{4}$	1	$\frac{7}{8}$	$3\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6			$1\frac{3}{4}$	$\frac{1}{2}$	4				
	8					5	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	10	$\frac{3}{8}+$	$\frac{5}{8}$	$1\frac{1}{2}$	$\frac{3}{4}$	$5\frac{1}{2}$				
	12					$6\frac{1}{2}$				
14	4	$\frac{5}{16}+$	$\frac{1}{4}$	$1\frac{1}{8}$	$\frac{1}{2}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6					$4\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	8	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{8}$	$\frac{5}{8}$	5				
	10					6				
	12			$1\frac{1}{8}$	$\frac{1}{2}$	$6\frac{1}{2}$				
16	4	$\frac{5}{16}+$	$\frac{1}{4}$	$1\frac{3}{8}$	$\frac{5}{8}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6					$4\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{5}{8}$
	8	$\frac{5}{8}+$	$\frac{5}{8}$	$1\frac{7}{8}$	$\frac{5}{8}$	5				
	10					6				
	12	$\frac{7}{8}$	$\frac{1}{2}$			$6\frac{1}{2}$	$\frac{3}{4}$			
18	16			$1\frac{7}{8}$	$\frac{1}{2}$	$8\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{3}{4}$	
	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{8}$	$\frac{5}{8}$	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6					$4\frac{1}{2}$				
	8	$\frac{7}{8}$	$\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{2}$	$5\frac{1}{2}$	$\frac{3}{4}$			
	10					6				
20	12					$7\frac{1}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	
	16	$\frac{1}{4}$	$\frac{3}{8}$	$2\frac{1}{4}$	$1\frac{1}{4}$	8				
	20					9				
	4	$\frac{3}{8}+$	$\frac{5}{8}$	$1\frac{3}{8}$	$\frac{5}{8}$	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6					$4\frac{1}{2}$				
20	8					5	$\frac{3}{4}$			
	10	$\frac{7}{8}$	$\frac{1}{2}$	$1\frac{5}{8}$	$\frac{3}{4}$	6				
	12					7				
	16	$\frac{3}{4}$	$\frac{7}{8}$	$2\frac{1}{4}$	$1\frac{1}{8}$	8	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{4}$	
	20					10	1			

TABLE—(Continued).

Diam.	Face.	Rim.		Arm.		Hub.		Boss.		
		A	B	C	D	E	F	G	H	I
22"	4	$\frac{1}{8}$	$\frac{1}{8}$	$1\frac{1}{2}$	$\frac{5}{8}$	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6					$4\frac{1}{2}$	$\frac{3}{4}$			
	8					5				
	10	$\frac{7}{16} +$	$\frac{1}{2}$	$1\frac{3}{4}$	$\frac{1}{2}$	$6\frac{1}{2}$				
	12						$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	
24	16					$8\frac{3}{4}$	1			
	20	$\frac{3}{4} +$	$\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{1}{4}$	11	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	
	4	$\frac{7}{16}$	$\frac{1}{2}$	$1\frac{1}{8}$	$\frac{1}{2}$	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6					$4\frac{3}{4}$	$\frac{3}{4}$			
	8					$5\frac{1}{2}$				
26	10	$\frac{1}{4}$	$\frac{3}{8}$	$1\frac{7}{8}$		7	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	
	12									
	16					$9\frac{1}{2}$	1			
	20	$\frac{1}{8}$	$\frac{1}{2}$	$2\frac{3}{4}$	$1\frac{3}{8}$		$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	
	24					11				
28	4	$\frac{7}{16}$	$\frac{1}{2}$	$1\frac{1}{2}$	$\frac{3}{4}$	$4\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6					5				
	8					6	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	
	10	$\frac{1}{4}$	$\frac{3}{8}$	2	$\frac{7}{8}$	7				
	12					$7\frac{1}{2}$				
30	16					10	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	
	20	$\frac{1}{8} +$	$\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{7}{8}$	$10\frac{1}{2}$				
	24					11				
	4	$\frac{7}{16} +$	$\frac{1}{2}$	$1\frac{3}{4}$	$\frac{3}{4}$	$4\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6					$5\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	
32	8					7				
	10					$7\frac{1}{2}$				
	12	$\frac{1}{4} +$	$\frac{3}{8}$	$2\frac{1}{8}$	$\frac{1}{2}$	8	1			
	16					10				
	20	$\frac{1}{8}$	$\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$	11	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	
30	24	$\frac{1}{8} +$	$\frac{1}{2}$	$1\frac{7}{8}$	$\frac{1}{2}$					
	4	$\frac{7}{16} +$	$\frac{1}{2}$	$1\frac{3}{4}$	$\frac{3}{4}$	$4\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6					$5\frac{1}{2}$	$\frac{7}{8}$			
	8					$6\frac{1}{4}$		$\frac{3}{4}$	$1\frac{3}{8}$	
	10					$6\frac{1}{2}$				
32	12	$\frac{7}{16}$	$\frac{7}{8}$	$2\frac{1}{4}$	1	8	1			
	16					$8\frac{1}{2}$				
	20					$11\frac{1}{2}$	$1\frac{1}{4}$	$\frac{7}{8}$	$1\frac{1}{2}$	
	24	$\frac{3}{8}$	$\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{5}{8}$	13	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{8}$
	4	$\frac{1}{4} +$	$\frac{3}{8}$	$2\frac{1}{8}$	$\frac{1}{2}$	$4\frac{1}{2}$				
32	6					$5\frac{1}{2}$				
	8					$6\frac{1}{2}$	1			
	10					$7\frac{1}{2}$				

TABLE—(Continued).

Diam.	Face.	Rim.		Arm.		Hub.		Boss.		
		A	B	C	D	E	F	G	H	I
34"	12	$\frac{1}{8}$	$\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	8	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	
	16					$9\frac{1}{2}$				
	20					11	$1\frac{1}{4}$			
	24					13				
	4	$\frac{1}{4}+$	$\frac{3}{8}$	$2\frac{1}{8}$	$\frac{1}{8}$	$4\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{8}$
	6					$5\frac{1}{2}$				
	8					$6\frac{1}{2}$	1			
	10					$7\frac{1}{2}$				
	12	$\frac{1}{8}$	$\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	$7\frac{3}{4}$	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	
	16					$9\frac{1}{2}$				
36	20					12	$1\frac{1}{4}$			
	24					13				
	4	$\frac{1}{4}+$	$\frac{3}{8}$	$2\frac{1}{8}$	$\frac{1}{8}$	$4\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{8}$
	6					$5\frac{1}{2}$				
	8					$6\frac{1}{2}$				
	10					$7\frac{1}{2}$				
	12	$\frac{1}{8}$	$\frac{1}{8}$			$7\frac{3}{4}$	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	
	16			$2\frac{1}{8}$	$1\frac{1}{8}$	$10\frac{1}{4}$	$1\frac{1}{4}$			
	20					12				
	24					$13\frac{1}{2}$	$1\frac{3}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
40	8	$\frac{1}{8}$	$\frac{1}{8}$	$2\frac{1}{8}$	1	$6\frac{3}{4}$	1	$\frac{3}{4}$	$1\frac{5}{8}$	$\frac{3}{8}$
	12					$7\frac{3}{4}$	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	
	16	$\frac{1}{8}$	$\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	10	$1\frac{1}{4}$			
	20					$11\frac{1}{2}$	$1\frac{3}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
	24					$15\frac{1}{2}$	$1\frac{1}{2}$			
	8	$\frac{3}{8}$	$\frac{1}{8}$	$2\frac{1}{2}$	$1\frac{1}{4}$	$6\frac{3}{4}$	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	$\frac{3}{8}$
	12					8				
	16	$\frac{1}{8}$	$\frac{1}{2}$	3	$1\frac{1}{8}$	10	$1\frac{1}{4}$			
	20					12	$1\frac{3}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
	24			$3\frac{1}{2}$	$1\frac{3}{4}$	15	$1\frac{1}{2}$			
48	8	$\frac{3}{8}+$	$\frac{1}{8}$	$2\frac{3}{4}$		$7\frac{1}{2}$	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	$\frac{3}{8}$
	12					$8\frac{3}{4}$	$1\frac{1}{4}$			
	16	$\frac{3}{8}$	$\frac{1}{8}$	$3\frac{1}{4}$	$1\frac{1}{8}$	10	$1\frac{3}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
	20					12				
	24					15	$1\frac{1}{2}$			
	8	$\frac{3}{8}+$	$\frac{1}{8}$	$2\frac{3}{4}$		$7\frac{1}{2}$	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	$\frac{3}{8}$
	12					$8\frac{3}{4}$	$1\frac{1}{4}$			
	16	$\frac{3}{8}$	$\frac{1}{8}$	$3\frac{1}{4}$	$1\frac{1}{8}$	10	$1\frac{3}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
	20					12				
	24					15	$1\frac{1}{2}$			
54	12	$\frac{1}{8}+$	$\frac{1}{8}$	3	$1\frac{1}{8}$	$9\frac{3}{4}$	$1\frac{3}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
	16					$11\frac{1}{4}$	$1\frac{1}{2}$			
	20	$\frac{1}{8}$	$\frac{1}{8}$	$3\frac{5}{8}$	$1\frac{5}{8}$					
	24					15	$1\frac{3}{4}$	$1\frac{1}{4}$	2	
	12	$\frac{1}{8}$	$\frac{1}{2}$	$3\frac{1}{8}$	$1\frac{1}{8}$	10	$1\frac{3}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
	16					$11\frac{1}{4}$	$1\frac{1}{2}$			
	20	$\frac{1}{8}$	$\frac{1}{8}$	$3\frac{5}{8}$	$1\frac{5}{8}$					
	24					15	$1\frac{3}{4}$	$1\frac{1}{4}$	2	
	12	$\frac{1}{8}$	$\frac{1}{2}$	$3\frac{1}{8}$	$1\frac{1}{8}$	10	$1\frac{3}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
	16					$11\frac{1}{4}$	$1\frac{1}{2}$			
60	20	$\frac{1}{8}$	$\frac{5}{8}$	$3\frac{1}{2}$	$1\frac{3}{4}$	$12\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{1}{4}$	2	
	24					15				

TABLE—(Continued).

Diam.	Face.	Rim.		Arm.		Hub.		Boss.		
		A	B	C	D	E	F	G	H	I
66"	12	$\frac{1}{4}$	$\frac{1}{2}$	$3\frac{1}{8}$	$1\frac{1}{8}$	10	$1\frac{1}{8}$	1	$1\frac{3}{4}$	$\frac{1}{2}$
	16					$11\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{1}{4}$	2	
	20	$\frac{1}{2}$	$\frac{3}{4}$	$4\frac{1}{4}$	$1\frac{1}{2}$	$13\frac{1}{2}$	$1\frac{7}{8}$			
	24					15				
72	12	$\frac{3}{8}$	$\frac{1}{2}$	$3\frac{7}{8}$	$1\frac{1}{2}$	$10\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{1}{4}$	2	$\frac{1}{2}$
	16					$12\frac{1}{2}$	$1\frac{3}{4}$			
	20	$\frac{1}{2}$	$\frac{3}{4}$	$4\frac{5}{8}$	$2\frac{1}{8}$	$13\frac{1}{2}$	$1\frac{7}{8}$			
	24					15	2			

ROPE BELTING.

There is a growing tendency toward the substitution of hemp and cotton ropes for belting and line shafting as a means of transmitting power in large factories and shops. The advantages claimed for the rope-driving system are:

1. Economy; for a rope system is cheaper to install than either leather belting or shafting.
2. In the rope system there is less loss of power by slipping.
3. Flexibility; that is, the ease with which the power is transmitted to any distance and in any direction.

In this country, a single rope is carried round the pulley as many times as is necessary to produce the required power, and the necessary tension is obtained by passing the rope round a tension pulley weighted to give the desired tension.

The ropes used in rope transmission are either of hemp, manila, or cotton. Manila ropes are mostly used in this country. They are of three strands, hawser laid, and may be from $\frac{1}{4}$ in. to 2 in. in diameter.

The weight of ordinary manila or cotton rope is about $.3 D^2$ lb. per ft. of length, where D represents the diameter of the rope in inches. Letting w = the weight per foot of length, $w = .3 D^2$.

The breaking strength of the rope varies from 7,000 to 12,000 lb. per sq. in. of cross-section. The average value may be taken as $7,000 D^2$, when D is the diameter of rope.

For a continuous transmission, it has been determined by experiment that the best results are obtained when the tension in the driving side of the rope is about $\frac{1}{4}$ of the breaking strength. That is,

$$T_1 = \text{tension in tight side} = \frac{7,000 D^2}{35} = 200 D^2.$$

The ropes run in V-shaped grooves, and the coefficient of friction is, of course, greater than on a smooth surface. The coefficient for grooves with sides at an angle of 45° may be taken at from .25 to .33.

The horsepower that can be transmitted by a single rope running under favorable conditions is given by the formula

$$H = \frac{v D^2}{825} \left(200 - \frac{v^2}{107.2} \right),$$

in which H = horsepower transmitted;

D = diameter of rope in inches;

v = velocity of rope in feet per second.

The maximum power is obtained at a speed of about 84 ft. per sec. For higher velocities, the centrifugal force becomes so great that the power is decreased, and when the speed reaches 145 ft. per sec. the centrifugal force just balances the tension, so that no power at all is transmitted. Consequently, a rope should not run faster than about 5,000 ft. per min., and it is preferable on the score of durability to limit the velocity to 3,500 ft. per min.

EXAMPLE.—A rope flywheel is 26 ft. in diameter, and makes 55 rev. per min. The wheel is grooved for 35 turns of $1\frac{1}{2}$ " rope. What horsepower may be transmitted?

SOLUTION.—Velocity in feet per second =

$$v = \frac{26 \times \pi \times 55}{60} = \frac{4,492}{60} = 74.9 \text{ ft.}$$

Applying the formula,

$$H = \frac{v D^2}{825} \left(200 - \frac{v^2}{107.2} \right),$$

the horsepower transmitted by one rope or turn is

$$\frac{74.9 \times (1\frac{1}{2})^2}{825} \left(200 - \frac{(74.9)^2}{107.2} \right) = 30.16.$$

Then, $30.16 \times 35 = 1,055.6 =$ horsepower transmitted by the 35 ropes.

EXAMPLE.—How many times should a 1" rope be wrapped around a grooved wheel in order to transmit 200 horsepower, the speed being 3,500 ft. per min.?

SOLUTION.— 3,500 ft. per min. = $\frac{3,500}{60} = 58\frac{1}{2}$ ft. per sec.

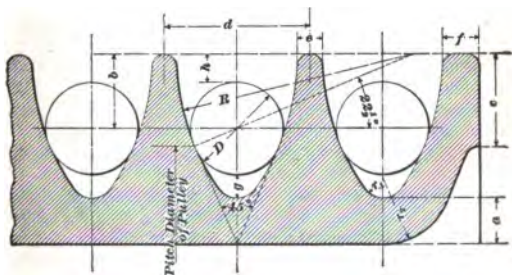
Applying the formula, the horsepower transmitted with one turn is,

$$H = \frac{58\frac{1}{2} \times 1^2}{825} \left[200 - \frac{(58\frac{1}{2})^2}{107.2} \right] = 11.9.$$

Hence, $200 \div 11.9 = 16.8$, say 17 turns.

Rope pulleys differ from belt pulleys only in their rims. The inclination of the sides of the grooves may vary from 30° to 60° . The more acute the angle, the greater the coefficient and, consequently, the wear on the rope.

A section of a grooved rim in which the sides of the grooves are formed with circular arcs is shown in the figure.



The proportions for this rim are as follows, using the diameter D of the rope as a unit:

$$a = \frac{1}{4} D;$$

$$b = \frac{3}{4} D + \frac{1}{16}'';$$

$$c = D;$$

$$d = 1.6 D;$$

$$e = \frac{1}{4} D + \frac{1}{16}'';$$

$$f = \frac{1}{4} D + \frac{1}{16}'';$$

$$g = \frac{1}{4} D;$$

$$h = \frac{1}{4} D + \frac{1}{16}''.$$

The radii r_1 and r_2 are to be found by trial; they should be of such lengths as to make the curves drawn by them tangent to the required lines.

The long radius R is determined by drawing a line through the center of the rope at an angle of $22\frac{1}{4}^\circ$ with the horizontal, and producing it until it intersects a line drawn through the tops of the dividing ribs; then, with this point of intersection as a center, draw the curve forming the side of the groove tangent to the circumference of the rope.

The advantage claimed for this groove is that the rope will turn more freely in it, thus presenting new sets of fibers to the sides of the grooves and increasing the life of the rope.

The diameter of a rope pulley should be at least 30 times the diameter of the rope. Good results are obtained when the diameters of pulleys and idlers on the driving side are 40 times, and those on the driven side 30 times, the rope diameter. Idlers used simply to support a long span may have diameters as small as 18 rope diameters, without injuring the rope.

When possible, the lower side of the rope should be the driving side, for in that case the rope embraces a greater portion of the circumference of the pulley, and increases the arc of contact.

When the continuous system of rope transmission is used, the tension pulley should act on as large an amount of rope as possible. It is good practice to use a tension pulley and carriage for every 1,200 ft. of rope, and have at least 10% of the rope subjected directly to the tension.

Aside from the grooved rim, rope pulleys are constructed the same as other pulleys. They may be cast solid, in halves, or in sections. The pulley grooves must be turned to exactly the same diameter; otherwise, the rope will be severely strained.

TRANSMISSION OF POWER BY WIRE ROPE.

Wire rope for transmitting power is made up of 6 strands twisted about a hemp core, each strand being composed of either 7 or 19 wires, according to the size of the sheaves, the 19-wire rope being employed in cases where it is impracticable to use the larger sheaves required by the 7-wire rope. Where the conditions, however, do not preclude the use of the

proper size of sheaves, the 7-wire rope is to be recommended in preference to the other, except sometimes on very short spans, where 19-wire rope is to be preferred, composed of the same size of wires as the smaller 7-wire rope, such as would ordinarily be used to transmit the power, and run under a tension corresponding to the smaller rope, or considerably below the maximum safe tension of the rope used. This is done in order to avoid stretching, which would otherwise occur, and the consequent use of mechanical appliances for preserving the necessary tension.

In flying transmission, where the rope makes a single half lap at each end, the sheaves are usually made of cast iron, with rims having grooves lined with segments of rubber and leather, dipped in tar, and laid in alternately, upon which the rope tracks. The diameters of the minimum sheaves, corresponding to a maximum efficiency, are as follows, according to a prominent manufacturer:

Diam. of sheave for 7-wire steel rope, 77 times diam. of rope.

Diam. of sheave for 19-wire steel rope, 46 times diam. of rope.

Diam. of sheave for 7-wire iron rope, 160 times diam. of rope.

Diam. of sheave for 19-wire iron rope, 96 times diam. of rope.

In long-distance transmissions, where the rope makes 2 or more half laps at each end about a pair of drums or several sheaves, the rims may be lined with wood or the rope may be run in plain turned grooves.

The horsepower capable of being transmitted is determined by the general formula:

$$N = [c D^2 - .000006 (w + g_1 + g_2)] v,$$

in which

D = diameter of rope in inches;

v = velocity of rope in feet per second;

w = weight of rope in pounds;

g_1 = weight of terminal sheaves and shafts;

g_2 = weight of intermediate sheaves and shafts;

c = constant depending on the material of which rope is made, the character of the filling or surface material in the sheaves or drums upon which the rope tracks, and the number of half laps at each end.

The values of c for from 1 up to 6 half laps for steel rope are given in the following table:

c for Steel Rope on	Number of Half Laps at Each End.					
	1	2	3	4	5	6
Iron.	5.61	8.81	10.62	11.65	12.16	12.56
Wood.	6.70	9.93	11.51	12.26	12.66	12.83
Rubber and Leather.	9.29	11.95	12.70	12.91	12.97	13.00

The values of c for iron ropes are one-half the above. It is apparent from this table that, when more than 3 half laps are made, the character of filling or surface in contact is immaterial so far as slipping is concerned.

Where the distance is comparatively short, as in most flying transmissions, the effect of the weight of the rope and sheaves is so slight that it may be neglected, and we have the general rule, that *the actual horsepower capable of being transmitted by a wire rope approximately equals c times the square of the diameter of the rope in inches, multiplied by the speed of the rope in feet per second.*

The tension of the rope is measured by the amount of sag or deflection at the center of the span, and the deflection corresponding to the maximum safe working tension is determined by the following formulas, in which s represents the span in feet:

Deflection.	Steel Rope.	Iron Rope.
Still rope at center, in ft.....	$h = .00004 s^2$	$h = .00008 s^2$
Driving portion, running, in ft... h_1	$= .000025 s^2$	$h_1 = .00005 s^2$
Slack portion, running, in ft... h_2	$= .0000875 s^2$	$h_2 = .000175 s^2$

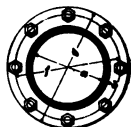
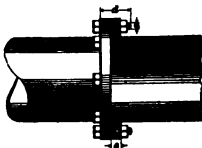
In very long transmissions it often happens that the conditions will not allow of the required amount of tension to drive properly with but a single half lap on the pulley. In such cases it is customary to give the rope a sufficient number of half turns around successive grooves in the driving pulley and a series of guide pulleys that serve to lead the rope from one groove on the driving pulley to the next.

With this arrangement a guide pulley at one end of the

line is usually made to serve the purpose of a *tension pulley* by being mounted in a movable frame that can be drawn by means of a screw or a weight so as to give the rope the desired tension.

PIPE FLANGES.

The figure shows the method of flanging and bolting the ends of two cast-iron pipes. The dimensions of the flanges for the various sizes of pipes are given in the following table:



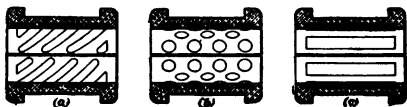
STANDARD PIPE FLANGES. n = number of bolts.

a	b	c	d	n	e	f	g
2.0	.409	$\frac{5}{8}$	2.000	4	$\frac{5}{8}$	4.75	6.00
2.5	.429	$\frac{5}{8}$	2.250	4	$\frac{11}{8}$	5.50	7.00
3.0	.448	$\frac{5}{8}$	2.500	4	$\frac{3}{4}$	6.00	7.50
3.5	.466	$\frac{5}{8}$	2.500	4	$\frac{11}{8}$	7.00	8.50
4.0	.486	$\frac{3}{4}$	2.750	4	$\frac{11}{8}$	7.50	9.00
4.5	.498	$\frac{3}{4}$	3.000	8	$\frac{11}{8}$	7.75	9.25
5	.525	$\frac{3}{4}$	3.000	8	$\frac{11}{8}$	8.50	10.00
6	.563	$\frac{3}{4}$	3.000	8	1	9.50	11.00
7	.600	$\frac{3}{4}$	3.250	8	$\frac{11}{8}$	10.75	12.50
8	.639	$\frac{3}{4}$	3.500	8	$\frac{11}{8}$	11.75	13.50
9	.678	$\frac{3}{4}$	3.500	12	$\frac{11}{8}$	13.25	15.00
10	.713	$\frac{7}{8}$	3.625	12	$\frac{11}{8}$	14.25	16.00
12	.790	$\frac{7}{8}$	3.750	12	$\frac{11}{8}$	17.00	19.00
14	.864	1	4.250	12	$\frac{13}{8}$	18.75	21.00
15	.904	1	4.250	16	$\frac{13}{8}$	20.00	22.25
16	.946	1	4.250	16	$\frac{17}{8}$	21.25	23.50
18	1.020	$\frac{11}{8}$	4.750	16	$\frac{17}{8}$	22.75	25.00
20	1.090	$\frac{11}{8}$	4.750	20	$\frac{17}{8}$	25.00	27.50
22	1.180	$\frac{11}{4}$	5.500	20	$\frac{17}{8}$	27.25	29.50
24	1.250	$\frac{11}{4}$	5.500	20	$\frac{17}{8}$	29.50	32.00
26	1.300	$\frac{11}{4}$	5.750	24	2	31.75	34.25
28	1.380	$\frac{11}{4}$	6.000	28	$2\frac{1}{8}$	34.00	36.50
30	1.480	$\frac{13}{8}$	6.250	28	$2\frac{1}{8}$	36.00	38.75
36	1.710	$\frac{13}{8}$	6.500	32	$2\frac{3}{8}$	42.75	45.75
42	1.870	$\frac{11}{2}$	7.250	36	$2\frac{5}{8}$	49.50	52.75
48	2.170	$\frac{11}{2}$	7.750	44	$2\frac{3}{4}$	56.00	59.50

LINING FOR SEATS.

Seats for large bearings are often lined with Babbitt metal, or anti-friction metal. It has been found by experience that a bearing will run cooler when so lined, probably because the Babbitt metal, being softer, accommodates itself to the journal more readily than the more rigid gun metal.

Some of the common methods of lining the seats are shown in the figure. At (a) the Babbitt metal is shown cast



into shallow helical grooves; at (b), into a series of round holes; and at (c), into shallow rectangular grooves. Consequently, the journal rests partly on the brass and partly on the Babbitt metal.

In cheap work, very frequently the seats are made entirely of Babbitt metal. A mandrel the exact size of the journal is placed inside the bearing, and the melted Babbitt metal is poured around it. In better work a smaller mandrel is used, and the metal is hammered in, the bearing being then bored out to the exact size of the journal.

CYLINDERS AND STEAM CHESTS.

Fig. 1 shows a cylinder designed for a simple slide-valve engine. The front head *A* is cast solid with the cylinder. The method of fastening to the frame *B* is clearly shown.

The principal dimensions of this cylinder may be determined from the following proportions:

D = diameter of cylinder;

L = length of stroke + thickness of piston + twice the piston clearance;

C = length of stroke + distance from outer edge to outer edge of piston rings — $(.01 D + .125'')$;

a = 5.5 t ;

$$b = 4.2 i;$$

$$c = i;$$

$$d = i;$$

$$e' = \text{net area of a single cylinder-head bolt whose nominal diameter is } e = \frac{A P}{4,000 n},$$

where A = area of cylinder head in square inches;

P = steam pressure;

n = number of bolts.

The pitch of the bolts may be from 4.5 to 5.5 in., but should never be more than $5f$.

$$f^A = 1.5 i;$$

$$g = .04 D + .125''. \text{ Take the nearest nominal size pipe tap.}$$

h = twice the outside diameter of drain pipe.

$$i = .0003 P D + .375'', \text{ where } P \text{ is the steam pressure. If the steam pressure is less than 100 lb., make } P = 100.$$

$$j = .85 i;$$

$$k = 4 i;$$

$$l = .75 i;$$

$$m = 1.01 D + .125'';$$

$$n = m + 6e, \text{ never less. Here, } e \text{ is the nominal diameter of the bolt.}$$

$$o = \text{the nominal diameter of steam-chest bolts. The net area of a single steam-chest bolt} = \frac{A' P}{4,000 n'},$$

where A' = area of steam chest;

n' = number of bolts in steam chest.

$$p = 2.75 o;$$

$$q = 1.5 r;$$

$$r = 1.25 i;$$

$$s = i. \text{ This is required only when the length of the port is greater than 12 in.}$$

$$t = 1.25 i. \text{ When } D \text{ is greater than 24 in., use 4 bolts in the standard and make } t = 1.1 i.$$

$$u = 1.5 i;$$

$$v = .25'' \text{ (constant).}$$

The dimensions of the *steam ports*, *exhaust ports*, and other steam passages depend on the velocity of the flow of steam. The ports and passages must be large enough to allow the steam to follow up the advancing piston without loss of

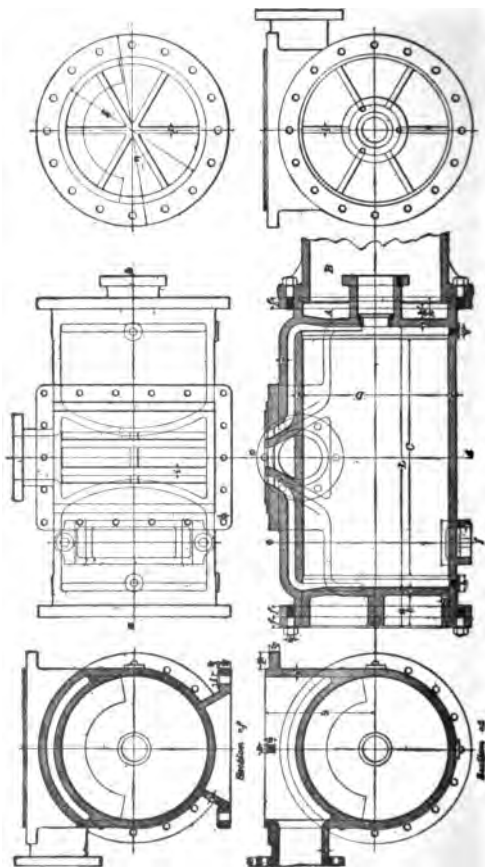


Fig. 1.

pressure. The maximum allowable velocity of the steam in the passages, when they are short, is about 160 ft. per sec. But, with the ordinary ratio between the length of connecting-rod and length of crank, the average velocity is about five-eighths of the maximum. Hence, the allowable average velocities are 100 to 125 ft. per sec. for long and short passages respectively.

Let l = length of port in inches;

b = breadth of port in inches;

A = area of cylinder;

S = average piston speed in feet per second;

v = average velocity of steam in feet per second.

Then, area of port $\times$ velocity of steam = area of piston $\times$ velocity of piston, or $lbv = AS$; whence,

$$lb = \frac{AS}{v}.$$

For long indirect passages, take $v = 100$; and for short direct passages, take $v = 125$.

The constant 100 may be used for v , when designing plain slide-valve engines of the ordinary type, which cut off late in the stroke, and 125 may be used for high-speed engines with early cut-off, and for the Corliss type.

The area of the exhaust port or ports may be from $1\frac{1}{4}$ to $2\frac{1}{4}$ times the area of a steam port.

The area of the cross-section of the steam pipe is approximately equal to the area of the steam port; likewise, the area of the exhaust pipe should be equal to that of the exhaust port.

The length l of the port may be $.6D$ to $.9D$ for slide-valve engines, and about $.9D$ to D for the Corliss type.

The height w , Fig. 1, of the valve seat must be such that the area of the most contracted part of the exhaust port is not less than 75% of the area of the steam port.

THE STEAM CHEST.

Fig. 2 shows a steam chest for the cylinder illustrated in Fig. 1. The principal dimensions are to be determined by the following proportions, which are based on the thickness i of the cylinder walls, and on the travel and dimensions of the valve:

- a = length of valve + travel of valve + twice the clearance between the valve and the steam chest at ends of valve travel;
 b = breadth of valve + twice the clearance between one valve and steam chest;
 c = .75 i ;

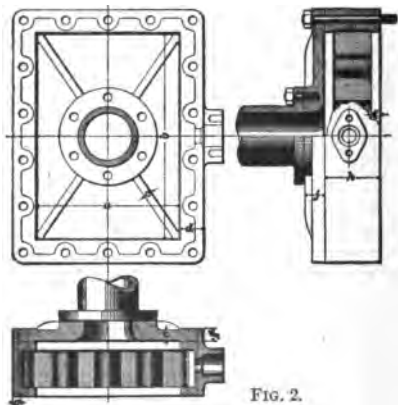


FIG. 2.

- d = $2.75 o$, where o is the nominal diameter of the steam-chest bolts, as in Fig. 1;
 e = $.04 \sqrt{A'} + .125''$ for all areas above 100 sq. in.
 A' = area of steam chest, outside measurement, in square inches;
 f = $1.3 e$;
 g = $.85 i$;
 h = height of valve + necessary clearance;
 t = $.85 i$;
 j = $2.5 i$.

NOTE.—When the area of the steam-chest cover is less than 100 sq. in., its thickness e may be made equal to i . If the area of the steam-chest cover exceeds 600 sq. in., the height of the ribs should be $3.5 i$, and their number should be increased.

Fig. 3 shows a design for a steam-chest cover when the steam-pipe flange is on one side of the steam chest. Determine the thickness e by the same formula and rules as for the cover in Fig. 2. The other dimensions are found as follows:

$$c = .75 e;$$

$$j = 2.6 c;$$

$$f = 1.3 e;$$

$$r = 6 e.$$

p should never exceed the distance in inches given by the formula $p = \sqrt{\frac{40 e_1^2}{p_g}}$, where e_1 is the numerator of the fraction expressing the thickness of the cover in sixteenths of an inch, and p_g is the gauge boiler pressure in pounds per square inch.

EXAMPLE.—Find the maximum pitch of the ribs for a cover $1\frac{5}{8}$ in. thick, subjected to a steam pressure of 160 lb. per sq. in.

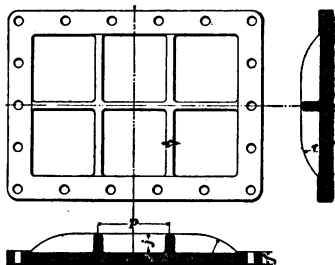


FIG. 3.

SOLUTION.—Substituting in the formula for p , we have

$$p = \sqrt{\frac{40 \times e_1^2}{p_g}} = \sqrt{\frac{40 \times 15^2}{160}} = 7.5 \text{ in.}$$

Fig. 4 shows a Corliss engine cylinder (that may be designed according to the following proportions:

D = diameter of cylinder.

$$a = 1.21 D + 2 e + 1.22'';$$

$$g = .9 e;$$

$$b = 2 D + 1.125'';$$

$$h = b + 2(c + g);$$

$$c = .048 D;$$

$$h' = h;$$

$$c' = .079 D;$$

$$i = 1.8 e;$$

$$d = .17 D;$$

$$j = e;$$

$$e = .0003 P D + .375'', \text{ if}$$

$$k = 1.2 e;$$

boiler pressure is above
100 lb.; otherwise, e

$$l = 1.7 x + 2'' - 1.2 e, \text{ where}$$

$$= .03 D + .375'';$$

x = diameter of piston
rod;

$$f = .82 e;$$

$$l' = .32 D, \text{ about;}$$

$$m = .25 D;$$

$$n = .32 D;$$

$$o = 1.25 e;$$

$$p = 1.3 e;$$

$$q = .25 D;$$

$$q' = .32 D;$$

$$r = 1.2 e;$$

$$s = 1.5 e;$$

$$t = (\text{see note});$$

$$u = e; \text{ take diameter nearest standard size bolt;}$$

$$v = 1.2 e; \text{ take diameter nearest standard size bolt;}$$

$$w = 1.7 x + 2.25'', \text{ where } x = \text{diameter of piston rod;}$$

$$y = D;$$

$$z = 1.5 e.$$

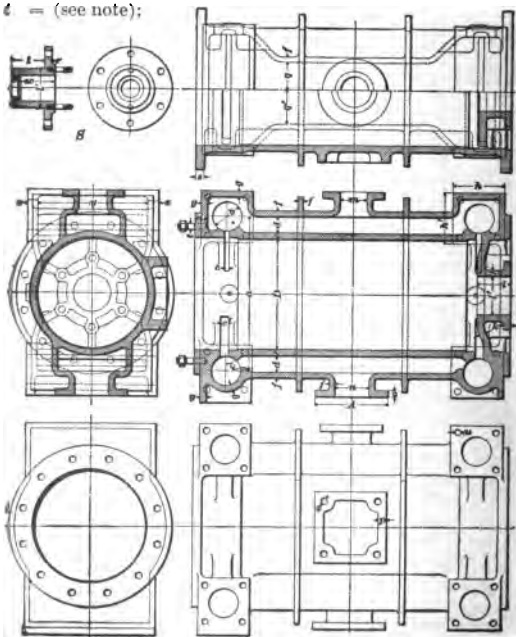


FIG. 4.

A is to be made according to proportions given on page 215.
Bolts to be made according to the same table.

NOTE.—The bolts for cylinder heads are to be calculated from the formula given for cylinder-head bolts in connection with Fig. 1.

In this cylinder the stuffingbox *S* is a separate piece that is to be bolted to the cylinder head.

CRANK-SHAFTS.

For high-speed, automatic short-stroke engines, the following formula corresponds with good practice:

$$d = .44 D + \frac{1}{4}''$$

where *d* is the diameter of shaft and *D* is the diameter of cylinders.

For the Corliss type, in which the stroke is equal to or greater than twice the diameter,

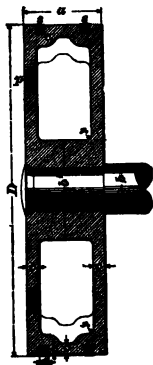
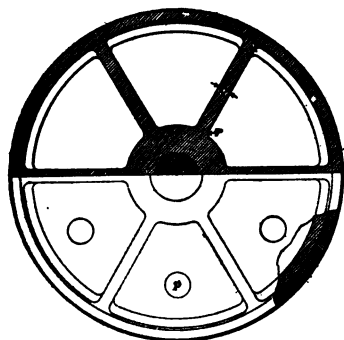
$$d = .34 D + 2\frac{1}{4}''$$

when *D* is equal to or greater than 16 in. When *D* is less than 16 in.,

$$d = \frac{1}{4} D.$$

PISTONS.

A form of piston that is much used is shown in the following figure. It consists simply of a hollow circular disk of cast iron.



The packing rings s, s are made of cast iron, and are split and sprung into place. Their elasticity causes them to press against the cylinder walls and thus prevent the leakage of steam.

The following proportions will give dimensions suitable for this piston:

$$\begin{aligned}
 D &= \text{diameter of cylinder in inches;} \\
 a &= .2 D + 1.5''; & e &= .75 c; \\
 b &= \text{diameter of piston rod;} & r &= .5 c; \\
 b' &= 2b; & p &= \text{core plug;} \\
 c &= .18\sqrt{2D} - .1875''; \text{ number of ribs} = .08(D + 34).
 \end{aligned}$$

CONNECTING-RODS.

The figure shows a *strap-end* connecting-rod. The straps e_1 and e_2 are fastened to the ends of the rods by means of the gibs a_1 and a_2 and the cotters b_1 and b_2 . The cotters are held in place by the setscrews s_1 and s_2 . Small steel blocks shown between the ends of the setscrews and the cotters are used to prevent injury of the cotter by the setscrews.

The rod, cotters, gibs, and straps may be made of either wrought iron or steel. The crankpin brasses are shown babbitted and wristpin brasses without babbitt. The brasses are adjusted by means of the cotters, which draw the straps farther on to the rod when they are driven in.

The dimensions for the rod are given by the following proportions:

For wristpin end:

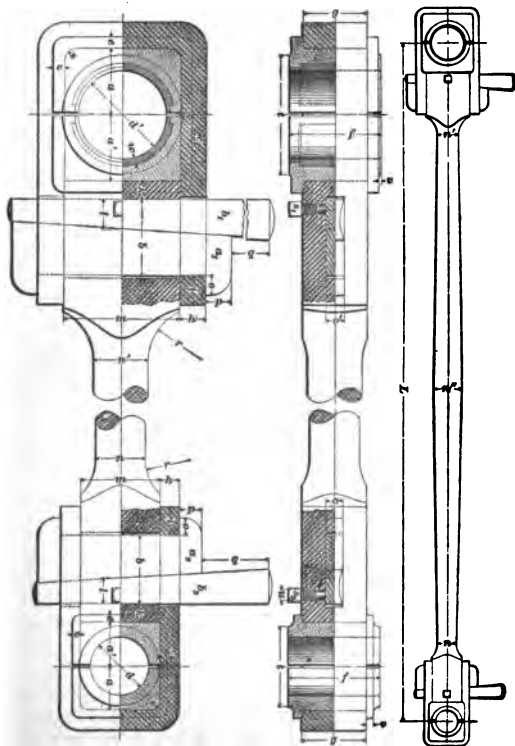
$$\begin{aligned}
 D &= \text{diameter of cylinder;} & c &= .25 b; \\
 d &= .2 D = \text{diameter of} & e &= .125 d; \\
 &\quad \text{wristpin;} & f &= .26 D + .5' \text{ for cylinders} \\
 n &= .155 D + .0625''; & &\quad \text{to } 26'' \text{ in diameter, and} \\
 x &= \frac{\pi}{4} n^2 = \text{a factor for use} & f &= .28 D \text{ for cylinders} \\
 &\quad \text{in finding proportions} & &\quad \text{above } 26'' \text{ in diameter;} \\
 &\quad \text{below;} & g &= 1.3 n; \\
 a &= .75 d + .125''; & h &= \frac{.5 x}{g - c}; \\
 a' &= .75 d + .125''; & i &= \frac{.32 x}{h}; \\
 b &= \sqrt{2.5 x};
 \end{aligned}$$

$$k = \frac{x}{1.8d};$$

$$l = .375 b;$$

$$o = .25 b;$$

$m = 1.35 d$ for wristpins up
to 3.5" in diameter,
and $m = 1.48 n$ for pins
above 3.5" in diameter;



$$p = .33 b;$$

$$q = 1.125 d \text{ for wristpins up to } 3.5'' \text{ in diameter, and}$$

$$q = 4'', \text{ constant, for pins above } 3.5'' \text{ in diameter;}$$

$$r = n;$$

$$s = .125 d;$$

$$t = 1.35 d;$$

$$u = .02 D + .25'';$$

$$v = .125 d.$$

The taper of the cotter is $\frac{1}{4}$ in. per foot.

Proportions for the crankpin end:

$$D = \text{diameter of cylinder in inches;}$$

$$d' = .28 D = \text{diameter of crankpin;}$$

$$n' = 1.1 n; (n = .155 D + .0625'');$$

$$x' = \frac{\pi}{4} n'^2 = \text{a factor used below;}$$

$$a = .75 d';$$

$$a' = .75 d';$$

$$b = \sqrt{2.5 x'};$$

$$c' = .25 b;$$

$$e = .125 d';$$

$$f = .26 D \text{ for cylinder diameters up to } 26'', \text{ and}$$

$$f = .28 D \text{ for cylinders above } 26'' \text{ in diameter;}$$

$$g = 1.3 n = \text{same as wristpin end;}$$

$$h = \frac{.5 x'}{g - c'};$$

$$i = \frac{.32 x'}{h};$$

$$k = \frac{x}{1.8 d} \text{ same as wristpin end;}$$

$$l = .375 b;$$

$$m = 1.3 d';$$

$$o = .25 b;$$

$$p = .33 b;$$

$$q = \text{same values as for wristpin end;}$$

$$r = 1.1 n;$$

$$s = .125 d;$$

$$t = 1.35 d';$$

$$v = .125' \text{ (constant);}$$

$$w = .02 D + .0625'';$$

$$n'' = n \left(\sqrt{\frac{L}{S}} - 22'' \right),$$

where L = length of rod, and

S = stroke, both in inches.

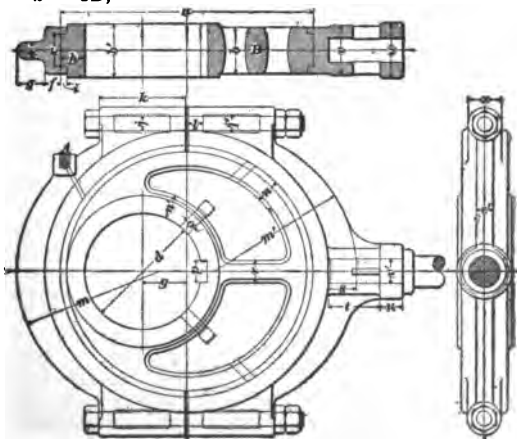
The taper of the cotter is $\frac{1}{4}$ in. per foot.

ECCENTRIC AND STRAP.

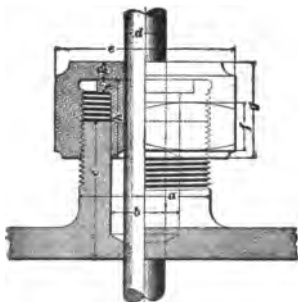
The figure shows an eccentric sheave and strap, both of cast iron. The eccentric sheave is cast solid, and must be slipped over end of shaft. The eccentric rod is held in a boss on the strap by a cotter. For eccentrics used with valve stems $\frac{1}{4}$ in. in diameter or less, holes for bolts j are not to be cored. A = boss for oil cup; B = cross-section of rib r .

The proportions are as follows:

D = diameter of valve stem;	$l = j$;
d = diameter of shaft;	$m = \frac{d + 2q + 2h + 2f}{2}$;
$a = d + 2q + 2h$;	$m' = m$;
$b = 2D + .125''$;	$n = D + .125''$;
$b' = 2.25D + .125''$;	$n' = D + .125''$;
$c = 1.5D$;	$o = .75j$;
$e = .75D$;	$p = D$;
$e' = .75D$;	q = eccentricity;
$f = .7D$;	$r = D$;
$g = 1.25d$;	$s = 1.25D$;
$h = D + .125''$;	$t = 2.25D + 1.25''$;
$i = .25D + .0625''$;	$u = D$;
j = area of bolt at root of thread = $.38 D^2$; use the nearest standard size bolt;	$v = 2.25D$;
$j' = j + .1875$;	$v' = 1.125D$;
$k = 4D$;	$w = 2.5D$;
	$x = 2.25j$.



STUFFINGBOXES.



The stuffingbox of the form shown in the figure is generally used for small work, such as the spindles of valves, etc. The outside of the stuffingbox is threaded to receive a hexagonal nut that fits over the gland. As the nut is screwed down, the gland is pressed downwards and compresses the packing.

The proportions used are:

$$d = \text{diameter of rod;}$$

$$a = 2.5d + .5'';$$

$$b = 1.5d + .125'';$$

$$c = 3d + .25'';$$

$$e = 3.5d + .625'';$$

$$f = d + .125'';$$

$$g = 2d + .25'';$$

$$h = 1.5d + .25'';$$

$$i = .25d + .0625'';$$

$$k = .5d.$$

This design may be used for rods up to $1\frac{1}{4}$ in. in diameter.

Make the number of threads per inch the same as for a bolt whose diameter is equal to the diameter of the rod.

GEARING.

The *circular pitch* of a gear-wheel is the distance in inches measured on the pitch circle from the center of one tooth to the center of the next tooth.

If the distance of the teeth of a gear thus measured were $2\frac{1}{4}$ in., we would say that the circular pitch was $2\frac{1}{4}$ in.

Let P = circular pitch;

D = diameter of pitch circle, in inches;

C = circumference of pitch circle, in inches;

N = number of teeth;

$\pi = 3.1416.$

$$\text{Then, } P = \frac{C}{N} \text{ or } \frac{\pi D}{N}. \quad N = \frac{C}{P} \text{ or } \frac{\pi D}{P}.$$

$$C = PN \text{ or } \pi D. \quad D = \frac{PN}{\pi} \text{ or } \frac{C}{\pi}.$$

$$\text{Addendum} = .3 P. \quad \text{Root} = .4 P.$$

The thickness of the teeth for a cut gear is equal to $.5 P$, and for a cast gear $.48 P$.

The *diametral pitch* of a gear-wheel is the name given to the quotient that is obtained by dividing the number of teeth in the wheel by the diameter of the pitch circle in inches; or, the diametral pitch may be defined as the number of teeth on the circumference of the gear-wheel for 1 in. diameter of pitch circle.

A gear with a pitch diameter of 5 in., and having 40 teeth is 8 pitch; one with the same pitch diameter and having 70 teeth is 14 pitch.

In the gear of 8 pitch there are 8 teeth on the circumference for each inch of the diameter of the pitch circle; and in one of 14 pitch there are 14 teeth on the circumference for each inch of the diameter of the pitch circle.

Let P = diametral pitch;

D = diameter of pitch circle, in inches;

N = number of teeth;

d = outside diameter;

l = length of tooth;

t = thickness of tooth;

$$P = \frac{N}{D}. \quad D = \frac{N}{P}. \quad N = PD. \quad d = \frac{N+2}{P}. \quad l = \frac{2.157}{P}. \quad t = \frac{1.57}{P}.$$

The circular pitch corresponding to any diametral pitch may be found by dividing 3.1416 by the diametral pitch; and the diametral pitch corresponding to any circular pitch may be found by dividing 3.1416 by the circular pitch.

(a) If the diametral pitch of a gear is 6, what is the corresponding circular pitch?

(b) If the circular pitch is 1.5708 in., what is the corresponding diametral pitch?

$$(a) \frac{3.1416}{6} = .5236 \text{ in.} \quad (b) \frac{3.1416}{1.5708} = 2.$$

DIAMETRAL PITCHES WITH THEIR CORRESPONDING CIRCULAR PITCHES.

Diametral Pitch, or Teeth, per Inch in Diameter.	Corresponding Circular Pitch.	Diametral Pitch, or Teeth, per Inch in Diameter.	Corresponding Circular Pitch.
1	3.1416	8	.3927
2	1.5708	9	.3491
3	1.0472	10	.3142
4	.7854	12	.2618
5	.6283	14	.2244
6	.5236	16	.1963
7	.4488	20	.1571

ELECTRICITY.

PRACTICAL UNITS.

The *volt* is the practical unit of electromotive force or electrical pressure. It is that electromotive force which will maintain a current of 1 *ampere* in a circuit whose resistance is 1 *ohm*.

The *electromotive force* of a Daniell's cell is 1.072 volts.

The *ampere* is the practical unit denoting the strength of an electric current, or the rate of flow of electricity. It is that strength of current or rate of flow which would be maintained in a circuit whose resistance is 1 *ohm* by an electromotive force of 1 *volt*.

One ampere decomposes .00009342 gram of water (H_2O) per second; or deposits .001118 gram of silver per second.

The *ohm* is the practical unit of resistance. It is that resistance which will limit the flow of an electric current under an electromotive force of 1 *volt* to 1 *ampere*.

The *legal ohm* is the resistance of a column of mercury 106 centimeters long and 1 square millimeter sectional area at 0° C.

One mile of pure copper wire $\frac{1}{8}$ in. in diameter has a resistance of 13.59 ohms at a temperature of 59.9° F.

To make the significance of these units clearer, take the analogous case of water flowing through a pipe under a pressure of a column of water. The force that causes the water to flow is due to the pressure or head; the flow or current of water is measured in *gallons per minute*; and the resistance that opposes or resists the flow of water is caused by the friction of the water against the inside of the pipe.

In electrotechnics, the electromotive force or electrical potential expressed in volts corresponds to the pressure or head of water; and the resistance in ohms to the friction in the pipe.

The unit that expresses the *rate of transmission of electricity per second* is called the *ampere*, while the flow of water is expressed in gallons per minute.

In either case the strength of current or rate of flow depends on the ratio between the pressure and the resistance; for, as the pressure increases, the current increases proportionately; and as the resistance increases, the current diminishes.

This relation, as applied to electricity, was discovered by Dr. G. S. Ohm, and has since been called *Ohm's law*.

Ohm's Law.—*The strength of the current in any circuit is directly proportional to the electromotive force in that circuit and inversely proportional to the resistance of that circuit, i. e., is equal to the quotient arising from dividing the electromotive force by the resistance.*

Let E = electromotive force in volts;
 R = resistance in ohms;
 C = strength of current in amperes.

Then $C = \frac{E}{R}$. $R = \frac{E}{C}$. $E = CR$.

EXAMPLE.—The electromotive force of a circuit is 110 volts, and its resistance is 55 ohms; what is the strength of current?

SOLUTION.— $E = 110$ volts. $R = 55$ ohms. $C = \frac{E}{R} = \frac{110}{55}$

= 2 amperes.

The unit by which electrical power is expressed is called the *watt*. It is that *rate of doing work* when a current of 1 ampere is passing through a conductor under an electromotive force of 1 volt, and is equal to $\frac{1}{746}$ of a horsepower.

- Let E = electromotive force in volts;
 C = strength of current in amperes;
 R = resistance in ohms;
 W = power in watts;
H. P. = horsepower.

$$W = E \times C = C^2 \times R = \frac{E^2}{R}.$$

$$\text{H. P.} = \frac{E \times C}{746} = \frac{C^2 \times R}{746} = \frac{E^2}{R \times 746} = \frac{W}{746}.$$

One *kilowatt* is equal to 1,000 watts: sometimes abbreviated to K. W.

Watt hour is a unit of work. It is used to indicate the expenditure of an electrical power of 1 watt for 1 hour.

EXAMPLE.—The resistance of a lighting circuit is 5 ohms and the electromotive force is 110 volts. (a) What is the amount of electrical power in watts required for this current? (b) What is the equivalent horsepower?

SOLUTION.— $E = 110$. $R = 5$.

$$\frac{E^2}{R} = \frac{110^2}{5} = 2,420 \text{ watts.}$$

$$\frac{E^2}{R \times 746} = \frac{110^2}{5 \times 746} = 3.244 \text{ H. P.}$$

Conductivity is the name given to the reciprocal of the resistance of any conductor. There is no unit by which to express conductivity.

NOTE.—The reciprocal of any number is unity divided by that number. Thus, the reciprocal of 2 is $\frac{1}{2}$ or .5.

CURRENTS.

RULES FOR DIRECTION OF CURRENT, ETC.

To determine the direction of a current in a conductor by the aid of a compass:

Rule.—If the current flows from the south pole over the needle to the north, the north end of the needle will point towards the west, as in Fig. 1. If the compass is placed over the conductor so that the current will flow from the south under the needle to the north, the north end of the needle will point towards the east, as in Fig. 2.

To determine the polarity of an electromagnet:

Rule.—In looking at the face of a pole (Fig. 3), if the current

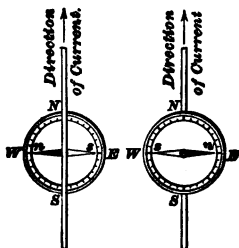


FIG. 1.

FIG. 2.

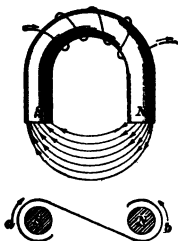


FIG. 3.

flows in the direction *a*, of the hands of a watch, it will be a south pole, and if in the opposite direction *b*, it will be a north pole.

To determine the direction of an induced current in a conductor that is moving in a magnetic field:

Rule.—Place thumb, forefinger, and middle finger of right hand, each at a right angle to the other two, as shown in Fig. 4; if the forefinger shows direction of lines of force and the thumb the direction of motion of conductor, then the middle finger will show the direction of the induced current.



FIG. 4.



FIG. 5.

NOTE.—The above rule will give the polarity of a dynamo.

To determine the direction of motion of a conductor carrying a current when placed in a magnetic field:

Rule.—Place thumb, forefinger, and middle finger of the left hand, each at a right angle to the other two, as shown in Fig. 5; if the forefinger shows the direction of the lines of force and the middle finger shows the direction of the current, then the thumb will show the direction of motion of the conductor.

NOTE.—The above rule will give the polarity of a motor.

DERIVED OR SHUNT CIRCUITS.

A circuit divided into two or more branches, each branch transmitting part of the current, is said to be a *derived circuit*; the individual branches are in multiple-arc, or parallel with each other.

To find the joint resistance of a derived circuit:

Rule.—As the conductivity of any conductor is equal to the reciprocal of its resistance, then the joint conductivity of two or more circuits in parallel is equal to the sum of the reciprocals of their separate resistances. The joint resistance of two or more circuits in parallel is equal to the reciprocal of their joint conductivity.

In a derived circuit of three branches, let r_1 , r_2 , and r_3 be the resistances of the three branches, respectively. Their joint conductivity, or the sum of the reciprocals of their resistances, is

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}, \text{ or } \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3}$$

Their joint resistance is, therefore,

$$\frac{1}{\frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3}}, \text{ or } \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}.$$

The joint resistance of a derived circuit with but two branches in parallel may be thus expressed:

$$\frac{\text{product of their resistances}}{\text{sum of their resistances}}.$$

EXAMPLE.—The resistances of two branches of a derived circuit are 20 and 30 ohms, respectively. Find their joint resistance.

SOLUTION.—

$$\frac{\text{product of their resistances}}{\text{sum of their resistances}} = \frac{600}{50} = 12 \text{ ohms.}$$

To find the strength of current in the separate branches of a derived circuit:

Rule.—A current is divided among the branches of a derived circuit in proportion to their conductivities—i. e., to the reciprocal of their resistances.

EXAMPLE.—If the resistances of the two branches *A* and *B* of a derived circuit are 20 and 30 ohms, respectively, and the total current in the main circuit is 60 amperes, what is the current in each? The conductivity of *A* is $\frac{1}{20}$ and of *B* $\frac{1}{30}$.

SOLUTION.—If C_1 represents the current in *A*, and C_2 represents the current in *B*, then,

$$C_1 : C_2 = \frac{1}{20} : \frac{1}{30}.$$

$$\text{Hence, } \frac{C_1}{C_2} = \frac{\frac{1}{20}}{\frac{1}{30}}, \text{ or } \frac{C_1}{C_2} = \frac{30}{20} = \frac{3}{2}.$$

$$\text{Now, } C_1 + C_2 = 60, \text{ or } C_2 = 60 - C_1.$$

$$\text{Substituting, } \frac{C_1}{60 - C_1} = \frac{3}{2};$$

$$C_1 = 36, \text{ and } C_2 = 24.$$

WIRING.

INTERIOR WIRING.

A *mil* is a unit of length used in measuring the diameters of wires, and is equal to .001 in.

A *circular mil* is a unit of area used in measuring the cross-sections of wires, and is equal to $\frac{.7854}{10^6}$ sq. in.

The sectional area of a wire expressed in circular mils is equal to the square of its diameter in mils.

Let c. m. = circular mils;

C = total current in amperes;

c = current in amperes to each lamp;

n = number of lamps in multiple;

v = volts lost in line;

r = resistance per foot of wire;

a = distance from dynamo to lamps.

The resistance of 1 ft. of commercial copper wire, 1 mil in diameter, at a temperature of 75° F., is 10.8 ohms.

A 16 c. p. (candlepower) 110-volt lamp takes about .5 ampere; a 16 c. p. 55-volt lamp takes about 1 ampere.

All calculations for size of wire must be checked by comparing with a table of safe carrying capacity (see table on pages 238 and 239), and the current value there given must not be exceeded.

To find the size of wire for 110-volt circuit with 16 c. p. lamps:

$$r = \frac{v}{n d}.$$

$$\text{For large cables, c. m.} = \frac{10.8 n d}{v}.$$

EXAMPLE.—Find the size of wire necessary for a circuit supplying current to 50 110-volt 16 c. p. lamps, 300 ft. from the dynamo, allowing a loss of 5% in line.

$$\text{SOLUTION.}—\text{Volts at dynamo} = \frac{110}{.95} = 115.8.$$

$$\text{Volts lost in line} = 115.8 - 110 = 5.8 = v.$$

$$\begin{aligned} \text{Then, } r &= \frac{v}{n d} = \frac{5.8}{50 \times 300} = .000386 \text{ ohm per ft.,} \\ &= .386 \text{ ohm per 1,000 ft.} \end{aligned}$$

The nearest size of wire, as given in the table on page 238, is No. 6 B. & S., and its current capacity is 35 amperes; therefore it is safe.

To find the size of wire for a 55-volt circuit with 16 c. p. lamps:

$$r = \frac{v}{2 n d}.$$

$$\text{For large cables, c. m.} = \frac{21.6 n d}{v}.$$

EXAMPLE.—What size of wire should be used for supplying current to 75 16 c. p. lamps on a 55-volt circuit, the distance from dynamo being 230 ft., and line loss, 4 volts?

SOLUTION.—

$$\begin{aligned} r &= \frac{v}{2 n d} = \frac{4}{2 \times 75 \times 230} = .000116 \text{ ohm per ft.,} \\ &= .116 \text{ ohm per 1,000 ft.} \end{aligned}$$

By referring to the table, (page 238) the nearest wire is found to be No. 1 B. & S., and its carrying capacity is greater than the current (75 amperes) that it is to conduct.

To find the size of wire for any circuit on a 2-wire system:

In general,
$$r = \frac{v}{C \times 2d};$$

or,
$$\text{c. m.} = \frac{10.8 \times 2d \times C}{v}.$$

EXAMPLE.—What wire should be used to carry 450 amperes a distance of 600 ft., the allowable drop being 6%, and the E. M. F. at the end of the circuit 115 volts?

SOLUTION.—Volts at dynamo $= \frac{115}{.94} = 122.3.$

Volts lost in line $= 7.3.$

Then,
$$\text{c. m.} = \frac{10.8 \times 2 \times 600 \times 450}{7.3} = 798,900.$$

Comparing this number with the table on page 239, giving current capacity of cables, it will be seen that it is within the prescribed limits.

These formulas may be used for feeders, mains, branch mains, service mains, and inside wiring on continuous-current circuits, and for secondary wiring on alternating systems.

To find the size of wire for a 110-volt circuit, 3-wire system, 16 c. p. lamps:

$$r = \frac{4v}{nd} \text{ for each wire.}$$

For large cables,

$$\text{c. m.} = \frac{2.7nd}{v} \text{ for each wire.}$$

In checking for carrying capacity, remember that the wire carries only one-half the current that would be used on a 2-wire system, as the voltage between the outside conductors is double the voltage at the terminal of 1 lamp.

EXAMPLE.—What should be the size of the conductors for a 3-wire system, when 132 110-volt, 16 c. p. lamps are installed at a distance of 210 ft. from the source of supply, the loss being 4 volts?

SOLUTION.—

$$\begin{aligned} r &= \frac{4 \times 4}{132 \times 210} = .000577 \text{ ohm per ft.,} \\ &= .577 \text{ ohm per 1,000 ft.} \end{aligned}$$

This would call for a wire between Nos. 7 and 8. The

current will be $\frac{132 \times .5}{2} = 33$ amperes; but this is too much for the wire to carry, and No. 6 B. & S. wire should be used, notwithstanding the somewhat less drop in volts that will result.

For continuous-current circuits, 5% loss is usually allowed, with full current from the dynamo to the lamps. For long distances a larger line loss may be allowed, if the dynamo is wound for that loss.

DIMENSIONS, WEIGHT, AND DISTANCE OF COPPER WIRE.

B. & S. Gauge.	Diameter in Mils (d). 1 mil = .001 in.	Area.	Weight and Length.		Resistance at 75° F. Ohms per 1,000 ft.	Current. Amperes.		B. & S. Gauge.
		Circular Mils (d ²).	Lb. per 1,000 Ft.	Ft. per Lb.		Exposed.	Concealed.	
0000	460.000	211,600.0	639.33	1.56	.049	300	175	0000
000	409.640	167,805.0	507.01	1.97	.062	245	145	000
00	364.800	133,079.0	402.09	2.49	.078	215	120	00
0	324.950	105,592.0	319.04	3.13	.098	190	100	0
1	289.300	83,694.0	252.88	3.95	.124	160	95	1
2	257.630	66,373.0	200.54	4.99	.156	135	70	2
3	229.420	52,634.0	159.03	6.29	.197	115	60	3
4	204.310	41,742.0	126.12	7.93	.248	100	50	4
5	181.940	33,102.0	100.01	10.00	.313	90	45	5
6	162.020	26,250.0	79.32	12.61	.395	80	35	6
7	144.280	20,817.0	62.90	15.90	.498	67	30	7
8	128.490	16,509.0	49.88	20.05	.628	60	25	8
9	114.430	13,094.0	39.56	25.28	.792			9
10	101.890	10,381.0	31.37	31.88	.999	40	20	10
11	90.742	8,234.1	24.88	40.20	1.260			11
12	80.808	6,529.9	19.73	50.69	1.589	30	15	12
13	71.961	5,178.4	15.65	63.91	2.003			13
14	64.084	4,106.8	12.41	80.59	2.526	22	10	14
15	57.068	3,256.7	9.83	101.65	3.186			15
16	50.820	2,582.9	7.80	128.17	4.017	15	5	16
17	45.257	2,048.2	6.19	161.59	5.066			17
18	40.303	1,624.3	4.91	203.76	6.388	10		18
19	35.890	1,288.1	3.89	257.42	8.055			19
20	31.961	1,021.5	3.08	324.12	10.158	5		20

CARRYING CAPACITY OF CABLES.

Area. Circular Mils.	Current. Amperes		Area. Circular Mils.	Current. Amperes.	
	Exposed.	Concealed.		Exposed.	Concealed.
200,000	299	200	1,200,000	1,147	715
300,000	405	272	1,300,000	1,217	756
400,000	503	336	1,400,000	1,287	796
500,000	595	393	1,500,000	1,356	835
600,000	682	445	1,600,000	1,423	873
700,000	765	494	1,700,000	1,489	910
800,000	846	541	1,800,000	1,554	946
900,000	924	586	1,900,000	1,618	981
1,000,000	1,000	630	2,000,000	1,681	1,015
1,100,000	1,075	673			

To find the size of wire on primary circuits for alternating system:

$$c. m. = \frac{10.8 \times 2 d \times C^1}{v}; \quad r = \frac{v}{C^1 \times 2 d}.$$

C^1 = the total current in amperes on primary circuit, and may be determined by dividing the total current on the secondary circuit by the product of the ratio and efficiency of conversion.

The ratio is generally 20 to 1 on a 1,000-volt apparatus when using 52-volt lamps, and 10 to 1 when using 100- to 110-volt lamps.

The efficiency of conversion can be taken as 95% in ordinary transformers.

EXAMPLE.—If the loss is 5%, find the size of wire necessary on a 1,000-volt primary circuit when the distance between the dynamo and transformer is 2,000 ft., and the dynamo is supplying current for 500 16 c. p. 52-volt lamps.

SOLUTION.—

$$\text{Volts at dynamo} = \frac{1,000}{.95} = 1,052, \text{ nearly.}$$

$$\text{Volts lost in line} = 52.$$

Assume the lamp efficiency to be 3.6 watts per c. p. Then, since the product of amperes and volts gives watts,

$$\text{Current to each lamp} = \frac{3.6 \times 16}{52} = 1.11 \text{ amperes.}$$

$$\text{Current on secondary} = 1.11 \times 500 = 555 \text{ amperes.}$$

$$\text{Total current on primary is } \frac{555}{.95 \times 20} = 29.21 \text{ amperes.}$$

Therefore,

$$\text{c. m.} = \frac{10.8 \times 2d \times C^1}{v} = \frac{10.8 \times 4,000 \times 29.21}{52} = 24,267.$$

$$\text{And } r = \frac{v}{C^1 \times 2d} = \frac{52}{29.21 \times 4,000} = .000445 \text{ ohm per ft., or .445 ohm per 1,000 ft. This gives No. 6 B. \& S. See page 238.}$$

For alternating systems under ordinary conditions, 5% loss at full load from dynamo to transformer on primary circuit is a maximum, although some dynamos are specially wound for 10% loss. A loss of from 1% to 2% may be allowed on secondary circuits from transformer to lamps.

INCANDESCENT LAMPS.

Let c = current in amperes to each lamp;

E = electromotive force in volts;

$$R = \frac{E}{c} = \text{resistance of lamp when hot;}$$

c. p. = candlepower of lamp;

W. per c. p. = watts per c. p. (often called *lamp efficiency*).

$$\text{W. per c. p.} = \frac{c \times E}{c. p.}$$

$$\text{The number of candles per electrical H. P.} = \frac{746}{\text{W. per c. p.}}$$

$$c = \frac{\text{W. per c. p.} \times c. p.}{E}$$

As the commercial efficiency of good dynamos is about 90%, the calculations of candles per electrical H. P. must be multiplied by .90 to give the number of candles per mechanical H. P.

LAMP EFFICIENCIES.

3.1 watts per c. p., or 12 lamps, 16 c. p., to 1 mechanical H. P.

3.6 watts per c. p., or 10 lamps, 16 c. p., to 1 mechanical H. P.

4.0 watts per c. p., or 8 lamps, 16 c. p., to 1 mechanical H. P.

NOTE.—Lamps of an efficiency of 3.1 watts per c. p. should not be used where the voltage averages, for any length of time, more than 2½ high; lamps of 3.6 watts per c. p. should not be used where the voltage averages more than 4½ high; and lamps of an efficiency of 4 watts per c. p. should be used where the regulation of the plant receives little or no attention. If these cautions are not followed, the life of the lamp will be greatly diminished.

Size of Wire for Arc-Light Circuits.—For ordinary distances, or small currents, use No. 8 B. & S. wire. For longer distances, or large currents, use No. 6 B. & S. wire.

BELL WIRING.

The simple bell circuit is shown in Fig. 1, where *p* is the push button, *b* the bell, and *c, c* the cells of the battery connected up in series.

When two or more bells are to be rung from one push button, they may be joined up in parallel across the battery wires as in Fig. 2 at *a*

and *b*, or they may be arranged in series as in Fig. 3. The battery *B* is indicated in each diagram by short parallel lines,

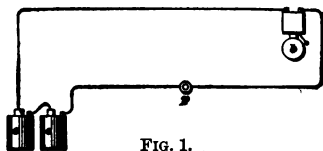


FIG. 1.

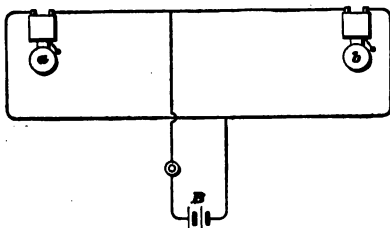


FIG. 2.

this being the conventional method. In the parallel arrangement of the bells, they are independent of each other, and the failure of one to ring would not affect the others; but in the

series grouping all but one bell must be changed to a single-stroke action, so that each impulse of current will produce only one movement of the hammer. The current is then

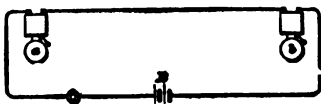


FIG. 3.

interrupted by the vibrator in the remaining bell, the result being that each bell will ring with full power. The only change necessary to

produce this effect is to cut out the circuit-breaker on all but one bell by connecting the ends of the magnet wires directly to the bell terminals.

When it is desired to ring a bell from one of two places some distance apart, the wires may be run as shown in Fig. 4. The pushes p, p' are located at the required points, and the battery and bell are put in series with each other across the wires joining the pushes.

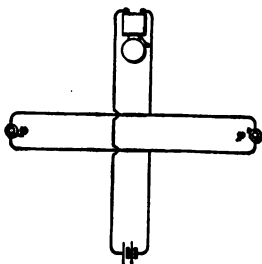


FIG. 4.

A single wire may be used to ring signal bells at each end of a line, the connections being given in Fig. 5. Two batteries are required, B and B' , and a key and bell at each station. The keys k, k' are of the double-contact type, making connections normally between

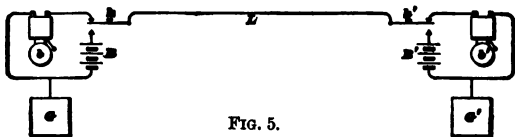


FIG. 5.

bell b or b' and line wire L . When one key, as k , is depressed, a current from B flows along the wire through the upper contact of k' to bell b' and back through ground plates G', G .

When a bell is intended for use with burglar-alarm apparatus, a constant-ringing attachment may be introduced, which closes the bell circuit through an extra wire as soon as the trip at door or window is disturbed. In the diagram, Fig. 6, the main circuit, when the push *p* is depressed, is through the automatic drop *d* by way of the terminals *a*, *b* to the bell and battery. This current releases a pivoted arm which, on falling, completes the circuit between *b* and *c*, establishing a new path for the current by way of *e*, independent of the push *p*.

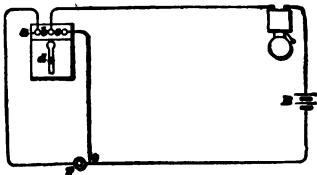


FIG. 6.

For operating electric bells, any good type of open-circuit battery may be used. The Leclanché cell is largely used for this purpose, also several types of dry cells.

ANNUNCIATOR SYSTEM.

The wiring diagram for a simple annunciator system is shown in Fig. 1. The pushes 1, 2, 3, etc. are located in the various rooms, one side being connected to the battery wire

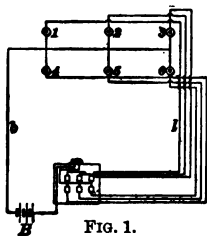


FIG. 1.

b, and the other to the leading wire *l* in communication with the annunciator drop corresponding to that room. A battery of 2 or 3 Leclanché cells is placed at *B* in any convenient location. The size of wire used throughout may be No. 18 annunciator wire.

A return-call system is illustrated in Fig. 2, in which there is one battery wire *b*, one return wire *r*, and one leading wire *l*, *l*₂, etc.

for each room. The upper portion of the annunciator board is provided with the usual drops, and below these are the

return-call pushes. These are double-contact buttons, held normally against the upper contact by a spring. When in this position, the closing of the circuit by the push button in any room, such as No. 4, rings the office bell and releases No. 4 drop, the path of the current in this case being from

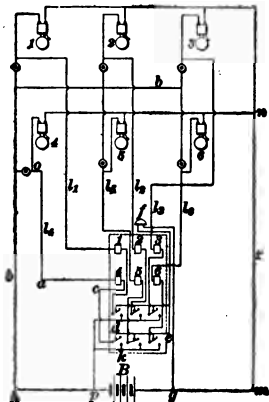


FIG. 2.

push 4 to *a-c-d-e-f-g-B-h-b* back to the push button. On the return signal being made by pressing the button at the lower part of the annunciator board, the office-bell circuit is broken at *d*, and a new circuit formed through *k* as follows: From the battery *B* to *g-m-r-n-o-a-c-k-p* to battery, the room bell being in this circuit. A general fire-alarm may be added to this system, consisting of an automatic clockwork apparatus for closing all the room-bell circuits at once, or as many at a time as a battery can ring. When this system is

installed, the battery wire should be either No. 14 or No. 16. Four or five Leclanché cells are usually required in this case.

It will be seen that the connections are so arranged that the room bell will ring when the push in that room is pressed. If this be not desired, a double-contact push may be substituted, so that the room-bell circuit is broken at the same time that the circuit is made through the annunciator. This double push should be so connected that the circuit is normally complete through the bell, the leading wire being connected to the tongue and the battery wire being connected to the second contact point, which is normally out of circuit.

EXTRACT FROM THE REGULATIONS OF THE UNDERWRITERS' ASSOCIATION.

Incandescent Wires.—Conducting wires, carried over or attached to buildings, must be (a) at least 7 ft. above the highest point of flat roofs, and (b) 1 ft. above the ridge of pitch roofs; (c) when in proximity to other conductors likely to divert any portion of the current, they must be protected by guard irons or wires, or a proper additional insulation, as the case may require.

For entering buildings, (a) wires with an extra-heavy waterproof insulation must be used; (b) they must be protected by drip loops, (c) also protected from abrasion by awning frames; (d) be at least 6 in. apart; (e) the holes through which they pass in the outer walls of such buildings must be bushed with a non-inflammable, waterproof, insulating tube, and (f) should slant upward toward the inside.

(a) Wires must never be left exposed to mechanical injury, or to disturbance of any kind. (b) Wires must not be fastened by metallic staples. (c) When wires pass through walls, floors, partitions, timbers, etc., glass tubing, or so-called "floor insulators," or other moisture-proof, non-inflammable insulating tubing must be used. (d) At all outlets to and from cut-outs, switches, fixtures, etc., wires must be separated from gas pipes or parts of the building by porcelain, glass, or other non-inflammable insulating tubing, (e) and should be left in such a way as not to be disturbed by the plasterers. (f) Wires of whatever insulation must not in any case be taped, or otherwise be fastened, to gas piping. (g) If no gas pipes are installed at the outlets, an approved substantial support must be provided for the fixtures.

In crossing any metal pipes, or any other conductor, (a) wires must be separated from the same by an air space of at least $\frac{1}{4}$ in., where possible, and (b) so arranged that they cannot come in contact with each other by accident. (c) They should go over water pipes, where possible, so that moisture will not settle on the wires.

In unfinished lofts, between floors and ceilings, in partitions, and other concealed places, wires must (a) be kept free of contact with the building; (b) be supported on glass,

porcelain, or other non-combustible insulators; (c) have at least 1 in. clear air space surrounding them; (d) be at least 10 in. apart, when possible; and (e) should be run singly on separate timbers or studding. (f) When thus run in perfectly dry places, not liable to be exposed to moisture, a wire having simply a non-combustible insulation may be used.

Soft rubber tubing is not desirable as an insulator.

Care must be taken that the wires are not placed above each other in such a manner that water could make a cross-connection.

On all loops of incandescent circuits, safety catches must be used on both sides of the loop, and switches on such loops should be double-poled.

Wires must not be fished (a) for any great distance, and (b) only in cases where the inspector can satisfy himself that the above rules have been complied with. (c) Twin wires must never be employed in this class of concealed work.

Dynamo Machines.—Dynamo machines must be located in dry places, not exposed to flyings or easily combustible material, and insulated upon wooden foundations. The machines must be provided with devices that shall be capable of controlling any changes in the quantity of the current; and if the governors are not automatic, a competent person must be in attendance near the machine whenever it is in operation.

Each machine must be used with complete wire circuits; and connections of wires with pipes or the use of circuits in any other method, are absolutely prohibited.

The whole system must be kept insulated, and tested *every day* with a magneto for ground connections in ample time before lighting, to remedy faults of insulation, if they are discovered; and proper testing apparatus must in each case be provided. This applies to both central station and isolated plants.

Testing circuits for grounds with a battery and bell is not considered a reliable test.

Preference is given to switches constructed with a lapping connection, so that no electric arc can be formed at the switch when it is changed; otherwise the stands of switches, where

powerful currents are used, must be made of some incombustible substances that will withstand the heat of the arc when the switch is changed.

Motors.—Wires for motors should be run exactly as for lamps on similar circuits.

On low-tension circuits, where motors are run in multiple, safety catches must be used on each side of the circuit.

On high-tension circuits the same restrictions apply as for arc lamps, and suitable cut-outs must be provided.

Motors must be treated as dynamos as regards insulation, flyings, dampness, etc.

NOTE.—If the regulations of the Underwriters' Association are not followed in wiring buildings, the wiring is liable to be condemned by the Insurance Inspectors and the policy canceled.

WIRE TABLES.

WEIGHT OF UNDERWRITERS' LINE WIRE, INSULATED.

No. B. & S.	Pounds per 1,000 Feet.	Feet per Pound.
0000	800	1.25
000	666	1.50
00	500	2.00
0	363	2.75
1	313	3.20
2	250	4.00
3	200	5.00
4	144	6.9
5	125	8.0
6	105	9.5
7	87	11.5
8	69	14.5
10	50	20.0
12	31	32.0
14	22	45.0
16	14	70.0
18	11	90.0

EQUIVALENT SECTIONAL AREA OF WIRES, B. & S. GAUGE.

Gauge No.	No. of Wires. Gauge No.	No. of Wires. Gauge No.	No. of Wires. Gauge No.	No. of Wires. Gauge No.	No. of Wires. Gauge No.	No. of Wires. Gauge No.	Gauge No. Gauge No.
0000	2- 0	4- 3	8- 6	16- 9	32-12	64-15	
000	2- 1	4- 4	8- 7	16-10	32-13	64-16	
00	2- 2	4- 5	8- 8	16-11	32-14	64-17	1 and 3
0	2- 3	4- 6	8- 9	16-12	32-15	64-18	2 and 3
1	2- 4	4- 7	8-10	16-13	32-16		3 and 5
2	2- 5	4- 8	8-11	16-14	32-17		4 and 6
3	2- 6	4- 9	8-12	16-15	32-18		5 and 7
4	2- 7	4-10	8-13	16-16			6 and 8
5	2- 8	4-11	8-14	16-17			7 and 9
6	2- 9	4-12	8-15	16-18			8 and 10
7	2-10	4-13	8-16				9 and 11
8	2-11	4-14	8-17				10 and 12
9	2-12	4-15	8-18				11 and 13
10	2-13	4-16					12 and 14
11	2-14	4-17					13 and 15
12	2-15	4-18					14 and 16
13	2-16						15 and 17
14	2-17						16 and 18
15	2-18						

The above table indicates the number of smaller wires required to give a sectional area equal to one larger size wire, the figures between the horizontal lines corresponding to each other. For example: It requires two wires, No. 0, or 4 wires, No. 3, etc., to give a sectional area equal to 1 wire, No. 0000. Again: it requires two wires, No. 13, or 4 wires, No. 16; or 2 wires, 1 No. 12 plus 1 No. 14, to give a sectional area equal to 1 No. 10.

COMPARATIVE SIZES OF WIRES, B. & S. AND BIRMINGHAM GAUGES.

Diameter. Inches.	B. & S.	Birmingham.
.460	0000	
.454		0000
.425		000
.4096	000	
.380		00
.3648	00	
.340		0
.3249	0	
.3000		1
.2893	1	
.284		2
.259		3
.2576	2	
.238		4
.2294	3	
.22		5
.2043	4	
.203		6
.1819	5	
.18		7
.165		8
.162	6	
.148		9
.1443	7	
.134		10
.1285	8	
.12		11
.1144	9	
.109		12
.1019	10	
.095		13
.0907	11	
.083		14

**COMPARATIVE SIZES OF WIRES, B. & S. AND BIRMINGHAM
GAUGES—(Continued).**

Diameter, Inches.	B. & S.	Birmingham.
.0808	12	
.0720	13	15
.0650		16
.0641	14	
.0580		17
.0571	15	
.0508	16	
.0490		18
.0453	17	
.0420		19
.0403	18	
.0359	19	

NOTE.—B. & S. gauge is generally used in America.

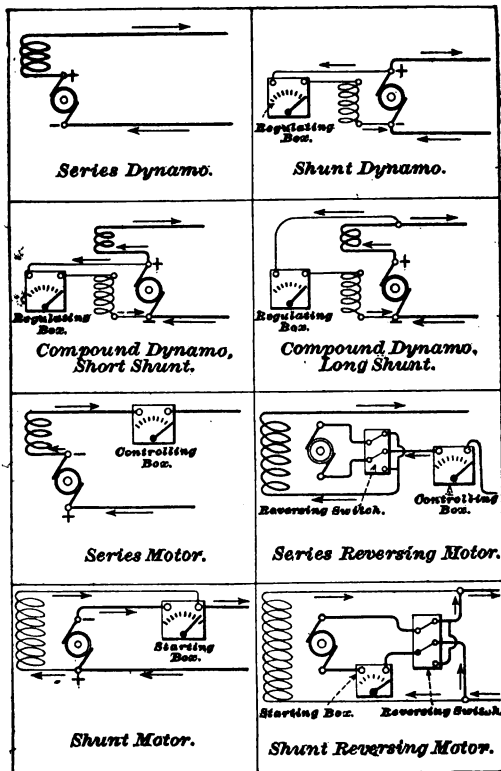
COMPARISON OF PROPERTIES OF ALUMINUM AND COPPER.

	Aluminum.	Copper.
Conductivity (for equal sizes)	.54 to .63	1.
Weight (for equal sizes)	.33	1.
Weight (for equal length and resistance)	.48	1.
Price (per pound) Aluminum, 29c.; Copper, 16c. (bare wire)	1.81	1.
Price (equal length and resistance, bare line wire)	.868	1.
Temperature coefficient per degree F.	.002138	.002155
Resistance of mil-foot (20° C.)	18.73	10.5
Specific gravity	2.5 to 2.68	8.89 to 8.93
Breaking strength (equal sizes)	1.	1.

RESISTANCE OF PURE COPPER WIRE.

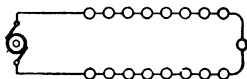
No. B. & S.	Resistance at 75° F.			
	R. Ohms per 1,000 Feet.	Ohms per Mile.	Feet per Ohm.	Ohms per Pound.
4-0	.04904	.25891	20,392.90	.00007653
3-0	.06184	.32649	16,172.10	.00012169
00	.07797	.41168	12,825.40	.00019488
0	.09827	.51885	10,176.40	.00030734
1	.12398	.65460	8,066.00	.00048920
2	.15633	.82543	6,396.70	.00077784
3	.19714	1.04090	5,072.50	.00123700
4	.24858	1.31248	4,022.90	.00196660
5	.31846	1.65507	3,190.20	.00312780
6	.39528	2.08706	2,529.90	.00497280
7	.49845	2.63184	2,006.20	.00790780
8	.62849	3.31843	1,591.10	.01257190
9	.79242	4.18400	1,262.00	.01998530
10	.99948	5.27726	1,000.50	.03170460
11	1.26020	6.65357	793.56	.05054130
12	1.58900	8.39001	629.32	.08036410
13	2.00370	10.57980	499.06	.12778800
14	2.52660	13.34050	395.79	.20318000
15	3.18600	16.82230	313.87	.32307900
16	4.01760	21.21300	248.90	.51373700
17	5.06600	26.74850	197.39	.81683900
18	6.38800	33.72850	156.54	1.29876400
19	8.05550	42.53290	124.14	2.06531200
20	10.15840	53.63620	98.44	3.28437400

CONNECTIONS FOR DYNAMO-ELECTRIC MACHINES

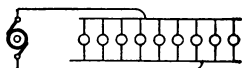


In the diagrams showing the connections of dynamo-electric machines, the heavy coils represent the series winding on the field magnets through which the entire current of the machine passes; the lighter coils represent the shunt winding on the field magnets through which only part of the main current passes.

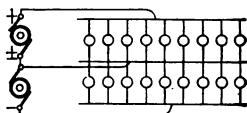
Lamps connected in series.



Lamps connected in multiple-arc or parallel.



Edison three-wire system.



DYNAMOS AND MOTORS.

MOTOR CIRCUITS.

To find the size of wire on stationary motor circuits:

Let c. m. = circular mils;

e = E. M. F. of motor in volts;

v = loss of volts in line;

d = distance from generator to motor in feet;

k = efficiency of motor;

10.8 ohms is the resistance of 1 ft. of commercial copper wire 1 mil in diameter.

$$\text{c. m.} = \frac{\text{H. P. of motor} \times 746 \times 2d \times 10.8}{evk}$$

APPROXIMATE MOTOR EFFICIENCY.

$\frac{1}{2}$ to $1\frac{1}{2}$ H. P. inclusive = 75% efficiency.

3 to 5 H. P. inclusive = 80% efficiency.

$7\frac{1}{2}$ to 10 H. P. inclusive = 85% efficiency.

15 H. P. and upwards = 90% efficiency.

Under ordinary circumstances, 10% loss from generator to motor is a maximum on stationary motor circuits.

EXAMPLE.—What is the size of wire necessary for a circuit on which a 10 H. P. 500-volt motor is running, when the distance between the motor and generator is 2,000 ft. and the loss is 5%?

SOLUTION.—Volts at generator, $\frac{500}{.95} = 526$, nearly.

Volts lost in line, $526 - 500 = 26$.

In the table on page 253, the approximate efficiency of a 10 H. P. motor is given as 85%.

$$\text{c. m.} = \frac{10 \times 746 \times 4,000 \times 10.8}{500 \times 26 \times .85} = 29,165.$$

In the table on page 238, the nearest size of wire corresponding to this area is No. 6 B. & S. gauge.

The approximate weight and resistance per mile of round bare wire when d is the diameter in mils, are, for copper wire, $\frac{d^2}{62.5}$ lb. and $\frac{56,970}{d^2}$ ohms; for iron wire, $\frac{d^2}{72}$ lb. and $\frac{380,060}{d^2}$ ohms.

Copper wire is approximately $1\frac{1}{2}$ times the weight of an iron wire of the same diameter.

In determining the size of wire to be used for inside work, after finding the c. m., always refer to the table on page 238, and see that the wire obtained by the formula is sufficiently large to carry the current; if not, use larger wire, regardless of per-cent. loss. *For pole-line construction, never use wire smaller than No. 8 B. & S. gauge.*

DYNAMO DESIGN.

The fundamental principle of dynamo design is expressed by the formula

$$E = \frac{N C n}{10^8 \times 60}$$

in which

E = electromotive force in volts given by the dynamo;

N = number of lines of force used to magnetize the armature;

C = number of conductors in a bipolar machine, measured all round the outside of the armature (whether in one

or more layers), or in a multipolar machine, as measured from a point opposite one north pole to a corresponding point opposite the next succeeding north pole;

n = number of revolutions per minute of the armature.

For example, a 2-pole dynamo has 2,000,000 lines of force passing from the north pole through the armature to the south pole; there are 200 conductors on the surface of the armature, and the speed is 1,500 rev. per min. The electromotive force generated will then be

$$E = \frac{2,000,000 \times 200 \times 1,500}{100,000,000 \times 60} = 100 \text{ volts,}$$

If a 4-pole dynamo were used, having a 4-circuit armature and 4 sets of brushes, with 1,000,000 lines of force passing through any one pole piece, then the total number would be 2,000,000, because the same lines of force pass into a south pole that emerge from a north pole. With the same armature as above, the number of conductors to be counted is only 100, as taken from one north pole to the next, and the electromotive force is

$$E = \frac{2,000,000 \times 100 \times 1,500}{100,000,000 \times 60} = 50 \text{ volts.}$$

For determining the number of lines of force required in a specific case, the above formula may be reversed, and we have

$$N = \frac{E \times 10^8 \times 60}{C n}.$$

These lines of force have a circuit to traverse composed of three different paths. One of these is through the field magnet and yoke M , Fig. 1; next, through a double air gap G ; and, lastly, through the armature core A . A given density of lines of force may not be exceeded, this limit being for ordinary cast iron about 50,000 lines per square inch; for wrought-iron forgings or cast steel, about 90,000; and for soft sheet iron, 110,000.

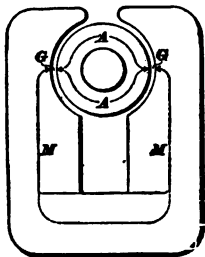


FIG. 2.

The ratio of magnetization to magnetizing force is called

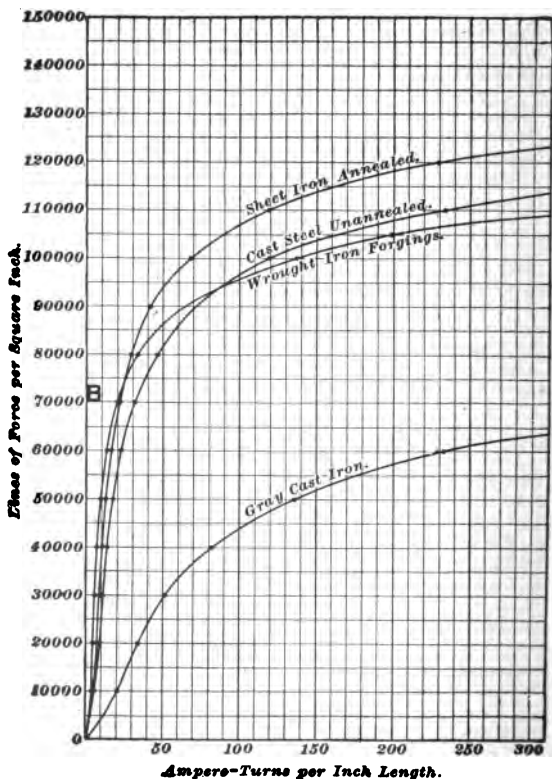


FIG. 2.

the *permeability*. The permeability of air is very low, the intensity of magnetization being a direct measure of the magnetizing force required; therefore, the air gap is usually made short.

In order to drive the lines of force through the magnetic circuit, magnetizing coils are wound on the cores at *M, M*. A certain number of ampere-turns will be required, depending on the density of the lines of force and the permeability of the different portions of the circuit. The number of turns may be found by taking a convenient current value, and dividing the ampere-turns by this. Reference to a wire table will then determine whether the resistance of the wire will be such that the terminal E. M. F. of the machine will supply the proper current. A margin should be allowed for regulating, and for the increase in resistance due to rise in temperature, which is about .4% for every degree centigrade, or .222% for every degree Fahrenheit above 75° F.

In the saturation curves of Fig. 2 are represented graphically the different values of the induction (**B**) in lines of force per square inch, corresponding to the magnetizing force expressed in ampere-turns per inch of length of circuit. Thus, to send 70,000 lines of force through a cast-steel core 1 sq. in. in cross-sectional area, would require about 30 ampere-turns for every inch in length of core. The 30 ampere-turns might be obtained by using a coil of 30 turns carrying 1 ampere, or 300 turns of $\frac{1}{10}$ ampere, etc. The number of lines of force *N* for any particular case being known, and also the allowable density **B**, which will vary somewhat with different samples of iron, the cross-sectional area $A = \frac{N}{B}$.

The ampere-turns to be added to the magnetizing coils to overcome the resistance of the air gap is

$$\text{A. T.} = \frac{H \times l}{3.192},$$

where **H** = number of lines of force per square inch;
and *l* = length of air gap (the two sides added together) in inches, usually a fraction.

It is necessary, in calculating the ampere-turns for the field circuit, to allow for leakage of lines of force through the

surrounding air, as the total number generated does not pass through the armature core. This leakage may amount to 30% or 40% of the whole, but is much less in well-designed machines.

For example, a bipolar dynamo has magnet cores having a mean length, with pole pieces, of 10 in. each; the yoke of the magnet is 13 in.; air gap, $\frac{1}{8}$ in. each side; armature core, 10 in. The magnetic density in the core is 85,000; air gap, 46,000; yoke, 65,000; armature core, 90,000 lines of force per square inch. If the fields are wrought-iron forgings, and the armature is built up of soft sheet iron, then the ampere-turns necessary will be:

	Length.	B	A.-T. per in.	Ampere- Turns.
Magnet cores	20 in.	85,000	44	880
Yoke	13 in.	65,000	16	208
Armature	10 in.	90,000	40	400
Air gap	$\frac{1}{8}$ in.	46,000		5,425
Total ampere-turns				6,913

In determining the size of wire to be used in the armature winding, a certain density of current may be assumed as the limit. This is usually expressed in circular mils or thousandths of an inch per ampere. For most purposes of design, a density of 600 circular mils per ampere may be allowed. In estimating the current passing through the armature, it must be remembered that the current of the outside circuit divides on reaching the armature, and passes through it along two paths in parallel with each other.

FAULTS OF DYNAMOS.

Reversal of Field.—Run the machine up to speed, and hold a small compass near each pole piece in succession. Their polarity should alternate all the way round.

Failure to Build Up.—This is probably due to reversal of shunt connections. Rock the brushes around until any one set occupies a position formerly occupied by the next set. If this should remedy the trouble, and such position is inconvenient, move them back and reverse connections of shunt

windings. If the failure of machine is due to want of residual magnetism, send a current from some external source through the fields. If it is due to a broken circuit, each coil may be tested separately with a battery and galvanometer or low-reading Weston voltmeter. Failure to generate may be due to the brushes being out of the neutral plane, which may be tested by moving them into different positions.

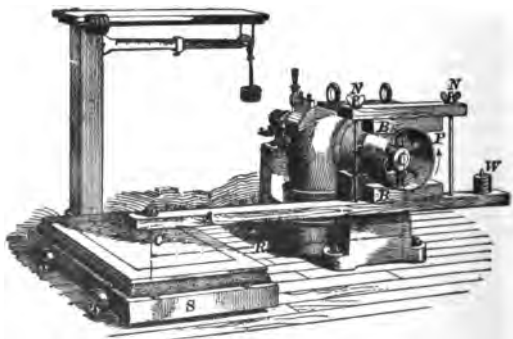
Heating.—This may be caused by a short-circuited armature coil. Allow the machine to cool, then run for a few minutes with no load, and stop. The defective coil will be found to be much hotter than the rest. It should be marked, and the armature taken out, when the coil may be rewound or otherwise repaired. If the heating is even, the load may be excessive and should be reduced. The effect may be due to eddy currents in the armature core, but this is a question of design in the first instance.

Sparking at Commutator.—If this be due to overload, the sparking cannot be cured except by reducing the load. The trouble may be due to improper position of brushes. Move the rocker-arm to one side or the other to determine this. If copper brushes (tangential) are used, they may be unevenly spaced round the commutator; each set of brushes should have the same relative position with regard to the respective pole tips. Sparking may be caused by an uneven commutator, in which case it should be smoothed with sandpaper (never emery) or turned down in the lathe. A broken connection at the commutator leads will produce flashing at each revolution, and one of the bars will show a burn extending nearly across it. The loose wire should be secured, or if broken, the commutator bars may be connected together with a piece of wire or a drop of solder as a temporary repair. As soon as possible a new coil should be put in. Sparking may also occur, in a multipolar machine, from the wearing away of the bearings, which produces eccentricity of the armature with respect to field, and consequent unequal magnetic induction at different points. A slight sparking at the brushes of the machine is not detrimental.

OUTPUT AND EFFICIENCY OF MOTORS.

A dynamo, when supplied with current from an external source, becomes a motor, turning the electrical energy into mechanical energy. The ratio between these two quantities, that is, between the input and output, determines the efficiency of the motor. The input may be found by measuring the current C with an ammeter, and the voltage E with a voltmeter, their product giving the power supplied in watts, $W = CE$. This quantity, divided by 746, gives the electrical horsepower, or E. H. P. = $\frac{W}{746}$.

The output is measured by means of a Prony brake (see figure). The motor pulley P is clamped between two blocks



of wood B, B , their pressure being regulated by the thumb-screws N, N , on the long bolts which hold them together. The lower block is extended to form an arm A of convenient length, and furnished with a sharp lagscrew C at the end. The lagscrew presses on the platform of a set of scales S , whereby its pressure may be determined. A counterbalance at W neutralizes the weight of the arm. When the pulley is revolved in the direction shown, the pressure on the scale will indicate the torque, or twisting power, developed, which

is expressed as the product of the pressure on the scale into the distance between the center of pulley and the point of the screw. If the length of arm $R = 2$ ft., and the pressure is 50 lb., the torque $T = 100$ ft.-lb. The horsepower may be determined by the following formula:

$$\text{H. P.} = \frac{2 \times 3.1416 TS}{33,000},$$

in which S is the speed of motor in revolutions per minute.

APPLICATIONS OF ELECTRIC MOTORS.

The same varieties of field and armature connections are used for motors as for dynamos, namely, series, shunt, and compound, and each type has distinguishing characteristics. The series motor is especially suitable for use in cases where a very high starting torque is required in order to obtain rapid acceleration under load, as, for instance, in street-railway work. Torque may be defined as the reaction of the current in the armature or moving part against the magnetic lines of force in the field magnets or stationary part. Strength of field is obtained by the current circulating through the magnet coils; consequently, the torque in a series motor will be a maximum when the current passing through is a maximum, as the same amount flows through armature and field. The opposition to the flow of current is the resistance of the circuit and the counter E. M. F. of the armature. When the current is applied, its value is determinable by Ohm's law for the first moment, supposing self-induction to be eliminated. The resistance of a series motor is usually so low that an additional resistance must be used at starting in order to prevent an excessive flow of current; but, as soon as the armature begins to revolve, the counter E. M. F. opposes and cuts down the current, and, consequently, the torque. The speed will continue to increase and the torque to decrease until the mechanical resistance to rotation balances the torque. If the motor is running light, the speed will rise continually, the counter E. M. F. will also increase and cut down the current, and the consequent reduction of field strength will require a still higher speed in order to develop

the necessary counter E. M. F., the final result being, probably, the bursting of the armature. The speed of a series motor under a constant load may be regulated by the somewhat wasteful method of introducing a resistance in series to reduce the speed, and by cutting out or shunting part of the field coils, to increase it. When two motors are used, they may be put in series at starting and connected in parallel for higher speeds. The series motor is well adapted for electric cranes, because it will automatically regulate its speed to the weight to be raised, exerting a very powerful torque at low speed for a heavy load.

The shunt motor will give a nearly constant speed for any variation in load, as long as the potential of current supply (the applied E. M. F.) is constant. This condition produces a constant field, as the shunt winding is directly across the main leads, and the speed of the motor will then be such that the difference between the E. M. F. of supply and the counter E. M. F., divided by the resistance of the armature, will be equal to the current passing through the armature. A change in the current will then produce but a relatively small change in the required counter E. M. F. of the motor, and the speed will only vary to that extent. As the load is put on, the motor tends to slow down; but this, by decreasing the counter E. M. F., allows more current to flow, thereby producing more torque to overcome the added mechanical resistance. Change of speed may be produced by varying the strength of the magnetic field, the weaker the field the higher the speed. If the load is constant, the torque will be decreased, but, if the load be correspondingly increased, the torque will remain nearly constant. Considerable weakening of the field is inadvisable, as it will cause destructive sparking at the commutator. The theoretically perfect method of speed regulation for a shunt motor is to provide a constant and independent field, and effect change of speed by varying the applied E. M. F. at the armature terminals without insertion of extra resistance. In this case the torque will always be proportional to the load, and the efficiency will be constant and independent of speed and torque. In the operation of such a system, certain complications are intro-

duced, inasmuch as it is necessary to install in connection with each motor a special dynamo with variable field, and this condition may therefore constitute a serious objection when the first cost of the plant is required to be low.

A differential compound winding may be used when a more nearly constant speed is wanted. The series turns on the field magnets are so connected as to oppose the shunt turns, and when an increase of load tends to cut down the speed, the additional current through the series turns weakens the field slightly, so that the same speed as before is required to generate the lower counter E. M. F.

Shunt motors are especially useful for machine tools, which require a constant speed irrespective of load, and may also be used on printing presses and similar machines where the load is more nearly uniform. When a variation in speed with load is immaterial, a cumulative compound winding may be employed, in which the series turns act with the shunt, thereby increasing the torque at starting, and affording some of the characteristics of both the shunt and series windings.

BATTERIES.

The simple primary battery consists of two elements, the *anode*, which is usually zinc, and the *cathode*, which may be carbon, both immersed in an exciting liquid called the *electrolyte*. The chemical action incident to the generation of current dissolves the zinc and liberates free hydrogen at the cathode, which adheres to the surface and reduces the E. M. F. of the battery. To overcome this effect, called *polarization*, a depolarizer is used which will take up the hydrogen as it is formed.

Depolarizers may be solid or liquid. When solid, the material is usually packed round the cathode, as in the case of the Leclanché cell; when the depolarizer is liquid, it may be prevented from mixing with the electrolyte by a porous partition, or, if their specific gravities differ considerably, they will remain separated one over the other in the jar. The following table gives the elements and depolarizers for different cells, with the E. M. F. in volts:

Name.	Anode.	Electrolyte.	Cathode.	Depolarizer.	E.M.F.	Remarks.
Volta	Zinc.....	Sulphuric acid (dilute)	Copper....	None	.9	Polarizes rapidly.
Law	Zinc.....	Sulphuric acid (dilute)	Carbon ...	None	1.35	
.....	Zinc.....	Sodium chloride (common salt)....	Carbon ...	None	1.08	
Grenet	Zinc.....	Sulphuric acid ⁴ } Potassium bi- ³ } chromate ¹⁸ } Water	Carbon ...	Electrolyte	1.9 to 2	For large currents.
Pabst	Wrought iron ...	Ferric chloride.....	Carbon ...		.78	Non-polarizing electrolyte.
Bunsen...	Zinc.....	Sulphuric acid (dilute)	Carbon ...	Nitric acid.....	1.89	Cathode and depolarizer in porous cup.
Fuller..	{ Amalgamated zinc in mercury }	Sulphuric acid (very dilute) or water.....	Carbon ...	Electropoison fluid diluted one-half..	2.14	Anode in porous cup.
Partz	Zinc.....	Sodium chloride or magnesium sulphate.....	Carbon ...	Bichromate solution (sulphochromic salt).....	1.9 to 2	Gravity cell. Resistance with sodium chloride, .5 ohm; with magnesium sulphate, 1 ohm.

Name.	Anode.	Electrolyte.	Cathode.	Depolarizer.	E.M.F.	Remarks.
D'Arsonval.....	Zinc.....	Caustic soda	Carbon ...	Ferric chloride	2.7	Porous cup used; pores become filled with ferric hydrate, an insoluble conductor.
Daniell ..	Zinc.....	Zinc sulphate	Copper....	Copper sulphate with copper-sulphate crystals....	1.07	Cathode and depolarizer in porous cup.
Gravity Daniell.	Zinc.....	Zinc sulphate. Sp. Gr. 1.0	Copper....	Copper sulphate with copper-sulphate crystals....	1.07	For closed-circuit work only; resistance 3 ohms.
Leclanché	Zinc.....	Ammonium chloride (saturated) ..	Carbon ...	Peroxide of manganese	1.48	Carbon and depolarizer in porous cup; resistance 4 ohms.
Lalande and Chaperon	Zinc.....	Caustic potash.....	Iron or Copper.	Cupric oxide	.7	Surface of electrolyte covered with layer of oil.
Edison-Lalande	Zinc.....	Caustic potash.....	Molded plates of cupric oxide and magnesium chloride held in copper frames		.7	Surface of electrolyte covered with layer of oil; resistance .07 ohm.
Chloride of mercury cell	Zinc.....	Saturated ammonium chloride	Carbon ...	Paste of mercurous chloride.....	1.45	Cathode and depolarizer in porous cups. For small currents.

Name.	Anode.	Electrolyte.	Cathode.	Depolarizer.	E.M.F.	Remarks.
Chloride- of-silver cell.....	Zinc.....	Ammonium chlo- ride (dilute)	Silver wire or plate with chlo- ride of silver.....		1.03	For medical work and testing.
Chloride- of-silver cell.....	Zinc.....	Zinc chloride.....	Silver wire or plate with chlo- ride of silver.....		1.02	
Chloride- of-silver cell.....	Zinc.....	Sodium chloride....	Silver wire or plate with chlo- ride of silver.....		.97	
.....	Zinc.....	Sulphuric acid (di- lute)	Carbon ...	Mercuric sulphate, mercurous sul- phate, or turpeth mineral	1.3 to 1.5	Depolarizer in form of paste. Poison- ous.
Latimer- Clark ...	Zinc.....	Paste of mercurous sulphate formed with zinc sulphate	Mercury.	Electrolyte	1.442	Standard cell, for very minute cur- rents.
Gouy	Zinc.....	Zinc sulphate 10% solution	Mercury.	Oxide of mercury..	1.39	Standard cell.
Weston ...	Cadmium amalgam	Cadmium sulphate	Mercury, with sulphate of mercury		1.019	Standard cell. No temperature coef- ficient.
Baille and Fery.....	Amalgamated zinc	Zinc chloride.....	Lead, with crystals of lead chloride.....		.5	Standard cell.

STORAGE BATTERIES.

Storage batteries or accumulators are composed of plates of prepared lead, placed side by side in glass cells or wooden boxes lined with rubber or lead, alternate plates being connected together, thus forming two sets, which constitute the positive and negative elements. The plates are entirely submerged in dilute sulphuric acid, specific gravity 1.17. The charging E. M. F. is about 2.5 volts per cell, so that, if 10 cells are connected in series, the required E. M. F. will be 25 volts. The discharging E. M. F. is usually taken as 1.9 volts, so that an installation to supply current at 115 volts should consist of $\frac{115}{1.9} = 61$ cells, with a few added to replace any that are out of order or to serve as regulators to vary the E. M. F. As soon as the battery is set up and the electrolyte added, the charging should commence, the first charge being continued a long while at a comparatively slow rate. Observe that the direction of current through the cell in charging is from the positive or brown plate to the negative or gray one. Discharging should be at a low rate, as rapid discharge leads to deterioration of the positive plates.

The rating of the capacity of accumulators is usually made on the basis of a discharge current that will cause the E. M. F. to fall to 1.8 volts in 10 hours, but it is well to stop discharging when the E. M. F. falls to 1.9 volts.

Storage-Battery Regulation.—In electric-lighting plants, an equalization of load on the dynamos is sometimes obtained by installing accumulators or storage batteries. Automatic or hand regulation may be employed, the usual method being to cut out one or more cells when the load is light and change the remainder, these cells being connected in again when the load rises. The following method obviates the many disadvantages of this system.

A shunt dynamo *d*, Fig. 1, supplies current to the lighting mains *m*, *n*, this current passing through the fields *c* of a low-voltage dynamo or *booster* *b*, driven by a shunt motor and connected across the mains in series with the battery *B*. The E. M. F. of the dynamo *d* is a little greater than that of the battery, so that it will charge the battery when there is no

external load. When all the lights are turned on, the booster field will be fully energized, and the E. M. F. of the booster

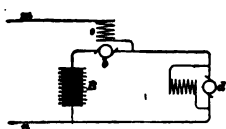


FIG. 1.

will be added to that of the battery, thereby causing the battery to discharge and assist the dynamo. At a medium load, the battery will be neutral, neither taking current nor discharging, while the dynamo is running at full load. Any increase that

may be made in the load will then be taken up by the battery.

In electric-railway plants the dynamos are usually over-compounded, thus giving a higher E. M. F. at the brushes at full load than at light load. In a case of this kind, a differential winding is employed, as shown in Fig. 2, which

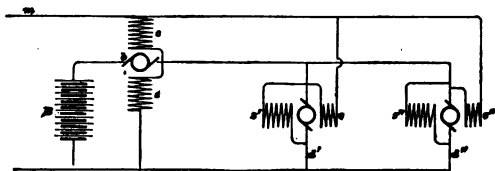


FIG. 2.

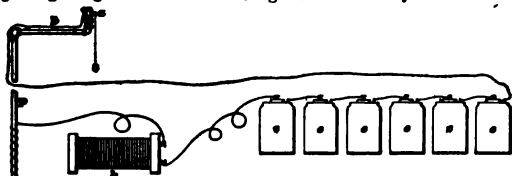
causes the booster to work both ways. On light loads a differential winding will assist the dynamos d' and d'' to charge the battery, raising the E. M. F. to the required value; but on heavy loads the series winding c will overpower the shunt s , and the battery will discharge into the outer circuit. The shunt field must be regulated so that the total charging and discharging that is done within a given time will balance each other, as the battery will otherwise tend either to overcharge or to undercharge. If the shunt field is strengthened, it will cause the batteries to charge, while if the field is weakened, it will cause the batteries to discharge at a lower value of the external load than before.

ELECTRODEPOSITION.

For electrodeposition of metals, low-resistance primary batteries giving from 2 to 10 volts may be used when the work is on a small scale. For larger work, accumulators may be employed, or the current may be taken directly from a low-voltage dynamo. The electroplating bath consists of a solution that has little or no chemical action on the objects to be plated, and that are suspended in it and electrically connected to the negative pole of the battery. The anode is a plate of the metal that it is desired to transfer; it is also submerged in the solution and connected through a resistance, if necessary, to the positive pole of the battery. For deposition of copper, the bath is made by taking 4 parts saturated solution of sulphate of copper mixed with 1 part of water containing one-tenth its volume of sulphuric acid. The current used must not exceed 18 amperes per square foot of surface of cathode. For nickel, use the double sulphate of nickel and ammonia, specific gravity 1.03; the current density must be low, and the solution should be neutral or slightly alkaline, as an acid bath will cause the nickel to peel off. For silver, the bath is a solution of cyanide of silver dissolved in cyanide of potassium. For gold, use cyanide of gold dissolved in cyanide of potassium. This solution is kept at 150° F. while in use.

ELECTRIC GAS LIGHTING.

The arrangement of the apparatus required for electric gas lighting is shown in the figure. A battery of about 6



Leclanché cells *c, c*, etc., joined up in series, is connected to one terminal of a spark coil *k*, the other terminal of which is soldered to a gas pipe *p*. The wire from the free end of the

battery is carried up through the house, and branches are run to the burners as at *b*, wherever needed. The insulation of this wire must be very thorough, special precautions being taken when it is carried through or along the fixtures. The burners are provided with a chain *a* attached to a movable contact spring, which is drawn past the burner, producing a spark of sufficient intensity to ignite the gas if it is previously turned on.

In multiple gas lighting, a fine wire is run from one burner to another of a group, as on a chandelier, leaving a small air gap at each one, and a current of very high tension is used, generated by a small frictional machine, causing a spark at each burner. The last contact in a series of burners is connected to the gas pipe.

THE WHEATSTONE BRIDGE.

A diagrammatic sketch of the Wheatstone bridge is shown in Fig. 1. This instrument is widely used for the determination of unknown resistances, and consists of such an arrangement of three circuits, *M*, *N*, *P*, of variable resistance, that

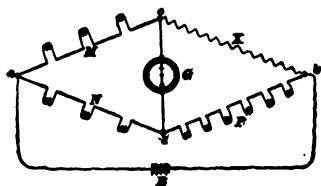


FIG. 1.

the value of a fourth may be found from their relation. This unknown resistance is connected between the points *b* and *c*, and the battery *B* between *a* and *b*. The variable resistances are then so adjusted that there shall be no

difference of potential between *c* and *d*, which form the terminals of the galvanometer *G*. The drop in potential from *a* to *c* will then be the same as from *a* to *d*, and *ac* bears the same proportion to *acb* as *ad* bears to *adb*. From this it follows that $ac : ad = cb : db$, or the unknown resistance $X = \frac{MP}{N}$.

For a certain test, the ratio of the arms, $\frac{M}{N} = \frac{10}{100}$. On

adjusting the resistance P , a balance is obtained when it is equal to 7,800 ohms. Then,

$$X = \frac{10 \times 7,800}{100} = 780 \text{ ohms.}$$

A commercial form of bridge is shown in Fig. 2. The same letters of reference are used as in the preceding diagram. Two keys, K and K' , are added, to be used in closing the

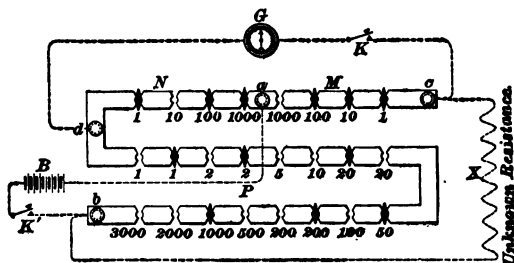


FIG. 2.

circuits. Resistances are put in by withdrawing the plugs. In the arm N there is a resistance of 10 ohms; in M , 1,000 ohms; in P , 5,838 ohms. If the galvanometer G indicates a balance, the value of the unknown resistance

$$X = \frac{1,000 \times 5,838}{10} = 583,800 \text{ ohms.}$$

CABLE TESTING.

Test for Capacity.—A condenser of known capacity k is charged by a battery and discharged through a galvanometer, producing a deflection d_1 . The cable, having an unknown capacity k_2 , is charged and discharged in similar manner, giving a deflection d_2 . Then $k_2 = k_1 \frac{d_2}{d_1}$. The connections for the test are shown in Fig. 1. A plug commutator p may be used to make connection with the insulated line wire L or

with one side of the condenser *c*, by putting a plug in 1 or 2.

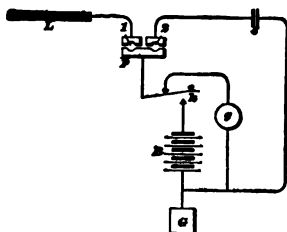


FIG. 1.

On depressing the key *k*, contact is made with one pole of the battery *B*, having about 100 cells; on releasing the key, the discharge from the line or the condenser passes through the galvanometer to the ground at *G*.

EXAMPLE.—The deflection through a condenser of 1.5 microfarads (mfd.) was 82 divisions,

and through a cable, 154 divisions. Find the capacity of cable.

SOLUTION.—From the formula given,

$$k_2 = 1.5 \times \frac{154}{82} = 2.8 \text{ microfarads.}$$

Voltmeter Method of Testing Insulation.—An ordinary Weston voltmeter with a range of 150 volts has a resistance of about 19,000 ohms. If, then, this instrument is connected across a 110-volt circuit, it will indicate the resistance of the circuit, that is, of itself, since the resistance of the armature and leads is very low. If *v* is the voltage across the mains, *r* the resistance of the voltmeter, and *x* the voltmeter reading, then the resistance to be determined, $R = \frac{vr}{x}$. When the voltmeter is put across the mains, *v* = 110, *r* = 19,000, and *x* = 110. The only resistance in the circuit is the voltmeter itself, for $R = \frac{110 \times 19,000}{110} = 19,000$ ohms. If we now put in series with the voltmeter a high resistance, thereby reducing the reading to 2 divisions, the total resistance $R = \frac{110 \times 19,000}{2} = 1,045,000$ ohms. From this we must subtract the voltmeter resistance in order to find the added resistance, which is $1,045,000 - 19,000 = 1,026,000$ ohms. A deflection of one division gives 2,071,000 ohms. To obtain higher readings, a special high-resistance voltmeter should be used. The connections are made as shown in Fig. 2, where *V* is the

voltmeter, F the feeder, and D the source of current. If I is the insulation resistance of a feeder, the corrected formula becomes

$$I = \frac{vr}{x} - r.$$



FIG. 2.

When a voltmeter is used having a resistance of 1 megohm (1,000,000 ohms), then a deflection of 1 division, when connected up as shown, would give an insulation resistance

$$I = \frac{110 \times 1,000,000}{1} - 1,000,000 = 109 \text{ megohms.}$$

Loss-of-Charge Method of Cable Testing.—The core of the cable must first be put to earth a sufficient length of time to be thoroughly clear from any charge due to previous electrification; then the far end is freed, and connections are

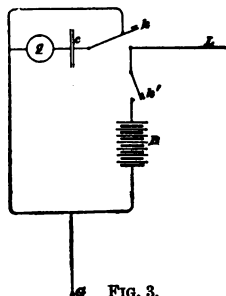


FIG. 3.

made as shown in Fig. 3. On depressing the key k , the cable is put to earth through the condenser c , which should be of very small capacity, say one-fiftieth of a microfarad. Both the cable L and the condenser c are then charged from the battery B by depressing the key k' , and on releasing k , the condenser is discharged through the ballistic galvanometer g , a moment being chosen when the galvanometer is at zero, showing

that the charge is steady. The deflection produced (d_1) represents the full charge held by the cable. The key k is then again depressed, and cable and condenser are charged for, say, half a minute, after which the battery is disconnected at k' , and leakage of the charge is allowed to take place for perhaps 5 minutes. Selecting a moment when the charge is steady, indicating an even distribution, the key k is raised, and the condenser discharged through the

galvanometer. The deflection (d_2) obtained will be less than the first one, owing to the leakage of charge during the 5 minutes, and will therefore be a measure of the conducting power of the cable covering, or its insulation resistance. The ratio of these two deflections, d_1 and d_2 , will ordinarily be sufficient to indicate the condition of the cable without further calculation; the exact insulation resistance may be found by the following formula,

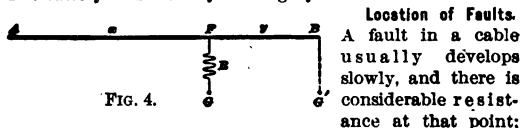
$$I = \frac{26.06 t}{K \log \frac{d_1}{d_2}},$$

where I = insulation resistance of the cable in megohms;
 t = time in minutes during which the charge is allowed to leak;
 K = capacity of the cable in microfarads;
 d_1 = initial discharge deflection;
 d_2 = final deflection after t minutes.

EXAMPLE.—In a loss-of-charge insulation test, the initial deflection was 238 divisions, and the deflection after 5 minutes' leakage was 137 divisions. The capacity of the cable being 1.8 microfarads, what was the insulation resistance?

$$\text{SOLUTION.— } I = \frac{26.06 \times 5}{1.8 \times \log \frac{238}{137}} = 308.1 \text{ megohms.}$$

The battery used in this test may be about 100 chloride-of-silver cells, or the same number of Leclanché cells. In the latter case it will be better to make the electrolyte of only about one-fifth the usual strength, to prevent creeping of the salts, as only very small currents are required for these tests. The battery must be very thoroughly insulated.



therefore, in determining the location of the fault, its resistance must be taken into account. Let AB , Fig. 4, be the cable and let a fault F connect to the ground at G through

a resistance R . When the end B of the cable is insulated, the resistance is measured at the station A , and is equal to the resistance of that portion of the cable between the station and the fault *plus* the resistance of the fault, that is, $x + R$. B is then grounded at G' , and the resistance is

$$x + \frac{yR}{y + R}.$$

Let $x + R = r.$

Let $x + \frac{yR}{y + R} = r'.$

Let $x + y = r''.$

Then, $x = r' - \sqrt{(r - r')(r'' - r')};$

$$y = r'' - r' + \sqrt{(r - r')(r'' - r')}.$$

If L = length of cable in feet, the distance from A to the fault is

$$\frac{Lx}{x + y}.$$

EXAMPLE.—The resistance of a cable in good condition is 3 ohms. A fault develops, and, on testing, the resistance through it is 160 ohms, the far end of the cable being insulated. When the far end is grounded, the resistance is 2.95 ohms. What is the distance to the fault, the length of cable being 5,180 ft.?

SOLUTION.— $r = 160, r' = 2.95, r'' = 3.$

Then, $x = 2.95 - \sqrt{157.05 \times .05} = .15 \text{ ohm.}$

$$y = 3 - 2.95 + \sqrt{157.05 \times .05} = 2.85 \text{ ohms.}$$

The distance to the fault = $\frac{5,180 \times .15}{2} = 259 \text{ ft.}$

SURVEYING.

COMPASS SURVEYING.

The *magnetic bearing* of a line is the angle that the line makes with the magnetic needle. The length of a line, together with its bearing, is termed a *course*. To take the bearing of a line, set the compass directly over a point in it, at one extremity, if possible. This may be done by means of a plumb-bob suspended from the compass.

Bring the compass to a perfectly level position. Let a flagman hold a rod carefully plumbed at another point of the line, preferably the other extremity, if he can be distinctly seen. Direct the sights upon this rod and as near the bottom of it as possible. Always keep the same end of the compass ahead—the north end is preferable, as it is readily distinguished by some conspicuous mark, usually a *fleur-de-lis*—and always read the same end of the needle, that is, the north end of the needle if the north point of the compass is ahead, and *vice versa*. Before reading the angle, see that the eye is in the direct line of the needle, so as to avoid the error that would otherwise result from *parallax*, or apparent change of the position of the needle, due to looking at it obliquely.

The angle is read and recorded by noting, *first*, whether the N or S point of the compass is nearest the end of the needle being read; *second*, the number of degrees to which it points; and *third*, the letter E or W nearest the end of the needle being read.

Let *AB* in Fig. 1 be the direction of the magnetic needle, *B* being at the north end. Let the sights of the compass be directed along the line *CD*. The north point of the compass will be seen to be nearest the north end of the needle which is to be read. The needle, which has remained stationary while the sights were being turned to *CD*, now points to 45° between the *N* and *E* points, and the angle is read *north forty-five degrees east* (*N* 45° *E*).

A sure test of the accuracy of a bearing is to set up the compass at the other end of the line. i. e., the end first sighted

line AD produced, is found by sighting to A , or, what is still better, to the point D , if that point can be seen. The number of Sta. (Station) E , namely, $4 + 40$, and the bearing of AE are then recorded by the compassman. By this time the chief of party has located the point F , and the flag is in place for sighting. The axmen, if there is work for them to do, are now put in line by the head chainman; the axmen clear only so much as would interfere with rapid chaining. The bearing of the line EF having been recorded, the compass is moved quickly to F , replacing the target left by the flagman, leveled up, and directed toward the point G , which is already located. The chainmen reaching F , its number $11 + 20$ is recorded by the compassman and the instrument sighted to G and the work continued as before.

FORM FOR KEEPING NOTES.

A plain and convenient form for keeping compass notes is the form given on page 279, which is a record of the survey platted in Fig. 2. The first column of the table contains the station numbers, the notation running from the bottom to the top of the page. By means of this arrangement, the lengths of the courses are found by subtracting the number of the station of one compass point from the number of the station of the next succeeding compass point. Before work has commenced on the plat, the subtractions are made and the lengths of the courses are written in red ink between the station numbers.

The second column contains the bearings of the lines. The bearing recorded opposite to a station is the bearing at the course between the given station and the one next above. Thus, the bearing recorded opposite Sta. 0 is $75^{\circ} 00' W$, and is the bearing of the line extending from Sta. 0 to Sta. $4 + 40$ next above. The length of the course is the difference between 0 and $4 + 40$ equal to 440 ft. The bearing recorded opposite to $4 + 40$ is $N 25^{\circ} 00' W$. It is the bearing of the line extending from Sta. $4 + 40$ to Sta. $11 + 20$ next above. Its length is found by subtracting $4 + 40$ from $11 + 20$ equal to 680 ft., and so on.

In the third column, under the head of remarks, are recorded notes of reference, topography, and any information that may aid in platting or subsequent location.

Station.	Bearing.	Remarks.
47 + 75		End of line.
35 + 75	N 25° 40' E	
27 + 50	N 14° 10' E	
20 + 35	N 2° 30' W	Woodland.
11 + 20	N 15° 10' W	
4 + 40	N 25° 00' W	
0	N 75° 00' W	Sta. 0 is at P. C. of 14° curve to left at Bellford Sta. O. & P. R. R.

TRANSIT SURVEYING.

The Vernier.—A *vernier* is a contrivance for measuring smaller portions of space than those into which a line is actually divided. The divided circle of the transit is graduated to half degrees, or 30'. The graduations on the verniers run in both directions from its zero mark, making two distinct verniers, one for reading angles turned to the right and the other for reading those turned to the left. In reading the vernier, the observer should first note in which direction the graduations of the divided



FIG. 1.

circle run. In Fig. 1 the graduations increase from left to right and extend from 57° to 91°. Next, he should note the point where the zero mark of the vernier comes on the divided circle. In Fig. 1 the zero mark comes between 74° and 74½°. Now, as the circle graduations read from left to right, we read the right-hand vernier and find that the 23d graduation on the vernier coincides with a graduation on the

divided circle and the vernier reads $23'$, which we add to 74° , making a reading of $74^\circ 23'$, an angle to the *left*. In Fig. 2 the graduations on the circle increase from right to left, and we accordingly read the left-hand vernier. The zero mark of the vernier comes between $67\frac{1}{4}^\circ$ and 68° . Reading the vernier, we

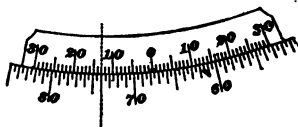


FIG. 2.

find that the 13th graduation on the vernier coincides with a graduation on the circle and the vernier reads $13'$. Accordingly, we add to $67\frac{1}{4}^\circ$, the reading = $13'$, making a total reading of $67^\circ 43'$, an angle to the *right*.

Setting Up the Instrument.—In setting up a transit, three preliminary conditions should be met as nearly as possible:

1. The tripod feet should be firmly planted.
2. The plate on which the leveling screws rest should be level.
3. The plumb-bob should be directly over the given point.

When these three conditions are met, the completion of the operation is quickly performed with the leveling screws.

How to Prolong a Straight Line.—Let AB , in Fig. 3, be a straight line which it is required to prolong or "produce."

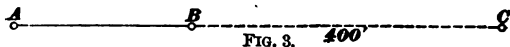


FIG. 3.

The line can be prolonged in two ways: by means of *foresight* or by means of *backsight*.

1. By *foresight*, set up the transit at A and sight to B ; measure 400 ft. from B in the opposite direction from A . Then, by means of signals, move the flag to the right or left until the vertical cross-hair shall exactly bisect the flag held at C . Then, the line BC will be the prolongation of the line AB .

2. By *backsight*, set the transit at B and sight to A . Reverse the telescope, and having measured 400 ft. from B in the opposite direction from A , set the flag at C ; then will the line BC be the line AB produced.

Horizontal Angles and Their Measurement.—A horizontal angle is one the boundary lines of which lie in the same horizontal plane. Let A , B , and C , in Fig. 4, be three points, and let it be required to find the horizontal angle formed by the lines AB and AC joining these points. Set up the instrument precisely over the point A , and carefully level it. Set the vernier at zero, and place flags at B and C . Sight to the flag at

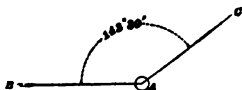


FIG. 4.

B and set the lower clamp. Then, by means of the lower tangent screw, cause the vertical cross-hair to exactly bisect the flag at B . Loosen the upper clamp. With a hand on either standard, turn the telescope in the same direction as that of the hands of a watch until the flag at C is covered or nearly covered by the vertical cross-hair. Clamp the upper plate, and with the upper tangent screw bring the line of sight exactly on the flag at C . The arc of the graduated circle traversed by the zero point of the vernier will be the measure of the angle BAC , as $143^{\circ} 30'$. The points A , B , and C are not necessarily in the same horizontal plane, but the level plate of the instrument projects them into the horizontal plane in which it revolves.



FIG. 5.

A Deflected Line.—A deflected line, or “angle line,” is a consecutive series of lines and angles. The direction of each line is referred to the line immediately preceding it, the latter being, in imagination, produced, and the angle measured between it and the next line actually run. The angles are recorded R° or L° , according as they are turned to the right or left of the prolongation of the immediately preceding line. An example of a deflected line is shown in Fig. 5; it starts from the head block of switch at Centon Station, O. & P. R. R.

Set up the transit at A with vernier at zero. Sight to a flag

held at *F* on the center line of the track, O. & P. R. R. Loosen the vernier clamp, the point *B* being determined, and turn the telescope until the point *B* is distinctly seen; clamp the vernier, and accurately sight to flag held at *B*; the angle reads $32^{\circ} 30'$ and is recorded $R^{\circ} 32^{\circ} 30'$, with a sketch showing the connection. The bearing of the line *AB* cannot be taken at *A* on account of the attraction of the rails. The point *A* is in the head block of the switch (which is designated by the abbreviation *H. B.*) at Benton Station, O. & P. R. R. The instrument is now moved to *B*, the vernier set at zero and backsighted to *A*; the bearing of *AB*, viz., $N 75^{\circ} 00' E$, is taken, and the number of station *B*, viz., $2 + 90$, together with the bearing of *AB* recorded. The telescope is then reversed, pointing in the direction *BB'*. The point *C* being determined, the upper clamp is loosened and the telescope turned to the right and sighted to *C*. The reading is found to be $14^{\circ} 30'$ and recorded $R^{\circ} 14^{\circ} 30'$. It measures the angle *B'BC*. The bearing $N 89^{\circ} 20' E$ is then recorded. The instrument is next set up at *C*, the vernier set at zero, backsighted to *B*, and then reversed; the deflection to *D*, viz., $R^{\circ} 10^{\circ} 00'$ read and recorded, together with the number of the station at *C*, viz., $6 + 85$. This deflection measures the angle *C'CD* and gives the direction of the line *CD*. A good form of notes for such a survey is the following:

Station.	Deflection.	Mag. Bearing.	Decl. Bearing.	Remarks.
13+63				End of Line.
10+31	$L^{\circ} 30^{\circ} 00'$	$N. 69^{\circ} 25' E.$	$N. 69^{\circ} 30' E.$	
6+85	$R^{\circ} 10^{\circ} 00'$	$S. 80^{\circ} 30' E.$	$S. 80^{\circ} 30' E.$	
2+90	$R^{\circ} 14^{\circ} 30'$	$N. 89^{\circ} 20' E.$	$N. 89^{\circ} 30' E.$	
0		$N. 75^{\circ} 00' E.$		Sta. 0 at Benton Sta.

Checking Angles by the Needle.—In spite of the greatest care, errors in the reading and recording of angles will occur. The best check to such errors is the magnetic needle.

In Fig. 6, we have an example of the use of the needle in checking angles. The bearing of the line *AB*, which corresponds to *AB* in Fig. 5, is $N 75^{\circ} 00' E$, and is assumed to be correct. The bearing of the line *BC*, as read from the needle,

is $N\ 89^{\circ}\ 20'\ E$. Its *deduced bearing* is obtained as follows: To the bearing of the line AB , viz., $N\ 75^{\circ}\ 00'\ E$, we add the R° deflection $14^{\circ}\ 30'$; the sum is $89^{\circ}\ 30'$, which is recorded in the column headed *Ded. Bearing*. The deduced bearing, it will be seen, is 10 minutes greater than the magnetic bearing read from the needle. Had the deflection angle been recorded L° instead of R° , the deduced bearing would have been the difference between $75^{\circ}\ 00'$ and $14^{\circ}\ 30'$, which is $60^{\circ}\ 30'$, and would be recorded $N\ 60^{\circ}\ 30'\ E$.

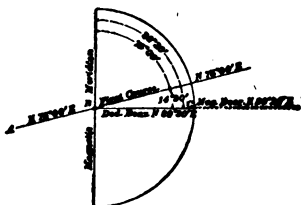


FIG. 6.

The magnetic bearing being $N\ 89^{\circ}\ 20'\ E$, would have at once revealed the error. The confusion of the directions R° and L° is the commonest source of error in recording deflections, though sometimes a mistake of 10 degrees is made in reading the vernier. Both angle and bearing should be read after they are recorded, and compared with the recorded readings

TRIANGULATION.

Triangulation is an application of the principles of trigonometry to the calculation of inaccessible lines and angles.

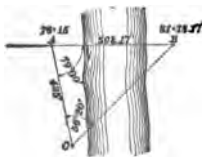


FIG. 1.

A common occasion for its use is illustrated in Fig. 1, where the line of survey crosses a stream too wide and deep for actual measurement. Set two points A and B on line, one on each side of the stream. Estimate roughly the distance AB . Suppose the estimate is 425 ft. Set another point C , making the distance AC equal to the estimated distance $AB = 425$ ft. Set the transit at A and measure the angle $BAC =$ say, $79^{\circ}\ 00'$. Next set up at the point C and

measure the angle $A \cdot C B =$ say, $56^{\circ} 20'$. The angle $A B C$ is then determined by subtracting the sum of the angles A and C from 180° ; thus, $79^{\circ} 00' + 56^{\circ} 20' = 135^{\circ} 20'$; $180^{\circ} 00' - 135^{\circ} 20' = 44^{\circ} 40' =$ the angle $A B C$. We now have a side and three angles of a triangle given, to find the other two sides $A B$ and $C B$. In trigonometry, it is demonstrated that, *in any triangle the sines of the angles are proportional to the lengths of the sides opposite to them*. In other words, $\sin A : \sin B = B C : A C$; or, $\sin A : \sin C = B C : A B$, and $\sin B : \sin C = A C : A B$.

Hence, we have $\sin 44^{\circ} 40' : \sin 56^{\circ} 20' = 425 : \text{side } A B$;

$$\sin 56^{\circ} 20' = .83228;$$

$$.83228 \times 425 = 353.719;$$

$$\sin 44^{\circ} 40' = .70298;$$

$$353.719 \div .70298 = 503.17 \text{ ft.} = \text{side } A B.$$

Adding this distance to $76 + 15$, the station of the point A , we have $81 + 18.17$, the station at B .

Another case is the following: Two tangents, $A B$ and $C D$ (see Fig. 2), which are to be united by a curve, meet at some inaccessible point E . Tangents are the straight portions of a



FIG. 2.

line of railroad. The angle $C E F$, which the tangents make with each other, and the distances $B E$ and $C E$ are required. Two points A and B of the tangent

$A B$, and two points C and D of the tangent $C D$, being carefully located, set the transit at B , and backsighting to A , measure the angle $E B C = 21^{\circ} 45'$; set up at C , and, backsighting to D , measure the angle $E C B = 21^{\circ} 25'$. Measure the side $B C = 304.2$ ft.

Angle $C E F$ being an exterior angle of triangle $E B C$ equals sum of $E B C$ and $E C B = 21^{\circ} 45' + 21^{\circ} 25' = 43^{\circ} 10'$; angle $B E C = 180^{\circ} - C E F = 136^{\circ} 50'$. From trigonometry, we have

$$\sin 136^{\circ} 50' : \sin 21^{\circ} 45' = 304.2 \text{ ft.} : C E;$$

$$\sin 21^{\circ} 45' = .37056;$$

$$.37056 \times 304.2 = 112.724352;$$

$$\sin 136^{\circ} 50' = .68412;$$

$$\text{side } C E = 112.724352 \div .68412 = 164.77 \text{ ft.}$$

Again, we find BE by the following proportion:

$$\sin 136^{\circ} 50' : \sin 21^{\circ} 25' = 304.2 : \text{side } BE;$$

$$\sin 21^{\circ} 25' = .36515;$$

$$.36515 \times 304.2 = 111.07863;$$

$$\sin 136^{\circ} 50' = .68412;$$

$$\text{side } BE = 111.07863 \div .68412 = 162.36 \text{ ft.}$$

A building H , Fig. 3, lies directly in the path of the line AB , which must be produced beyond H . Set a plug at B , and then turn an angle $DBC = 60^{\circ}$. Set a plug at C in the line BC , at a suitable distance from B , say, 150 ft. Set up at C , and turn an angle $BCD = 60^{\circ}$, and set a plug at D , 150 ft. from C . The point D will be in the prolongation of AB . Then, set up at D , and backsighting to C , turn the angle $CD D' = 120^{\circ}$. DD' will be the line required, and the distance BD will be 150 ft., since BCD is an equilateral triangle.

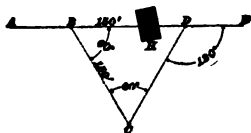


FIG. 3.

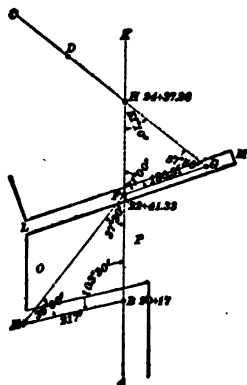


FIG. 4.

AB and CD , Fig. 4, are tangents intersecting at some inaccessible point H . The line AB crosses a dock OP , too wide for direct measurement, and the wharf LM . F is a point on the line AB at the wharf crossing. It is required to find the distance BH and the angle FHG . At B , an angle of $103^{\circ} 30'$ is turned to the left and the point E set 217' from B = to the estimated distance BF . Setting up at E , the angle BEF is found to be $39^{\circ} 00'$.

Whence, we find the ang.

$$BFE = 180^{\circ} - (103^{\circ} 30' + 39^{\circ}) = 37^{\circ} 30'.$$

From trigonometry, we have

$$\sin 37^{\circ} 30' : \sin 39^{\circ} 00' = 217 \text{ ft.} : \text{side } BF;$$

$$\sin 39^{\circ} 00' = .62932;$$

$$.62932 \times 217 = 136.56244;$$

$$\sin 37^{\circ} 30' = .60876;$$

$$\text{side } BF = 136.56244 \div .60876 = 224.33 \text{ ft.}$$

Whence, we find station F to be $20 + 17 + 224.33 = 241.33$. Set up at F and turn an angle $HFG = 71^{\circ} 00'$ and set up at a point G where the line CD prolonged intersects FG . Measure the angle $F GH = 57^{\circ} 50'$, and the side $FG = 180.3$. The angle $F HG = 180^{\circ} - (71^{\circ} + 57^{\circ} 50') = 51^{\circ} 10'$. From trigonometry we have

$$\sin 51^{\circ} 10' : \sin 57^{\circ} 50' = 180.3 : \text{side } FH.$$

$\sin 57^{\circ} 50' = .84650$; $.84650 \times 180.3 = 152.62395$; $\sin 51^{\circ} 10' = .77897$; side $FH = 152.62395 \div .77897 = 195.93 \text{ ft.}$; whence we find station H to be $24 + 37.26$.

CURVES.

Two lines forming an angle of 1° with each other will, at a distance of 100 ft. from the angular point, diverge by 1.745 ft.

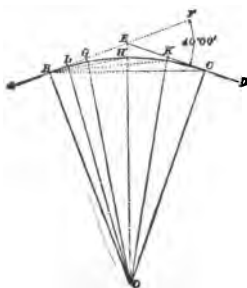


FIG. 1.

The *degree of a curve* is determined by that central angle which is subtended by a chord of 100 ft. Thus, if BOG (Fig. 1) is 10° and BG is 100 ft., $BGHKC$ is a 10° curve.

The *deflection angle* of a curve is the angle formed at any point of the curve between a tangent and a chord of 100 ft. The deflection angle is therefore *half the degree of the curve*. Thus, if the chord BG is 100 ft., the angle EBG is the deflection angle of curve $BGHKC$, and is half the angle BOG .

EXAMPLE.—Given, the deflection angle $EBG = D$ (Fig. 1) to find the radius $BO = R$.

SOLUTION.—Draw OL perpendicular to BG . In the right-angled triangle BOL , we have $\sin BOL = \frac{BL}{BO}$; but $BOL = EBG = D$, since OL , being perpendicular to the chord BG , bisects the arc BLG . But the angle $D = \frac{1}{2}BOG$; hence, angle $BOL = D$. $BL = 50$ ft., and the radius $BO = R$. Substituting these values in the given equation, we have $\sin D = \frac{50}{R}$; whence, $R \sin D = 50$, and $R = \frac{50}{\sin D}$.

For curves of from 1° to 10° , the radius may be found by dividing 5,730 ft. (the radius of a 1° curve) by the degree of the curve. The results obtained are sufficiently accurate for all practical purposes. For sharp curves, i. e., for those exceeding 10° , the above formula, viz., $R = \frac{50}{\sin D}$, should be used, especially if the radius is to be used as a basis for further calculation.

Tangent Distances.—When an intersection of tangents has been made and the intersection angle measured, the next question is the degree of curve that is to unite them, which being decided, the next step in order is the location of the points on the tangents where the curve begins and ends. These two points are equally distant from the point of intersection of the tangents, which is called the P. I. The point where the curve begins is called the *point of curve*, or the P. C., the point where the curve terminates is called the *point of tangent*, or the P. T. The distance of the P. C. and P. T. from the P. I. is called the *tangent distance*.

In Fig. 1, let AB and CD be tangents intersecting at the point E and forming an angle $CEF = 40^\circ 00'$ with each other. It is decided to unite these tangents by a 10° curve, whose radius is 573.7 ft. Call the angle of intersection I , the radius BO , R , and the tangent distance BE , T . From geometry we know that $BOC = CEF$, hence the angle $BOE = \frac{1}{2}CEF$. From the right triangle EBO , we have $\tan BOE = \frac{BE}{BO}$.

Substituting the above equivalents, we have $\tan \frac{1}{2}I = \frac{T}{R}$, or $T = R \tan \frac{1}{2}I$; $R = 573.7$; $\frac{1}{2}I = 20^\circ$; $\tan 20^\circ = .36397$;

$573.7 \times .36397 = 208.81$ ft. Measure back from the point *E* on both tangents the distance 208.81 ft. to the points *B* and *C*. Drive plug flush with the ground at both points and set accurate center points, marked by tacks, in both. Directly opposite each of these plugs drive a stake, called a *guard stake* because it guards or rather indicates where the plug is. The stake at *B*, if the numbering of the stations runs from *B* toward *C*, will be marked P. C., and the stake at *C* will be marked P. T.

To Lay Out a Curve With a Transit.—Having set the tangent points *B* and *C*, Fig. 1, set up the transit at *B*, the P. C. Set the vernier at zero and sight to *E*, the intersection point. Suppose *B* to be an even or "full station," say 18, and that it has been decided to set stakes at each hundred feet. Let the central angle BOG , measured by the 100-ft. chord BG , be 10° ; then, the deflection angle EBG , whose vertex *B* is in the circumference and subtended by the same chord BG , will be $\frac{1}{2} BOG$, or 5° . Turn an angle of 5° from *B*, which in this case will be to the right, measure a full chain 100 ft. from *B* and line in the flag at *G*; drive a stake at *G*, which will be marked 19. Turn off an additional 5° making 10° from zero, and at the end of another chain from *G*, at *H*, set at a stake marked 20. Continue turning deflections of 5° until 20° or one-half of the intersection angle is reached. This last deflection, if the work has been correctly done, will bring the head chainman to the point of tangent *C*. It is but rarely that the P. C. comes at a full station. When the P. C. comes between full stations it is called a *substation*, and the chord between it and the next full station is called a *subchord*. Had the P. C. come at a substation, say $17 + 32$, the deflection for the subchord of $100 - 32$, or 68 ft., the distance to the next station, is found as follows: The deflection for a full station, i. e., 100 ft., is $5^\circ = 300'$, and the deflection for 1 ft. is $\frac{300'}{100} = 3'$, and for 68 ft. the deflection will be $68 \times 3 = 204' = 3^\circ 24'$, which is turned off from zero and a stake set on line, 68 ft. from the transit, at station 18. The length of a curve uniting two given tangents whose intersection is determined, is found as follows:

Suppose $I = 32^{\circ} 40'$ and that the tangents are to be united by a 6° curve. $32^{\circ} 40'$ reduced to the decimal form is 32.667° ; as each central angle of 6° will subtend a 100-ft. chord or one chain, there will be as many such chords or chains as the number of times 6 is contained in 32.667 , which is 5.444 , that is, there will be 5.444 chains in the curve, or 544.4 ft., which is the required length of the curve. The P. C. and P. T. having been set and the station of the P. C. determined by actual measurement, say $58 + 71$, the station number of the P. T. is found by adding to $58 + 71$, the station number of the P. C., the calculated length of the curve 544.4 ft. $58 + 71 + 544.4 = 64 + 15.4$, the station of the P. T.

Tangent and Chord Deflections.—Let AB in Fig. 2 be a tangent, and $BCEH$ a curve commencing at B . Produce the tangent AB to the point D . The line CD is a *tangent deflection*, and is the perpendicular distance from the tangent to the curve. If the chord BC is produced to the point G , making $CG = BC = CE$, the distance GE is a *chord deflection* and is double the tangent deflection DC .

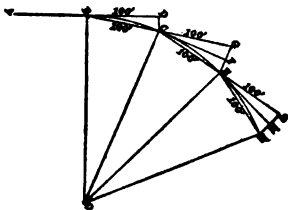


FIG. 2.

Given, the radius $BO = R$, Fig. 2, to find the chord deflection EG and the tangent deflection $CD = FE$.

The triangles OCE and CEG are similar, since both are isosceles, and the angle $GCE = \text{angle } COE$. Hence, we have $OC : CE = CE : EG$. Denoting the chord CE by c and the chord deflection EG by d , we have, from the above proportion, $R : c = c : d$. Therefore, $d = \frac{c^2}{R}$. To find the tangent deflection, draw CF to the middle point of EG . Then FE is equal to the tangent deflection, or DC . Hence, the tangent deflection is equal to one-half the chord deflection, or the tangent deflection $= \frac{c^2}{2R}$.

If the *P. C.* does not fall at a full station (and this is usually the case), compute the chord deflection by substituting for *c* in the formula for chord deflection $\frac{1}{2} c (c + c')$. Where *c'* is the length of the chord from the *P. C.* to the full station; or if the tangent deflection *f* for a chord of 100 feet has been previously found, the chord deflection for the second station beyond the *P. C.* is $d_o = f \left(1 + \frac{c'}{c}\right)$

Laying Out Curves Without a Transit.—During construction, the engineer is often called upon to restore center stakes on a curve when the transit is not at hand. This can be accomplished reasonably well with a tape, as follows:

In Fig. 3, *AB* is a tangent and *B*, at Sta. 8 + 25, is the *P. C.* of a 4° curve; a stake is required at each full station. The stakes at *A* and *B* are restored, determining the *P. C.* and the direction of the tangent. For a 4° curve the regular chord deflection for 100 feet is 6.98 ft., and the tangent deflection is $6.98 \div 2 = 3.49$ ft.



FIG. 3.

The distance from the *P. C.* to the next station *C* is 75 ft.; hence, the tangent deflection $CF = 75^2 \div (2 \times 5,730 + 4) = 1.96$ ft. The point *F* is found

by first measuring 75 feet from *B*, thus locating the point *C*, in the line with *AB*, then from *C* measuring $CF = 1.96$ feet, at right angles to *BC*; the point *F* thus determined will be Station 9. Next, the chord *BF* is prolonged 100 feet to *D*; as *BF* is only 75 feet, $DG = d_o = 3.49 \times \left(1 + \frac{100}{75}\right) = 6.11$ feet. This distance is measured at right angles to *BD*; the point *G* thus determined will be Station 10. The position of Station 11, the *P. T.*, is determined in the same manner, except that, as the chords *FG* and *GH* are each 100 feet long, the regular chord deflection of 6.98 feet is used for *EH*. A stake is driven at each station thus located.

To Determine Degree of Curve by Measuring a Middle Ordinate. In trackwork, it is often necessary to know the degree of a curve when no transit is available for measuring it. The degree can be found by measuring the middle ordinate of any

convenient chord, and multiplying its length by 8, which will give the chord deflection for that curve.

Let AB , in Fig. 4, be a 50-ft. chord, measured on the track, and let the middle ordinate ab be .44 ft. $.44 \times 8 = 3.52 =$ chord deflection for 50 ft., which, expressed in decimal parts of a full station, is .5; $.5^2 = .25$. The chord deflection for 100 ft. multiplied by .25 = the chord deflection for 50 ft., which we know by calculation to be 3.52 ft. Hence, $3.52 \div .25 = 14.08$ ft., the chord deflection for 100 ft., which, if divided by 1.745, the chord deflection for a 1° curve, gives a quotient of 8.07, nearly. The inference is that the curve is 8° .

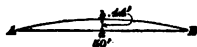


FIG. 4.

How to Keep Transit Notes.—A good form for location notes is the following:

Station	Deflection	Total Angle	Mag. Bearing	Dist. Bearing	Remarks	June 20, 1888
9						
8						
7						
6+00	S 84° P.T.	15° 00'	N. 36° 30' E.	N. 36° 15' E.		
5+00	4° 00'					
4	2° 00'					
3+00	2° 00'					
2	1° 00'					
1+00	2° 00'	5° 15'				
4	1° 00'				Int. Angle = 15° 00'	4° Curve R ^r
3+00	0° 00'				T = 188.61 ft.	Def. Angle for 60 ft. = 0°
2+00	P.C. 4° 25'				P.C. = 3+00	Def. Angle for 1 ft. = 1.5"
3					Length of Curve = 375 ft.	
2					P.T. = 4+00	
1						
0			N. 36° 30' E.	N. 36° 15' E.		

In the first column the station numbers are recorded. In the second column are recorded the deflections with the abbreviations P. C. and P. T., together with the degree of curve and the abbreviation R^r or L^r, according as the line curves to the right or left. At each transit point on the curve, the total or central angle from the P. C. to that point is calculated and recorded in the third column. This total angle is double the deflection angle between the P. C. and the transit point. In the above notes there is but one intermediate transit point between the P. C. and P. T. The

deflection from P. C. at Sta. 3 + 20 to the intermediate transit point at Sta. 4 + 50 is $2^{\circ} 36'$. The total angle is double this deflection, or $5^{\circ} 12'$, which is recorded on the same line in the third column. The record of total angles at once indicates the stations at which transit points are placed. The total angle at the P. T. will be the same as the angle of intersection, if the work is correct. When the curve is finished, the transit is set up at the P. T., and the bearing at the forward tangent taken, which affords an additional check upon the previous calculations. The magnetic bearing is recorded in the fourth column, and the deduced or calculated bearing is recorded in the fifth column.

LEVELING.

Examples in Direct Leveling.—The principles of direct leveling are illustrated in the figure.

Let *A* be the starting point, which has a known elevation of 20 ft. The instrument is set at *B*, leveled up and sighted to a rod held at *A*. The target being set, the reading, 8.42 ft., called a *backsight*, is the distance that the point where the line of sight cuts the rod is above the point *A*, and is to be added to the elevation of the point *A*. $20.00 + 8.42 = 28.42$ is called the *height of instrument* and is designated by *H. I.* The instrument being turned in the opposite direction, a point *C* is chosen, which must be below the line of sight. This point is called a *turning point*, and is designated by the abbreviation T. P. Drive a peg at *C*, or take for a turning point a point of rock or some other permanent object upon which the rod is held. The reading at this point is a *foresight*, and is to be subtracted from the height of the instrument at *B* to find the elevation of the point at *C*.

Let the rod reading be 1.20 ft. As this reading is a *foresight*, it must be subtracted from 28.42, the height of instrument at *B*; $28.42 - 1.20 = 27.22$ ft., the elevation of the point *C*. The leveler carries the instrument to *D*, which should be of such a height above *C* that, when leveled up, the line of sight will cut the rod near the top. The *backsight* to *C* gives a reading of 11.56 ft., which, added to 27.22 ft., the elevation

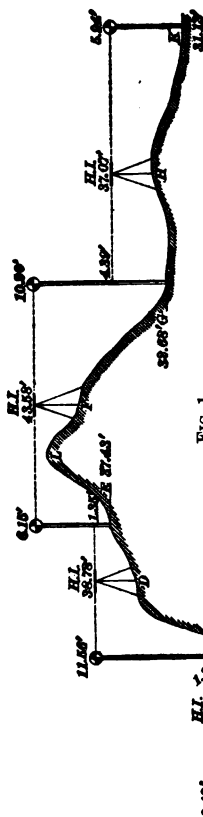


FIG. 1.

of *C*, gives 38.78 ft., the height of the instrument at *D*. The rodman then goes to *E*, a point where a foresight reading is 1.35, which, subtracted from 38.78, the H. I. at *D*, gives 37.43 ft., the elevation of *E*. The level is then set up at *F*, being careful that line of sight shall clear the hill at *L*. The back-sight, 6.15 ft., added to 37.43 ft., the elevation of *E*, gives 43.58 ft., the H. I. at *F*. The rod held at *G* gives a foresight of 10.90 ft., which, subtracted from 43.58 ft., the H. I. at *F*, gives 32.68 ft., the elevation at *G*. Again moving the level to *H*, the backsight to *G* of 4.39 ft., added to 32.68 ft., the elevation of *G*, gives 37.07 ft., the H. I. at *H*. Holding the rod at *K*, a foresight of 5.94, subtracted from 37.07, gives 31.13, the elevation of the point *K*. The elevation of the starting point *A* is 20.00 ft.; the elevation of the point *K* is found by direct leveling to be

31.13 ft., and the difference in the elevations of *A* and *K* is $31.13 - 20.00 = 11.13$ ft.; that is, the point *K* is 11.13 ft. higher than the point *A*.

Turning points previously mentioned are the points where back-sights and fore-

sights are taken. The backsights are plus (+) readings, and

are to be added; the foresights are minus (—) readings, and are to be subtracted. A point for a foresight having been determined, the rodman drives a peg firmly in the ground and holds the rod upon it. After the instrument is moved, set up, and a backsight taken, the peg is pulled up and carried in the pocket until another turning point is called for. Turning points should be taken at about equal distances from the instrument, in order to equalize any small errors in adjustment. In smooth country an ordinary level will permit of sights of from 300 to 500 ft.

To Keep Level Notes.—Many forms are used. The distinguishing feature of one of the best (see page 295) is a single column for all rod readings. The backsights being additive and the foresights subtractive readings, they are distinguished from other rod readings by the characteristic signs + (plus) and — (minus). The turning points, whose foresight readings are —, are further abbreviated T. P.

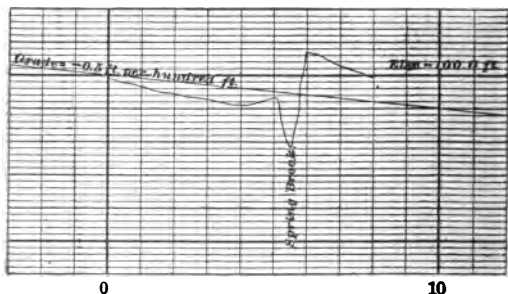
To Check Level Notes.—A well-known method of checking level notes provides for checking the elevations of turning points and heights of instrument only, which is sufficient, as all other elevations are deduced from them. The method depends on the fact that all backsights are additive (i. e. +) quantities, and all foresights are subtractive (i. e. —) quantities. The notes given on page 295 are checked as follows: The elevation of the bench mark at station 0 is 100.00 ft., to which all backsights, or + readings, are to be added and from this sum all foresights, or — readings, are to be subtracted. The sum of the backsights, with elevation of bench mark at 0, is 122.59. Sum of foresights is 24.27, and difference is 98.32 ft., the elevation of the turning point last taken. As soon as a page of level notes is filled, the notes should be checked and a check mark ✓ placed at the last height of instrument or elevation checked. When the work of staking out or cross-sectioning is being done, the levels should be checked at each bench mark on the line. After each day's work, the leveler must check on the nearest bench mark.

	+	—
Thus, 100.00	10.22	
5.61	2.52	
5.41	11.53	
11.57	24.27	
122.59		
24.27		
98.32		

1.	Station	Rod Reading.	Ht. Instrument.	Elevation.	Grade.	Cut.	Fill.	Remarks.	2.
	B. M.	+ 5.61	105.61	100.00				On root of white oak	Stump 10' L. Sta. 0.
	0	6.1		99.5					
	1	7.3		98.3					
	2	8.4		97.2					
	3	9.2		96.4					
	T. P.	-10.22		95.39					
		+ 5.41	100.80						
	4	6.3		94.5					
	5	4.2		96.6					
	5 + 50	11.5		89.3				Spring Brook.	
	T. P.	- 2.52		98.28					
		+11.57	109.85						
	6	6.2		103.6					
	7	8.5		101.3					
	8	10.1		99.7					
	T. P.	-11.53		98.32					

Profiles.—A profile represents a longitudinal projection of the line of survey. In it all abrupt changes in elevation are clearly outlined. Vertical and horizontal measurements are usually represented by different scales, to render irregularities of surface more distinct through exaggeration. For railroad work, profiles are commonly made to the following scales, viz., horizontal, 400 ft. = 1 in.; vertical, 20 ft. = 1 in.

A section of profile paper is shown in the following diagram. Every fifth horizontal line and every tenth vertical line is heavy. By the aid of these heavy lines, distances and elevations are quickly and correctly estimated and the work of platting greatly facilitated. The level notes



given in the preceding diagram are platted in the accompanying section. The elevation of some horizontal line is assumed. This elevation is, of course, referred to the datum plane, and is the base from which the other elevations are estimated. Every tenth station number is written at the bottom of the sheet under the heavy vertical lines. The profile is first platted in pencil and then inked in in black.

Grade Lines.—The principal use of a profile is to enable the engineer to establish a *grade line*, i. e., a line showing the slope of the road on which the amounts of excavation and embankment depend. The *rate* of a grade line is measured by the vertical rise or fall in each hundred feet of its length,

and is designated by the term *per cent.* Thus, a grade line that rises or falls 1 ft. in each hundred feet of its length is called an ascending or descending 1 per cent. grade, and is written + 1.0 or - 1.0 per hundred. A rise or fall of $\frac{1}{2}$ ft. in each hundred feet is called a 0.5 grade, and is written + 0.5 or - 0.5 per hundred. The grade line having been decided on, it is drawn in red ink.

EXAMPLE.—The elevation of station 20 is 140.0 ft.; between stations 20 and 100 there is an ascending grade of 75%. What is the elevation of the grade at station 71?

SOLUTION.—To obtain the elevation of the grade at station 71, we add to the elevation of the grade at station 20, or 140 ft., the total rise in grade between stations 20 and 71. Accordingly, $71 - 20 = 51$; $.75 \text{ ft.} \times 51 = 38.25 \text{ ft.}$; $140 \text{ ft.} + 38.25 \text{ ft.} = 178.25 \text{ ft.}$, the elevation of grade at station 71.

RADII AND CHORD AND TANGENT DEFLECTIONS.

The formulas used in the computation of the following table are as follows:

$$\text{For radius, } R = \frac{50}{\sin D}.$$

$$\text{For chord deflection, } d = \frac{c^2}{R}.$$

$$\text{For tangent deflection, tan deflection} = \frac{c^2}{2R}.$$

In these formulas, R is the radius of the curve, D is its deflection angle (equal to one-half the degree of curve), and c is the length of chord for which the chord or tangent deflection is to be determined. The chord and tangent deflections given in the table are computed for chords of 100 feet.

Thus, for a 6° curve the deflection angle is 3° , the sine of which is .052336. Hence, for the radius and chord deflection, we have

$$R = \frac{50}{.052336} = 955.37 \text{ ft.} \quad d = \frac{100^2}{955.37} = 10.467 \text{ ft.,}$$

as given in the table. The tangent deflection is always one-half the chord deflection.

TABLE OF RADII AND DEFLECTIONS.

Degree.	Radil.	Chord Deflection.	Tangent Deflection.	Degree.	Radil.	Chord Deflection.	Tangent Deflection.
0 5	68,754.94	.145	.073	3 25	1,677.20	5.962	2.981
10	34,377.48	.291	.145	30	1,637.28	6.108	3.054
15	22,918.33	.436	.218	35	1,599.21	6.253	3.127
20	17,188.76	.582	.291	40	1,562.88	6.398	3.199
25	13,751.02	.727	.364	45	1,528.16	6.544	3.272
30	11,459.19	.873	.436	50	1,494.95	6.689	3.345
35	9,822.18	1.018	.509	55	1,463.16	6.835	3.417
40	8,594.41	1.164	.582	4 0	1,432.69	6.980	3.490
45	7,639.49	1.309	.654	5	1,403.46	7.125	3.563
50	6,875.55	1.454	.727	10	1,375.40	7.271	3.635
55	6,250.51	1.600	.800	15	1,348.45	7.416	3.708
1 0	5,729.65	1.745	.873	20	1,322.53	7.561	3.781
5	5,288.92	1.891	.945	25	1,297.58	7.707	3.853
10	4,911.15	2.036	1.018	30	1,273.57	7.852	3.926
15	4,583.75	2.182	1.091	35	1,250.42	7.997	3.999
20	4,297.28	2.327	1.164	40	1,228.11	8.143	4.071
25	4,044.51	2.472	1.236	45	1,206.57	8.288	4.144
30	3,819.83	2.618	1.309	50	1,185.78	8.433	4.217
35	3,618.80	2.763	1.382	55	1,165.70	8.579	4.289
40	3,437.87	2.909	1.454	5 0	1,146.28	8.724	4.362
45	3,274.17	3.054	1.527	5	1,127.50	8.869	4.435
50	3,125.36	3.200	1.600	10	1,109.33	9.014	4.507
55	2,989.48	3.345	1.673	15	1,091.73	9.160	4.580
2 0	2,864.93	3.490	1.745	20	1,074.68	9.305	4.653
5	2,750.35	3.636	1.818	25	1,058.16	9.450	4.725
10	2,644.58	3.781	1.891	30	1,042.14	9.596	4.798
15	2,546.64	3.927	1.963	35	1,026.60	9.741	4.870
20	2,455.70	4.072	2.036	40	1,011.51	9.886	4.943
25	2,371.04	4.218	2.109	45	996.87	10.031	5.016
30	2,292.01	4.363	2.181	50	982.64	10.177	5.088
35	2,218.09	4.508	2.254	55	968.81	10.322	5.161
40	2,148.79	4.654	2.327	6 0	955.37	10.467	5.234
45	2,083.68	4.799	2.400	5	942.29	10.612	5.306
50	2,022.41	4.945	2.472	10	929.57	10.758	5.379
55	1,964.64	5.090	2.545	15	917.19	10.903	5.451
3 0	1,910.08	5.235	2.618	20	905.13	11.048	5.524
5	1,858.47	5.381	2.690	25	893.39	11.193	5.597
10	1,809.57	5.526	2.763	30	881.95	11.339	5.669
15	1,763.18	5.672	2.836	35	870.79	11.484	5.742
20	1,719.12	5.817	2.908	40	859.92	11.629	5.814

TABLE—(Continued).

Degree.	Radii.	Chord Deflection.	Tangent Deflection.	Degree.	Radii.	Chord Deflection.	Tangent Deflection.
° /				° /			
6 45	849.32	11.774	5.887	10 0	573.69	17.431	8.716
50	838.97	11.919	5.960	10	564.31	17.721	8.860
55	828.88	12.065	6.032	20	555.23	18.011	9.005
				30	546.44	18.300	9.150
7 0	819.02	12.210	6.105	40	537.92	18.590	9.295
5	809.40	12.355	6.177	50	529.67	18.880	9.440
10	800.00	12.500	6.250				
15	790.81	12.645	6.323	11 0	521.67	19.169	9.585
20	781.84	12.790	6.395	10	513.91	19.459	9.729
25	773.07	12.936	6.468	20	506.38	19.748	9.874
30	764.49	13.081	6.540	30	499.06	20.038	10.019
35	756.10	13.226	6.613	40	491.96	20.327	10.164
40	747.89	13.371	6.685	50	485.05	20.616	10.308
45	739.86	13.516	6.758				
50	732.01	13.661	6.831	12 0	478.34	20.906	10.453
55	724.31	13.806	6.903	10	471.81	21.195	10.597
				20	465.46	21.484	10.742
8 0	716.78	13.951	6.976	30	459.28	21.773	10.887
5	709.40	14.096	7.048	40	453.26	22.063	11.031
10	702.18	14.241	7.121	50	447.40	22.352	11.176
15	695.09	14.387	7.193				
20	688.16	14.532	7.266	13 0	441.68	22.641	11.320
25	681.35	14.677	7.338	10	436.12	22.930	11.465
30	674.69	14.822	7.411	20	430.69	23.219	11.609
35	668.15	14.967	7.483	30	425.40	23.507	11.754
40	661.74	15.112	7.556	40	420.23	23.796	11.898
45	655.45	15.257	7.628	50	415.19	24.085	12.043
50	649.27	15.402	7.701				
55	643.22	15.547	7.773	14 0	410.28	24.374	12.187
				10	405.47	24.663	12.331
9 0	637.27	15.692	7.846	20	400.78	24.951	12.476
5	631.44	15.837	7.918	30	396.20	25.240	12.620
10	625.71	15.982	7.991	40	391.72	25.528	12.764
15	620.09	16.127	8.063	50	387.34	25.817	12.908
20	614.56	16.272	8.136				
25	609.14	16.417	8.208				
30	603.80	16.562	8.281	15 0	383.06	26.105	13.053
35	598.57	16.707	8.353	10	378.88	26.394	13.197
40	593.42	16.852	8.426	20	374.79	26.682	13.341
45	588.36	16.996	8.498	30	370.78	26.970	13.485
50	583.38	17.141	8.571	40	366.86	27.258	13.629
55	578.49	17.286	8.643	50	363.02	27.547	13.773

TABLE—(Continued).

Degree.	Radli.	Chord Deflection.	Tangent Deflection.	Degree.	Radli.	Chord Deflection.	Tangent Deflection.
° /				° /			
16 0	359.26	27.835	13.917	18 10	316.71	31.574	15.787
10	355.59	28.123	14.061	20	313.86	31.861	15.931
20	351.98	28.411	14.205	30	311.06	32.149	16.074
30	348.45	28.699	14.349	40	308.30	32.436	16.218
40	344.99	28.986	14.493	50	305.60	32.723	16.361
50	341.60	29.274	14.637	19 0	302.94	33.010	16.505
17 0	338.27	29.562	14.781	10	300.33	33.296	16.648
10	335.01	29.850	14.925	20	297.77	33.583	16.792
20	331.82	30.137	15.069	30	295.25	33.870	16.935
30	328.68	30.425	15.212	40	292.77	34.157	17.078
40	325.60	30.712	15.356	50	290.33	34.443	17.222
50	322.59	31.000	15.500	20 0	287.94	34.730	17.365
18 0	319.62	31.287	15.643				

RETAINING WALLS.

On the Theory of Retaining Walls.—Let $abcd$, Fig. 1, be a retaining wall with battered face and vertical back. The top bc of the backing is level with the top of the wall. Let de represent the natural slope of the material composing the filling, viz., $1\frac{1}{2}$ horizontal to 1 vertical, which is the average of materials used for back filling.

It is assumed that the wall $abcd$ is heavy enough to resist sliding along its base and that it can fail only by overturning,

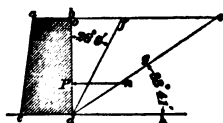


FIG. 1.

i. e., rotating about its toe c . Now, if the angle odc (between the vertical line od drawn from the inner bottom edge of the wall and the natural slope de) be bisected by the line df , the angle odf is called the *angle*, and the line df the *slope*, of *maximum pressure*. The triangular prism of earth odf is called the *prism of maximum pressure*, because, if considered as a

wedge acting against the back of the wall, it would exert a greater pressure against it than would the entire triangle ode of earth considered as a single wedge. For though the latter is more than double the weight of the former, yet it receives much greater support from the underlying earth. It has been proved by experiment that, if the triangle of earth ode is divided by any line df into wedges, the wedge that will press most against the wall is that formed when the line df divides the angle ode into two equal parts.

The angle odh formed by the vertical od and the horizontal dh is 90° . The angle of natural slope hde is $33^\circ 41'$; hence, the angle odf of maximum pressure is equal to $(90^\circ - 33^\circ 41') \div 2 = 28^\circ 09'$.

In making calculations, only *one foot* of the length of wall and of the *backing* is taken, so all that is necessary is to take the area of the section of the wall and backing. The material composing the backing is supposed to be perfectly dry and to possess no cohesive power, which is practically true of pure sand.

If we conceive the wall $abdc$, Fig. 1, to be suddenly removed, the triangle bdf of sand included between the line of maximum pressure df and the vertical back bd of the wall would slide downward, impelled by a force nP , acting in a direction nP at right angles to the side bd of the triangle, i. e., at right angles to the vertical back bd of the wall; the center of pressure being at P one-third of the distance between b and d measured from the bottom of the wall d . The amount of this force nP is:

$$\text{Perpendicular pressure} = \frac{\text{Wt. of triangle of earth } bdf \times of}{\text{vertical depth } od}.$$

This formula not only applies to walls with vertical backs, as in Fig. 1, but to those with inclined backs, as in Fig. 2, for inclinations as high as 6 in. horizontal to 1 ft. vertical, which is rarely met with and never exceeded.

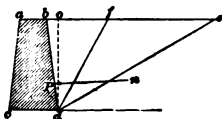


FIG. 2.

Friction Caused by Pressure of Backing.—If all the backing

hP measured to the same scale will give us the amount of their resultant, which is approximately the *single theoretical force* both in *amount* and *direction* that the wall has to resist. This force includes the wall friction. The force hP is always equal to the perpendicular force nP , divided by the cosine of the angle of wall friction. The cosine of the angle of wall friction is .832 and the value of the force hP may be expressed in the following formula:

Approximate theoretical pressure

$$= hP = \frac{\text{weight of triangle } bdf \times of}{\text{vertical height } od \times .832}.$$

When the back of the wall does not incline forward more than 6 in. horizontal to 1 ft. vertical, equal to an angle of about $26^{\circ} 34'$, the following formula by Trautwine is used, viz.:

Approximate theoretical pressure

$$= hP = \text{weight of triangle } bdf \times .643,$$

which includes friction of earth against the back of the wall.

To Find the Overturning and Resisting Forces.—*To find the overturning tendency of the earth pressure and the resistance of the wall against being overturned about its toe c , as a fulcrum (see Fig. 3).* Find the center of gravity g of the wall, and through g draw the vertical line gi . Produce the line of pressure hP , and draw cv at right angles to this line. To any convenient scale, lay off lt equal to the weight of the wall and to the same scale lm equal to the pressure hP . Complete the parallelogram $lmst$. The diagonal ls will be the resultant of the pressure and the weight of the wall. The stability of the wall will increase as the distance cr from the toe to the point where the resultant ls cuts the base, increases. To insure stability, cr must be greater than $\frac{1}{2}cd$.

The pressure hP , if multiplied by its leverage cv , will give the moment of the pressure about c , and the weight of the wall lt , multiplied by its leverage cr , will give the moment of the wall. The wall is secure against overturning in proportion as its moment exceeds that of the pressure.

For example, let the height of the wall $abcd$, in Fig. 3, be 9 ft.; the thickness at the base cd , 4.5 ft., and at the top ab , 2 ft.; and the batter of ac be 1 in. to the foot. The triangle of earth bdf has a base $bf = 6.57$ ft. and altitude $do = 9$ ft.

Taking the section as 1 ft. in thickness, we have the contents equal to $6.57 \times 9 \div 2 = 29.56$ cu. ft. Assuming the material to weigh 120 lb. per cu. ft., the weight of the triangle bdf is $29.56 \times 120 = 3,547$ lb.; $of = 4.81$ ft. $3,547 \times 4.81 = 17,061$. $17,061 \div od = 1,895.7$ lb. = the perpendicular pressure nP . Lay off on a line perpendicular to the back of the wall at P , to a scale of 2,000 lb. = 1 in., $nP = 1,895.7 \div 2,000 = .948$ in., the perpendicular pressure. Draw Ph , making the angle $nPh = 33^\circ 41'$. Draw nh intersecting hP in h ; then will nh to the same scale equal the friction of the earth against the back of the wall. Completing this parallelogram, $nhkP$, the diagonal $hP = 1,189$ in., which, to a scale of 2,000 lb. = 1 in., amounts to 2,278 lb., and is the resultant of the pressure and the friction.

Produce the resultant hP to u . We next find the center of gravity g of the wall $abcd$. The section of the wall is a trapezoid, and the center of gravity g is readily found as follows: Produce the upper base of the section to x , and make $ax = cd = 4.5$ ft. Then produce the lower base in the opposite direction to y , and make $dy = ab = 2$ ft. Join x and y . Find the middle points x' and y' of the upper and lower bases of the section. Join these points. The intersection g of the lines xy and $x'y'$ is the center of gravity of the trapezoid $abcd$.

The volume of the section of wall $abcd$ is readily found. The sum of top and bottom widths = $2.0 + 4.5 = 6.5$ ft. $6.5 \div 2 = 3.25$ ft. $3.25 \times 9 = 29.25$ cu. ft. $29.25 \times 154 = 4,504$ lb. (the weight per cubic foot of good mortar rubble = 154 lb.) = the weight of the section $abcd$. Draw through g a vertical line gi , and lay off on it, to a scale of 2,000 lb. to the inch, from the point l , where the line of gravity intersects the prolongation of the line of pressure hP , the length lt equal to 4,504 lb., the weight of the wall. Lay off from l on the prolongation of hP , lm equal to 2,278 lb. to the same scale. Complete the parallelogram $lmst$. The diagonal ls represents the resultant of the pressure and of the weight of the wall. The distance cr from the toe c to the intersection of the resultant ls with the base cd is more than one-third of the width of the base, which insures ample stability.

Pressure of the Backing on Surcharged Walls.—In Fig. 4 the surcharge of backing mbo slopes from b at its natural slope, and attains its maximum pressure where the slope of maximum pressure dk intersects the natural slope bm at f . Any additional height of surcharge does not increase this pressure. If the surcharge slopes from a , as shown by the line ap , or from any point between a and b , then the slope of maximum pressure must be extended, intersecting the slope from a in the

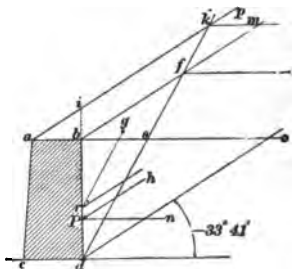


FIG. 4.

point k . The prism of maximum pressure will then be dik . The triangle of earth abi on the top of the wall exerts no pressure against the back of the wall, but adds to its stability.

Having found the weight of the triangle $bd f$, we have
approximate pressure = weight of triangle $bd f \times .643$,
 which includes the pressure of the backing and the friction of the earth against the back of the wall.

Draw Pn perpendicular to the back of the wall and draw hP making the angle $nPh = 33^\circ 41'$, the angle of wall friction. Then, hP will be the direction of the pressure. The point of application of this pressure will not always be at P , one-third of the height of bd measured from d , but above P , as at r , where a line drawn from the center of gravity g of the prism of maximum pressure dik (omitting any earth resting directly upon the top of the wall), and parallel to the line dk of maximum pressure, cuts the back bd of the wall. The center of pressure P will be at one-third the height of the wall when the sustained earth $db s$ or $db f$ forms a complete triangle, one of whose angles is at b , the inner top edge of the wall. For all other surcharges, the point of pressure will be above P .

TUNNEL SECTIONS.

Tunnel sections vary somewhat, according to the material to be excavated, but the general form and dimensions are much the same.

*Section of
Double-Track Tunnel.*

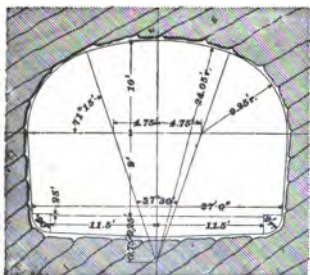


FIG. 1.

*Section of
Single-Track Tunnel.*

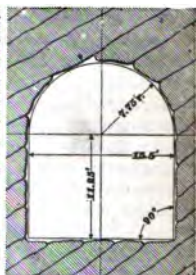


FIG. 2.

The general dimensions are as follows: For double track, from 22 to 27 ft. wide and from 21 to 24 ft. high, and for single track, from 14 to 16 ft. wide and from 17 to 20 ft. high (see Figs. 1 and 2).

In seamy or rotten rock the section is sufficiently enlarged to receive a lining of substantial rubble or brick masonry laid in good cement mortar. When the material has not sufficient consistency to sustain itself until the masonry lining is built, resort is had to timbering, which furnishes the necessary support.

CALCULATION OF 'EARTHWORK.

In calculating the quantity of material in excavation and embankment, two general methods are used, namely, the *end-area formula* and the *prismoidal formula*.

Calculation by the end-area method consists in multiplying the mean, or average, area in square feet of two consecutive sections by the distance in feet between them. Thus,

let A represent the area in square feet of one section; B , the area in square feet of the next section; C , the number of feet between the sections; and D , the total number of cubic feet in the prismoid lying between these sections. Then,

$$D = \frac{A+B}{2} \times C, \text{ approximately.}$$

The distance between sections should not be more than 100 ft., and should be less if the surface of the ground is irregular.

A more accurate result is obtained by the use of the prismoidal formula. In applying the prismoidal formula to the calculation of cubic contents, it is requisite to know the middle cross-section between each two that are measured on the ground. The dimensions of this middle section are the means of the dimensions of the end sections.

Calling one of the given sections A , the other B , the middle (not the mean) section M , the distance between the sections L , and the required contents S , we have, by the prismoidal formula,

$$S = \frac{L}{6} (A + 4M + B).$$

EXAMPLE.—Two sections are represented by Figs. 1 and 2, and are denoted by the letters A and B . The perpendicular distance between them is 50 ft. It

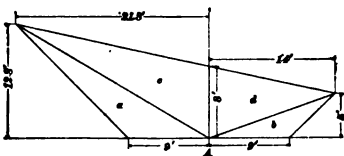


FIG. 1.

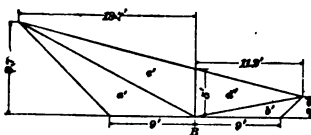


FIG. 2.

is required to find the cubical contents of the prismoid

SOLUTION.—The section given in Fig. 1 is composed of the four triangles a , b , c , and d . The triangles a and b have equal bases of 9 ft., the half width of the roadway; hence, if we

take half the sum of their altitudes and multiply it by the common base we shall have the sum of the areas of the triangles *a* and *b*.

The triangles *c* and *d* have a common base 8 ft., the center cut of the section, and if we take the half sum of the side distances and multiply it by 8 ft., we shall obtain the areas of the triangles *c* and *d*. Taking the dimensions of section *A* given in Fig. 1, we have

$$\text{Areas of triangles } a + b = \frac{12.8 + 5}{2} \times 9 = 80.1 \text{ sq. ft.}$$

$$\text{Areas of triangles } c + d = \frac{21.8 + 14}{2} \times 8 = 143.2 \text{ sq. ft.}$$

$$\text{Total area of section } A = 223.3 \text{ sq. ft.}$$

Taking the dimensions of the section *B* given in Fig. 2, we have

$$\text{Areas of triangles } a' + b' = \frac{9.7 + 2.2}{2} \times 9 = 53.55 \text{ sq. ft.}$$

$$\text{Areas of triangles } c' + d' = \frac{18.7 + 11.2}{2} \times 5 = 74.75 \text{ sq. ft.}$$

$$\text{Total area of section } B = 128.3 \text{ sq. ft.}$$

In applying the prismoidal formula we calculate the area of a section midway between the given sections, and for its

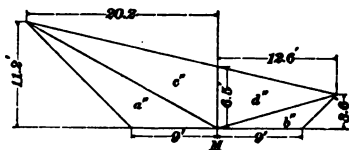


FIG. 3.

dimensions we take the mean of the dimensions of the given sections. These dimensions will be as follows:

$$\text{Center cut, } \frac{8 + 5}{2} = 6.5 \text{ ft.}$$

$$\text{Right-side distance, } \frac{14 + 11.2}{2} = 12.6 \text{ ft.}$$

$$\text{Left-side distance, } \frac{21.8 + 18.7}{2} = 20.25 \text{ ft.}$$

With dimensions thus found, construct the section M shown in Fig. 3.

The area of section M is computed by the same method as that used with sections A and B in Figs. 1 and 2, and is as follows:

$$\text{Area of triangles } a'' + b'' = \frac{11.2 + 3.6}{2} \times 9 = 66.6 \text{ sq. ft.}$$

$$\text{Area of triangles } c'' + d'' = \frac{20.2 + 12.6}{2} \times 6.5 = 106.6 \text{ sq. ft.}$$

$$\text{Total area of section } M = 173.2 \text{ sq. ft.}$$

Denoting the distance between the sections by L and the cubical contents of the prismoid by S , we have, by substituting in the prismoidal formula,

$$S = \frac{L}{6} (A + 4M + B).$$

$$S = \frac{50}{6} (223.3 + 4 \times 173.2 + 128.3) = 8.703 \text{ cu. ft.} = 322.3 \text{ cu. yd.}$$

TRACKWORK.

Curving Rails.—When laying track on curves, in order to have a smooth line, the rails themselves must conform to the curve of the center line. To accomplish this, the rails must be curved. The curving should be done with a rail bender or with a lever, preferably with the former.

To guide those in charge of this work, a table of middle and quarter ordinates for a 30-ft. rail for all degrees of curve should be prepared.

The following table of middle ordinates for curving rails is calculated by using the formula

$$m = \frac{c^2}{8R},$$

in which m = middle ordinate;

c = chord, assumed to be of the same length as the rail;

R = radius of the curve.

The results obtained by this formula are not theoretically correct, yet the error is so small that it may be ignored in practical work.

In curving rails, the ordinate is measured by stretching a cord from end to end of the rail against the gauge side, as shown in Fig. 1. Suppose the rail AB is 30 ft. in length, and



FIG. 1.

the curve 8° . Then, by the previous problem, the middle ordinate at a should be $1\frac{1}{8}$ in. To insure a uniform curve to the rails, the ordinates at the quarters b and b' should be tested. In all cases the quarter ordinates should be three-quarters of the middle ordinate. In Fig. 1, if the rail has been properly curved, the quarter ordinates at b and b' will be $\frac{3}{4} \times 1\frac{1}{8}$ in. = $1\frac{1}{4}$ in.

MIDDLE ORDINATES FOR CURVING RAILS.

Degree of Curve.	Length of Rail.					
	30 ft.	28 ft.	26 ft.	24 ft.	22 ft.	20 ft.
	in.	in.	in.	in.	in.	in.
1	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
3	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
4	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
5	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
6	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$
7	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{2}$
8	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
9	$2\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
10	$2\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
11	$2\frac{1}{8}$	$2\frac{1}{4}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
12	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
13	$3\frac{1}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
14	$3\frac{1}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
15	$3\frac{1}{8}$	$3\frac{1}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
16	$3\frac{1}{4}$	$3\frac{1}{4}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$\frac{1}{2}$
17	4	$3\frac{1}{2}$	3	$2\frac{1}{8}$	$2\frac{1}{8}$	$\frac{1}{2}$
18	$4\frac{3}{8}$	$3\frac{1}{4}$	$3\frac{3}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$\frac{1}{2}$
19	$4\frac{7}{8}$	$3\frac{7}{8}$	$3\frac{7}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$\frac{1}{2}$
20	$4\frac{1}{2}$	$4\frac{1}{8}$	$3\frac{3}{8}$	3	$2\frac{1}{8}$	$\frac{1}{2}$

In trackwork it is often necessary to ascertain the degree of a curve, though no transit is available for measuring it. The following table contains the middle ordinates of a 1° curve for chords of various lengths:

The lengths of the chords are varied, so that a longer or shorter chord may be used, according as the curve is regular or not.

The table is applied as follows: Suppose the middle ordinate of a 44-ft.

Length of Chord. Feet.	Middle Ordinate of a 1° Curve. Inches.
20	$\frac{1}{8}$
30	$\frac{1}{4}$
44	$\frac{1}{2}$
50	$\frac{3}{8}$
62	1
100	$2\frac{1}{2}$
120	$3\frac{1}{4}$

chord is 3 in. We find in the table that the middle ordinate of a 44-ft. chord of a 1° curve is $\frac{1}{2}$ in. Hence, the degree of the given curve is equal to the quotient of $3 \div \frac{1}{2} = 6^\circ$ curve.

Elevation of Curves.—To counteract the centrifugal force developed when a car passes around a curve, the outer rail is elevated. The amount of elevation will depend on the radius of the curve and the speed at which trains are to be run. There is, however, a limit in track elevation as there is a limit in widening gauge, beyond which it is not safe to pass.

The best authorities on this subject place the maximum elevation at one-seventh the gauge, or about 8 in. for standard gauge of 4 ft. 8 $\frac{1}{2}$ in. The gauge on a 10° curve elevated for a speed of 40 miles an hour should be widened to 4 ft. 9 $\frac{1}{2}$ in.

All curves, when possible, should have an elevated approach on the straight main track, of such length that trains may pass on and off the curve without any sudden or disagreeable lurch.

A good rule for curve approaches is the following: For each half inch or fraction thereof of curve elevation, add 30 ft., for 1 rail length, to the approach; that is, if a curve has an elevation of 2 in., the approach will have as many rail lengths as the number of times $\frac{1}{2}$ is contained in 2, or 4. The approach will, therefore, have a length of 4 rails of 30 ft. each, or 120 ft.

The following table for elevation of curves is a compromise between the extremes recommended by different engineers. It is a striking fact that experienced trackmen never elevate track above 6 in. and many of them place the limit at 5 in.

Degree of Curve.	Length of Approach. Feet.	Elevation. Inches.	Width of Gauge.	Speed of Train. Miles per Hour.
1	60	1	4' 8 $\frac{1}{2}$ "	60
2	120	2	4' 8 $\frac{1}{2}$ "	60
3	150	2 $\frac{1}{2}$	4' 8 $\frac{1}{2}$ "	60
4	180	2 $\frac{3}{4}$	4' 8 $\frac{1}{2}$ "	55
5	180	3	4' 8 $\frac{1}{2}$ "	50
6	210	3 $\frac{1}{4}$	4' 8 $\frac{1}{2}$ "	45
7	210	3 $\frac{1}{2}$	4' 9"	40
8	240	3 $\frac{3}{4}$	4' 9"	35
9	240	4	4' 9"	30
10	270	4 $\frac{1}{4}$	4' 9"	25
11	270	4 $\frac{1}{2}$	4' 9 $\frac{1}{4}$ "	20
12	270	4 $\frac{3}{4}$	4' 9 $\frac{1}{4}$ "	15
13	240	4 $\frac{1}{2}$	4' 9 $\frac{1}{4}$ "	10
14	240	4 $\frac{1}{4}$	4' 9 $\frac{1}{4}$ "	10
15	240	4	4' 9 $\frac{1}{4}$ "	10
16	240	4	4' 9 $\frac{1}{2}$ "	10

The Elevation of Turnout Curves.—The speed of all trains in passing over turnout curves and crossovers is greatly reduced, so that an elevation of $\frac{1}{2}$ in. per degree is amply sufficient for all curves under 16°. On curves exceeding 16°, the elevation may be held at 4 in. until 20° is reached, and on curves extending 20°, $\frac{3}{4}$ in. of elevation per degree may be allowed until the total elevation amounts to 5 in., which is sufficient for the shortest curves.

The Frog.—The frog is a device by means of which the rail at the turnout curve crosses the rail of the main track. The frog shown in Fig. 2 is made of rails having the same cross-section as those used in the track. The wedge-shaped part *A* is the *tongue*, of which the extreme end *a* is the *point*. The space *b*, between the ends *c* and *d* of the rails, is the *mouth*, and the channel that they form at its narrowest point *e* is the *throat*. The curved ends *f* and *g* are the *wings*.

That part of the frog between A and A' is called the *heel*. The width h of the frog is called its *spread*. Holes are drilled

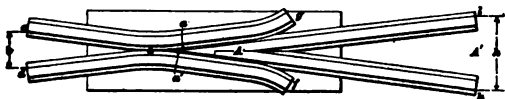


FIG. 2.

in the ends of the rails c , d , k , and l to receive the bolts used in fastening the rail splces, so that the rails of which the frog is composed form a part of the continuous track.

The Frog Number.—The number of a frog is the ratio of its length to its breadth; i. e., the quotient of its length divided by its breadth.

Thus, in Fig. 2, if the length $a'l$, from point to heel of frog is 5 ft., or 60 in., and the breadth h of the heel is 15 in., the number of the frog is the quotient of $60 \div 15 = 4$. Theoretically, the length of the frog is the distance from a to the middle point of a line drawn from k to l ; practically, we take from a to l as the distance. As it is often difficult to determine the exact point a of the frog, a more accurate method of determining the frog number is to *measure the entire length dl of the frog from mouth to heel, and divide this length by the sum of the mouth width b and the heel width h . The quotient will be the exact number of the frog.*

For example, if, in Fig. 2, the total length dl of the frog is 7 ft. 4 in., or 88 in., and the width h is 15 in., and the width b of the mouth is 7 in., then the frog number is $88 \div (15 + 7) = 4$. Frogs are known by their *numbers*. That in Fig. 2 is a No. 4 frog.

The Frog Angle.—The frog angle is the angle formed by the gauge lines of the rails, which form its tongue. Thus, in Fig. 2, the frog angle is the angle $la'k$. The amount of the angle may be found as follows: The tongue and heel of

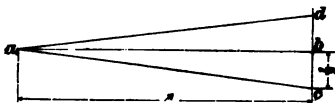


FIG. 3.

the frog form an isosceles triangle (see Fig. 3). By drawing a line from the point *a* of the frog to the middle point *b* of the heel *cd*, we form a right-angled triangle, right-angled at *b*. The perpendicular line *ab* bisects the angle *a*, and, by trigonometry, we have $\tan \frac{1}{2} a = \frac{bc}{ab}$. The dimensions of the frog point given in Fig. 3 are not the same as those given in Fig. 2, but their relative proportions are the same, viz., the length is four times the breadth. The length $ab = 4$ and the width $cd = 1$; hence, $bc = \frac{1}{2}$. Substituting these values, we have $\tan \frac{1}{2} a = \frac{\frac{1}{2}}{4} = \frac{1}{8} = .125$. Whence, $\frac{1}{2} a = 7^{\circ} 7\frac{1}{2}'$ and

$a = 14^{\circ} 15'$; that is, the angle of a No. 4 frog is $14^{\circ} 15'$.

Frog numbers run from 4 to 12, including half numbers, the spread of the frog increasing as the number decreases.



FIG. 4.

The Parts of a Turnout.—The several parts of a turnout are represented in Fig. 4. The distance *pf* from the P. C. of the turnout curve to the point of frog is called the *frog distance*. The radius *co* of the turnout curve, the frog distance, the

frog angle, and the frog number bear certain relations to one another, which are expressed by the following formulas:

Tangent of half frog angle = gauge + frog distance.

Frog number = $\sqrt{\text{radius } co \div \text{twice the gauge}}$.

Frog number = $1 \div \frac{1}{2} \text{ the tangent of } \frac{1}{2} \text{ the frog angle}$.

Radius *co* = twice the gauge $\times$ square of the frog number.

Radius *co* = (frog distance *pf* $\div$ sine of frog angle) $-\frac{1}{2}$ the gauge.

Radius *co* = gauge $\div$ (1 - cosine of frog angle) $-\frac{1}{2}$ the gauge.

Frog distance *pf* = frog number $\times$ twice the gauge.

Frog distance *pf* = gauge *pq* $\div$ tangent of $\frac{1}{2}$ the frog angle.

Frog distance *pf* = (radius *co* + half the gauge) $\times$ sine of frog angle.

Middle ordinate (approximate) = $\frac{1}{4}$ the gauge.

Each side ordinate (approximate) = $\frac{1}{8}$ the middle ordinate
= $\frac{1}{32}$ (or .188) of the gauge.

Switch length (approximate) =

$$\sqrt{\frac{\text{throw in feet} \times 10,000}{\tan \text{ deflection for chords of 100 ft. for radius } co \text{ of turnout curve}}}$$

The tangent deflection may be obtained from the table on pages 298-300.

TURNOUTS FROM A STRAIGHT TRACK.

Gauge, 4 ft. 8½ in. Throw of switch, 5 in.

Frog Number.	Frog Angle.	Turnout Radius.	Degree of Turnout Curve.	Frog Distance.	Middle Ordinate.	Side Ordinate.	Stub Switch Length.
	° ' "	Feet.	° ' "	Feet.	Feet.	Feet.	Feet.
12	4 46	1,356	4 14	113.0	1.177	.883	34
11½	4 58	1,245	4 36	108.3	1.177	.883	32
11	5 12	1,139	5 02	103.6	1.177	.883	31
10½	5 28	1,038	5 31	98.9	1.177	.883	29
10	5 44	942	6 05	94.2	1.177	.883	28
9½	6 02	850	6 45	89.5	1.177	.883	27
9	6 22	763	7 31	84.7	1.177	.883	25
8½	6 44	680	8 26	80.0	1.177	.883	24
8	7 10	603	9 31	75.3	1.177	.883	22
7½	7 38	530	10 50	70.6	1.177	.883	21
7	8 10	461	12 27	65.9	1.177	.883	20
6½	8 48	398	14 26	61.2	1.177	.883	18
6	9 32	339	16 58	56.5	1.177	.883	17
5½	10 24	285	20 13	51.8	1.177	.883	15
5	11 26	235	24 32	47.1	1.177	.883	14
4½	12 40	191	30 24	42.4	1.177	.883	13
4	14 14	151	38 46	37.7	1.177	.883	11

The switch lengths in the above table merely denote the *shortest* length of *stub switch* that will at the same time form part of the turnout curve, and give 5 in. throw. *Point or split switches* require a throw of not more than 3½ in., though many have a throw of 5 in., with an equal space between the gauge lines at the heel. The heels of a split switch, which occupy the same position as the toes of a stub switch, should

be placed at the point where the tangent deflection or offset is 5 in. The point where the tangent deflection is but $4\frac{1}{4}$ in. will answer for many rail sections, but for those above 65 lb. per yd., 5 in. should be taken.

In the table on pages 298-300, tangent deflections for chords of 100 ft. are given for all curves up to 20° ; and for a curve of higher degree, the tangent deflection may be found by applying the formula $\tan \text{ deflection} = \frac{c^2}{2R}$.

In complicated trackwork, where space is limited, curves must be chosen to meet the existing conditions, and not with reference to particular frog angles, in which case the frogs are called *special* frogs and are made to fit the particular curve used. The determination of the frog distance, switch length, and frog angle may be understood by referring to Fig. 5.

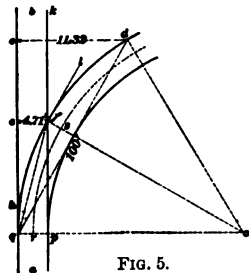


FIG. 5.

Let the main track ab be a straight line; the gauge $pq = 4 \text{ ft. } 8\frac{1}{4} \text{ in.} (= 4.71 \text{ ft.})$; the degree of the turnout curve $= 13^\circ$; the chord $qd = 100 \text{ ft.}$; cd = the tangent deflection of the chord qd ; and pf = the frog distance. From the table on page 299, we find the

tangent deflection for a chord 100 ft. long of a 13° curve is 11.32 ft. Then, from Fig. 5, we have the proportion

$$cd : ef = \overline{qc}^2 : \overline{qe}^2.$$

Now, in curves of large radius, qc and qd are assumed to be equal. Also, $qe = pf$, the frog distance, and substituting these equivalents we have the proportion

$$cd : ef = \overline{qd}^2 : \overline{pf}^2.$$

Substituting the above given quantities in the proportion, we have

$$11.32 : 4.71 = 100^2 : \overline{pf}^2;$$

whence,

$$\overline{pf}^2 = \frac{100^2 \times 4.71}{11.32},$$

and the frog distance, $pf = 64.5 \text{ ft.}$

If the space between the gauge lines at the heels of a split switch be taken at 5 in. = .42 ft., the distance from the P. C. of the turnout curve to the heel of the switch may be found as follows:

In Fig. 5, let h , the tangent offset at the heel of the switch = .42 ft., we have the proportion

$$cd : h = \overline{q} \overline{d}^2 : \overline{q} h^2,$$

and substituting known values, we have

$$11.32 : .42 = 100^2 : \overline{q} h^2,$$

whence,
$$\overline{q} h^2 = \frac{10,000 \times .42}{11.32} = 371.02,$$

and
$$\overline{q} h = 19.26 \text{ ft.}$$

This locates the heel of a *split switch* and the toe of a *stub switch*.

The *frog angle* is the angle kfl (see Fig. 5) formed by the gauge line of the main rail fk and the tangent to the outer rail qf of the turnout curve at the point where the two rails intersect. This angle is equal to the central angle qof . The arcs qf and rs are assumed to be of the same length. The turnout curve being 13° , the central angle for a chord of 1 ft.

is $\frac{13 \times 60}{100} = 7.8'$, and the central angle for 64.5 ft. the *frog distance*, is $7.8' \times 64.5 = 8^\circ 23'$, the frog angle for a 13° curve. By this process the frog distance, switch length, and frog angle may be calculated for curves of any radius.

To Lay Out a Turnout From a Curved Main Track.—There are two cases:

CASE I.—When the two curves deflect in opposite directions, illustrated in Fig. 6.

CASE II.—When the two curves deflect in the same direction, illustrated in Fig. 7.

In Fig. 6, the curve ab is $3^\circ 30'$, and it is proposed to use a No. 8 frog. By reference to the table on page 315, we find that the degree of curve corresponding to a No. 8 frog is $9^\circ 31'$. Accordingly, we use a turnout curve ac , whose degree when added to the degree of curve of the main track shall equal the degree required for a No. 8 frog; i. e., we use a 6° turnout curve, which is within 1 minute of the required degree, and close enough for practical purposes. We know that for

curves of moderate radii, i. e., from 1° up to 12° , the tangent deflections or offsets increase as the degree of the curve. That is, the tangent deflection of a 2° , 4° , and 6° is two, four, and six times, respectively, that of a 1° curve. In the accompanying cuts illustrating the location of frogs and switches, each curve is represented by two lines indicating the rails, whereas only the center lines of the curves are run in on the ground. In Fig. 6, the line cd is tangent to the center lines of the curves. These center lines do not appear in the cut.

Again referring to Fig. 6, if a tangent cd be drawn at c , the point common to the center lines of the curves, the sum of the deflections of both curves from the common tangent will be equal, in this case, to the tangent deflection

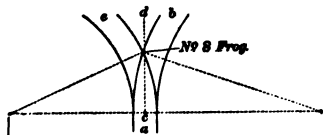


FIG. 6.

tion of a $9^\circ 30'$ curve from a straight line.

Accordingly, to find the frog distance for a 6° turnout curve from a $3^\circ 30'$ curve, the curves being in opposite directions, as shown in Fig. 6, we find the tangent deflection of a $9^\circ 30'$ curve for a chord of 100 ft. This deflection is 8.28 ft., as given in the table on page 299.

Assuming the gauge of track to be standard, viz., 4 ft. 8½ in. = 4.71 ft., and denoting the required frog distance by x , we have the following proportion:

$$8.28 : 4.71 = 100^2 : x^2,$$

$$\text{whence, } x^2 = \frac{10,000 \times 4.71}{8.28} = 5,688.4,$$

and the frog distance, $x = 75.42$ ft.

We use the tangent deflection for a $9^\circ 30'$ curve, which very nearly equals the tangent deflection for a $9^\circ 31'$ curve, thus saving the labor of a calculation; this will not appreciably affect the result.

We locate the heel of the switch in the same way, using for the second term of the proportion, .42 ft., the distance between the gauge lines at the heel, instead of 4.71 ft., the gauge of the track.

In Fig. 7, which comes under Case II, both curves deflect in the same direction, and the rate of their deflection from each other is equal to the rate of the deflection of a curve whose degree is equal to the difference of the degrees of the two curves from a tangent.

Let the main-track curve ab be 5° , and the turnout curve ac be 10° . Then, the rate of deflection or divergence of the 10° curve from the 5° curve equals the divergence of a $(10^\circ - 5^\circ) = 5^\circ$ curve from a straight track or tangent.

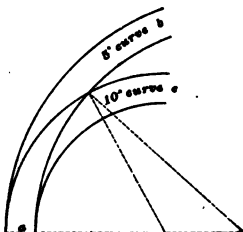


FIG. 7.

Accordingly, we find, in the table on page 298, the tangent deflection for a 5° curve for a chord of 100 ft. = 4.36 ft. Denoting the required frog distance by x , we have the following proportion: $4.36 : 4.71 = 100^2 : x^2$,

$$\text{whence, } x^2 = \frac{10,000 \times 4.71}{4.36} = 10,802.8,$$

and the frog distance, $x = 103.9$ ft.

Distances are not calculated nearer than to tenths of a foot.

How to Lay Out a Switch.—In laying out a switch, locate the frog so as to cut the least possible number of rails. Where there is some latitude in the choice of location, the P. C. of the turnout curve can be located so as to bring the frog near the end of a rail.

To do this, take from the table on page 315 the frog distance corresponding to the number of the frog to be used. Locate approximately the P. C. of the turnout curve, and measure from it, along the main-track rail, the tabular frog distance. If this brings the frog point near the end of the rail, the P. C. of the turnout curve may be moved so as to require the cutting of *but one* main-track rail. Measure the total length of the frog, and deduct it from the length of the rail to be cut, marking with red chalk on the flange of the rail the point at which the rail is to be cut. Measure the width of the frog at the heel, and calculate the distance from the heel

to the theoretical point of frog. For example, if the width of the frog at the heel is $8\frac{1}{4}$ in., and a No. 8 frog is to be used, the theoretical distance from the heel to the point of frog is $8.5 \times 8 = 68$ in. = 5 ft. 8 in. Measure off this distance from the point, marking the heel of the frog. This will locate the point of the frog, which should be distinctly marked with red chalk on the flange of the rail. It is a common practice to make a distinct mark on the web of the main-track rail, *directly opposite* the point of frog. This point being under the head of the rail, it is protected from wear and the weather. The P. C. of the turnout curve is then located by measuring the frog distance from the point of frog. From the table on page 315, we find the frog distance for a No. 8 frog is 75.3 ft., and the switch length, i. e., distance from P. C. of turnout curve to heel of split switch or toe of stub switch, is 22 ft.

If a *stub switch* is to be laid, make a chalk mark on both main-track rails on a line, marking the center of the head-block. A more permanent mark is made with a center punch. Stretch a cord touching these marks, and drive a stake on each side of the track, with a tack in each. This line should be at right angles to the center line of the track, and the stakes should be far enough from the track not to be disturbed when putting in switch ties. Next, cut the switch ties of proper length; draw the spikes from the track ties, three or four at a time, and remove them from the track, replacing them with switch ties, and tamping them securely in place. When all the long ties are bedded, cut the main-track rail for the frog, being careful that the amount cut off is just equal to the length of the frog. If, by increasing or decreasing the length of the lead 5% , it is possible to avoid cutting a rail, do not hesitate to do so, especially for frogs above No. 8.

Use full-length rails (30 ft.) for moving, or switch, rails, and be careful to leave a joint of proper width at the head-chair. Spike the head-chairs to the head-block so that the main-track rails will be in perfect line. Spike from 8 to 11 ft. of the switch rails to the ties, and slide the cross-rods on to the rail flanges, spacing them at equal intervals. The cross-rods are placed between the switch ties, which should not

be more than 15 in. from center to center of tie. The switch ties, especially those under the moving rails, should be of *sawed oak timber*. Southern pine is a good second choice. Attach the connection-rod to the head-rod and to the switch stand. With these connections made, it is an easy matter to place the switch stand so as to give the proper throw of the switch.

It is common practice to fasten the switch stand to the head-block with track spikes, but a better fastening is made with bolts. The stand is first properly placed, and the holes marked and bored, and the bolts passed through from the under side of the head-block. This obviates all danger of movement of the switch stand in fastening, which is liable to occur when spikes are used, and insures a *perfect throw*.

The use of track spikes is quite admissible when holes are bored to receive them, in which case a half-inch auger should be used for standard track spikes. The switch stand should, when possible, be placed facing the switch, so as to be seen from the engineer's side of the engine—the right-hand side.

Next stretch a cord from *a*, Fig. 8, a point on the outer main-track rail opposite the P. C. of the turnout curve, to *b'*, the point of the frog. This cord will take the position of the chord of the arc of the outer rail of the turnout curve. Mark the middle point *c* and the quarter points *d* and *e*. Whatever the degree of the turnout curve, the distance from the middle point *c* of the chord to the arc *ab'* is 1.18 ft., and the distances from the quarter points *d* and *e* are .88 ft.; hence, at *c* lay off the ordinate 1.18 ft., and at both *d* and *e* the ordinate .88 ft., three-quarters of the middle ordinate. These offsets will mark the gauge line of the rail *ab'*. Add to these offsets the distance from the gauge line to outside of the rail flange, and mark the points on the switch ties. Spike a lead rail to these marks, and place the other at easy track gauge from it. Spike the rails of the turnout as far as the point of frog to exact gauge, unless the gauge has been widened owing to the sharpness of the curve. Beyond the



FIG. 8.

point of frog the curve may be allowed to vary a little in gauge to prevent a kink showing opposite the frog. In case the gauge is widened at the frog, increase the guard-rail distance an equal amount. For a gauge of 4 ft. 8½ in., place the side of the guard rail that comes in contact with the car wheels at 4 ft. 6½ in. from the gauge line of the frog. This gives a space of 1½ in. between the main and guard rails.

In case the gauge is widened ¼ or ½ in., increase the guard-rail distance an equal amount.

When the turnout curve is very sharp, it will be necessary to curve the switch rails, to avoid an angle at the head-block. The lead rails should be carefully curved before being laid, and great pains should be taken to secure a perfect line.

If a *point*, or *split*, *switch* is to be laid, the order of work is nearly the same. The same precautions must be taken to avoid the unnecessary cutting of rails, with the additional precaution of keeping the switch points clear of rail joints, as the bolts and angle splices will prevent the switch points from lying close to the stock rails. As already stated, these conditions can usually be met where there is some range in the choice of the location of the switch. Where there is none, the main-track rails must be cut to fit the switch.

Having located the point of frog, the P. C. of the turnout curve, and the heel line of the switch, measure back from the heel line a distance equal to the length of the switch rails, and place on the flange of each rail a chalk mark to locate the ends of the switch points. This will also locate the head-block. Prepare switch ties of the requisite number and length, and place them in the track in proper order. As in the case of stub switches, see to it that all long switch ties are in place before cutting the rail for placing the frog; also, that the ends of the lead rails, with which the switch points connect, are exactly even; otherwise, the switch rods will be skewed, and the switch will not work or fit well. Fasten the switch rods in place, being careful to place them in their proper order, the head-rod being No. 1. Each rod is marked with a center punch, the number of the punch marks corresponding to the number of the rod.

Couple the switch points with the lead rails, and place the

sliding plates in position, securely spiking them to the ties. Connect the head-rod with the switch stand, and close the switch, giving a clear main track.

Adjust the stand for this position of the switch, and bolt it fast to the head-block. Next, crowd the stock rail against the switch point so as to insure a close fit, and secure it in place with a rail brace at each tie; then continue the laying of the rails of the turnout.

If there is no engineer to lay out the center line of the turnout, the section foreman can put in the lead from ordinates, as explained in Fig. 8. In modern railroad practice, however, most trackwork is done under the direction of an engineer, in which case the center line of the turnout is located with a transit. This insures a correct line and expedites work. For ordinary curves, center stakes at intervals of 50 ft. are sufficient, excepting between the P. C. of the turnout and the point of frog, where there should be a center stake at each interval of 25 ft. Place a guard rail opposite the point of frog on both main track and turnout. The guard rail should be 10 ft. in length; this is an economical length for cutting rails, as each full-length rail makes three guard rails.

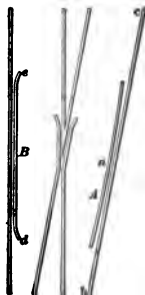


FIG. 9.

Two styles of guard rails are shown in Fig. 9. That shown at B is in general use, but the style shown at A is growing in favor. The latter is curved throughout its entire length. At its middle point *a*, directly opposite the point of frog, the guard rail is spaced $1\frac{1}{2}$ in. from the gauge line of the turnout rail *bc*. From this point the guard rail diverges in both directions, giving at each end a flangeway of 4 in. This allows the wheels full play, excepting at the point of frog, where the guard rail is exactly adjusted to the track gauge, and holds the wheels in true line, preventing them from *climbing*, or *mounting*, the frog. The style of guard rail shown at B, though still much used, has two objectionable features:

viz., first, the abruptly curved ends *d* and *e* often receive an almost direct blow from the wheel flanges, which causes a car to lurch violently; and second, the flangeway of uniform width, though proper for the main track when straight, as in Fig. 9, is unsuited for sharp curves on either a main track or a turnout, as it compels the wheels to follow a curved line; whereas the normal position of the wheel base of each truck is that of a chord of, or a tangent to, the curve. These two defects alone produce what is known as a *rough-riding frog*, even though the frog is well lined and ballasted.

Location of Crotch Frog.—A *crotch*, or *middle*, frog is a frog placed at the point where the outer rails of both turnouts of

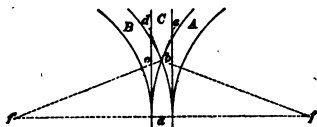


FIG. 10.

a three-throw switch cross each other. When both turnouts are of the same degree, the crotch frog comes midway between the main-track rails. Its location

and angle may be determined as follows: Let the turnout curves *A* and *B*, Fig. 10, be each $9^{\circ} 30'$, uniting with the main track *C* by a three-throw switch. Let *a* be the P. C. common to both curves, and *b*, the location of crotch, or middle, frog.

It is evident that the point of the crotch frog should be exactly midway between the gauge lines of the main-track rails, and if the gauge is 4 ft. 8½ in. = 4.71 ft., the point of the crotch of the frog will be $\frac{4.71}{2} = 2.35$ ft. from each rail.

Now, the problem is to find the frog distance from *a*, the P. C., to the point *c*, where the tangent deflection will equal 2.35, or half the gauge. From the table on page 299, we find the tangent deflection of a $9^{\circ} 30'$ curve is 8.28 ft. Applying the principle explained in connection with Fig. 5, and letting *x* represent the required frog distance, we have the following proportion:

$$8.28 : 2.35 = 100^2 : x^2;$$

whence,
$$x^2 = \frac{100^2 \times 2.35}{8.28} = 2,838.2 \text{ ft.},$$

and the required frog distance $x = 53.3$ feet, nearly.

Now, there are two curves starting at the common point *a*; the outer rails intersect at *b*, and the angle *d b e*, formed by tangents drawn at the point of intersection, is the angle of the crotch, or middle frog. The angle is equal to the sum of the angles *a f b* and *a f' b*; that is, equal to double the central angle of either curve between the P. C. and the point of intersection *b*. The degree of the curve is $9^{\circ} 30' = 570'$, and the central angle or total deflection for each foot is $\frac{570'}{100} = 5.7'$; and for the frog distance of 53.3 ft., the central angle is $53.3 \times 5.7' = 303.8' = 5^{\circ} 03.8'$. The angle of the crotch frog is double this angle; i. e., $5^{\circ} 03.8' \times 2 = 10^{\circ} 07.6'$. The crotch frog should be accurately located and spiked in place before the lead rails are placed.

The one objection to the three-throw switch is the open joint at the head-block, the inevitable attendant of the stub switch, but its advantages are so great that it will continue to be used, especially in yard service.

Crossover Tracks.—A *crossover* is a track by means of which a train passes from one track to another. The tracks united are usually parallel, as are the tracks of a double-track road. Such a crossover is shown in Fig. 11. The tracks *a b* and *c d* are 13 ft. apart from center to center, which is the standard distance for double tracks. The crossover consists of two

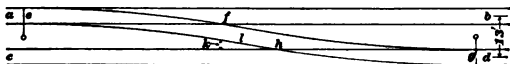


FIG. 11.

turnout curves, *e f* and *g h*. These curves are usually, though not necessarily, of the same degree. The curves terminate at the points of frog *f* and *h*, between which the track *f h* is a tangent. The essential point in laying out a crossover is to so place the frogs that the connecting track shall be tangent to both curves. In Fig. 11, suppose the frogs are No. 9, requiring $7^{\circ} 31'$ turnout curves.

From the table on page 315, we find the required frog distance is 84.7 ft., and the switch length 25 ft. As previously

noted, if there is considerable range in choice of location, the frogs can be so placed as to largely avoid the cutting of rails; but usually crossovers are required at certain precise places, and the rails must be cut as occasion demands. Having located the point of frog at f , we determine the point of the next frog at h , as follows: A No. 9 frog is one that spreads 1 in. in width to every 9 in. in length; and, as the track between the frog points is straight, the distance fh between these points will be as many times 9 in. as is the space k between the tracks at the frog point f . The main-track centers are 13 ft. apart, making the space between the gauge lines of the inside rails 8 ft. $3\frac{1}{4}$ in. As it is the rail l of the turnout that joins the second frog at h , we subtract the gauge, 4 ft. $8\frac{1}{4}$ in. from 8 ft. $3\frac{1}{4}$ in., leaving 3 ft. 7 in., the distance k , between the gauge line of the rail l , opposite the frog point f , and the gauge line of the nearest rail of the track $c d$. This distance multiplied by 9 in. will give the distance from the frog point f to the frog point h ; 3 ft. 7 in. = 43 in.; $43 \times 9 = 387$ in. = 32 ft. 3 in. Accordingly, having located the point or frog f , we mark a corresponding point on the nearest rail of the opposite track. From this point we measure along the rail the distance 32 ft. 3 in., locating the second frog point h , and again the frog distance 84.7 ft. to the P. C. of the second turnout curve at g .

If frogs of different numbers, say 7 and 9, were to be used, the distance between the frogs is found as follows:

As the No. 7 frog spreads 1 in. in 7 in., and the No. 9 frog 1 in. in 9 in., the two will together spread 2 in. in $7 + 9 = 16$ in., or 1 in. in 8 in. Now, if the rails to be united are 3 ft. 7 in., or 43 in., apart, as in the previous problem, the distance between the frog points will be $43 \times 8 = 344$ in. = 28 ft. 8 in.

In locating crossover tracks, regard should be paid to the direction in which the bulk of the traffic moves, and the crossover tracks should be so placed that loaded cars will be backed, not pushed, from one track to the other.

At all stations on double-track roads there should be a crossover to facilitate the exchange of cars and the making up of trains.

PERPETUAL CALENDAR—1797-1904.

3	4	5	6	0	1	2
June.	Sept. Dec.	April. July.	Jan. Oct.	May.	Aug.	Feb. Mar. Nov.
1797	1798	1799	1800	1801	1802	1803
	1804	1805	1806	1807		1808
1809	1810	1811		1812	1813	1814
1815		1816	1817	1818	1819	
1820	1821	1822	1823		1824	1825
1826	1827		1828	1829	1830	1831
	1832	1833	1834	1835		1836
1837	1838	1839		1840	1841	1842
1843		1844	1845	1846	1847	
1848	1849	1850	1851		1852	1853
1854	1855		1856	1857	1858	1859
	1860	1861	1862	1863		1864
1865	1866	1867		1868	1869	1870
1871		1872	1873	1874	1875	
1876	1877	1878	1879		1880	1881
1882	1883		1884	1885	1886	1887
	1888	1889	1890	1891		1892
1893	1894	1895		1896	1897	1898
1899	1900	1901	1902	1903		1904

Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44					

By means of the table given on the preceding page, the day of the week corresponding to any date between 1797 and 1904 inclusive, may be readily found. Before every leap year there is a blank space. To find the day of the week on which January 1 of any year fell, find that year in the table; glance down the column containing that year, and the day of the week at the foot of the column will be the day of the week required. Thus, to find on what day of the week January 1, 1895, fell, we find under 1895 in the table, Tuesday. For leap years, we look for day of week under the blank space before the year. Thus, January 1, 1896, fell on Wednesday, Wednesday being in the column containing the blank space before 1896. To find the day of the week for any other date, add (mentally) to the day of the month the first number under the day of the week that is contained in the column containing the year of the century; to this sum, add the number above the month at the top of the table. Find the number thus obtained in the columns of figures under the days of the week; the day of the week at the head of the column containing this number will be the day required. Thus, to find on what day of the week September 10, 1813, fell, we find 1813 in the table. The number under the day of the week in the column containing 1813 is 6, and the number above September at the top of the table is 4. Hence, $10 + 6 + 4 = 20$. The day of the week above 20, in the lower part of the table, is Friday.

For dates in January and February of leap years, take one day less, or add the number beneath the day of the week under the blank space preceding the year. Thus, for February 12, 1896, we have $12 + 4 + 2 = 18$, and the day of the week above 18 is Wednesday.

The table may also be used for fixing dates. Thus, Thanksgiving Day is the last Thursday in November; on what day of the month did it fall in 1897? Since the earliest day on which it can fall is the 24th, we find on what day of the week November 24 falls, and then count ahead to Thursday. Referring to the table, $24 + 6 + 2 = 32$; the day of the week above 32 is Wednesday, and since Thursday is one day later, it follows that Thanksgiving Day in 1897 fell on the 25th.

The International System

OF

EDUCATION BY MAIL

The method of correspondence instruction in the industrial sciences of the International Correspondence Schools was originated in 1891, by Thomas J. Foster, President of the Schools, and was first used in the Correspondence School of Mines.

Distinctive Features of the International System

1. Courses of Instruction for particular occupations, in which only such facts, processes, and principles are taught as are necessary to qualify the student therein.

2. Textbooks, Question Papers, and Drawing Plates, prepared for each Course; Principles applied in examples of practical value to the student; Frequent Revision, to keep pace with latest methods in trades and manufactures.

3. Thorough examination and correction of the written work of the student, and full, clear, and exact written explanations of all difficulties met with in studying.

ADMINISTRATION BUILDINGS OF

International Correspondence Schools

SCRANTON, PA.



Erected in 1898

**650,000 Students and Graduates. Faculty
of 358 Professors and Assistants**

ESTABLISHED 1891 .

THE ORIGINAL

Our Standing and Responsibility

The International Textbook Company, Proprietors of the International Correspondence Schools, is incorporated under the laws of Pennsylvania, and has a paid-up capital of \$3,000,000. Its administration buildings and its mammoth printery and instruction building were erected expressly for the purposes of correspondence instruction, at a cost of nearly a million dollars.

The administration buildings, two in number, are situated at 434-436 Wyoming Avenue, are five and four stories high, respectively, and contain the business departments of the Schools.

The printery and instruction building is situated at the corner of Wyoming Avenue and Ash Street. It is two and three stories high, and contains the instruction, illustrating, and printing departments of the Schools.

References

We refer to the commercial agencies, or to any bank officer, teacher, clergyman, or public official in Scranton, as to our responsibility and reputation. We will repay the traveling expenses of any person or committee who may come to Scranton and find that we have made misrepresentations as to the practicability and efficiency of our method of instruction and Courses of study, or as to our financial standing. We will, further, refer a person, intending to enroll, to a student in his locality, with whom he may correspond.

FROM J. A. LINEN, ESQ.

President First National Bank, Scranton, Pa.

**This Bank ranks third in the roll of honor
of National Banks in the United States**

We regard the Schools as an educational institution of a very superior character. The officers and managers are men of integrity and business capacity, and stand high in this community.

J. A. LINEN.

System of Instruction

Each Course is made up of a series of Instruction Papers, clearly written and fully illustrated. When the student enrolls, the first two Instruction Papers are sent to him. After carefully studying the first Instruction Paper, he writes his answers to the Examination Questions at the end of the Paper, sends his work to the Schools for examination and correction, and continues with his second Paper. When the sets of answers are received at the Schools, they are carefully examined. If an error is discovered, it is not only indicated in red ink, but, if considered advisable, a careful explanation of that particular problem is written on the back of the sheet.

Papers are passed if a mark of 90 per cent. has been attained. The answers are then returned, accompanied by a Percentage Slip and the third set of Papers. The student always has at least one Paper to study while the answers to the previous Paper are being examined and corrected.

How We Teach Drawing

The first Instruction Paper on Drawing and a mailing tube for returning the finished Plate are sent to the student. Detailed directions are given for the use of instruments and making the first Plate. Beginning with simple lines, the student is advanced to actual working drawings, to pen-and-ink rendering, to drawing from nature, from cast, and from the figure, and to water-color rendering.

Special Instruction

When a student desires assistance from his Instructors, he uses an "Information Blank" provided for the purpose. When considered necessary, or on request, a "Special Instructor" is assigned to give personal attention to his case, until the subject is completed. A Certificate of Progress is granted on the completion of each Division of a Course, and a Certificate of Proficiency, or Diploma, is awarded when the student passes his final examination.

Advantages of the System

1. You Study at Home.—You do not have to leave home to secure the education; the education comes to you.

2. No Time Lost From Work.—You can keep right on with your work, and study during spare hours. This is the only system of education that enables the student to combine education and experience by immediately using in his daily work the knowledge gained through his studies.

3. You Study When It Is Convenient.—Our Schools never close. You can study when and where you please. Failure is impossible to those that try.

4. We Teach Everywhere.—You can move from place to place while studying. We teach wherever the mails reach. We have students in every country.

5. No Books to Buy.—You have no textbooks to buy. We furnish all Instruction Papers, return envelopes, and Information Blanks. We prepay all postage on mail sent by us to the student. All that it need cost you, outside of the price of your Course, is the postage on matter sent to us.

6. Instruction Private.—Instruction is conducted privately. None need know you are a student except ourselves.

7. Only Spare Time Required.—Your studies need not interfere with business or social engagements.

8. Written Explanations.—Written explanations are always with you, and can be studied repeatedly.

9. Each Student a Class by Himself.—Our student's written examinations enable his Instructor to detect weak points and assist him. Each student is a class by himself, because the Instructor attends to him alone.

10. Study Assures Success.—The successful completion of any Course, and our Diploma, is assured to all that can read and write and will study as we direct.

11. Backward Students Assisted.—We take great pains with backward students; they become our best friends.

12. Prepared for Examinations.—Our Courses prepare the student for examinations; he learns to express himself clearly in writing, and remembers what he writes.

13. Open to All.—Our Schools are open to all—both sexes—"seven to seventy."

Bound Volumes Furnished to Students

Realizing the great value of the Instruction Papers and Drawing Plates to those that have studied them, and the desirability of preserving them for reference, the Schools have had them reprinted and bound into handsome half-leather volumes. The number furnished varies from one to



BOUND VOLUMES OF THE COMPLETE ARCHITECTURAL COURSE

ten volumes, according to the Course for which the student enrolls. He has the use of them as long as he lives up to his contract. Their contents correspond to the set of the Instruction Papers and Examination Questions sent to the student throughout his Course, and are supplied in addition to them. Each set gives the student thorough and complete instruction concerning the theory and practical application of the principles underlying the trade or profession on which they treat. Containing as they do the complete Course in permanent form, the volumes constitute an unequalled reference library, available at all times.

What the International Correspondence Schools Are Doing

First—Teaching Mechanics the Theory of Their Trades

In nearly every machine shop, drafting room, industrial plant, etc., we have students that have secured promotion and advances in salary through study in our Schools. The increased value of an employe that masters the theory of his trade or profession brings prompt and substantial recognition.

Experience in teaching 650,000 students has proved that any one that can read and write English is able to keep right on with his work, and at the same time acquire a technical education. You do not have to leave home or quit work. Our prices are low, our payments easy, and you have no textbooks to buy.

If you are dissatisfied with your salary, you can increase your earning capacity by study at home, and fit yourself for the highest positions in your trade or profession.

THROUGH THIS PLAN

Machinists and Apprentices	Have Become Mechanical Draftsmen.
Firemen and Oilers	Have Become Steam Engineers.
Linemen and Electricians	Have Become Electrical Engineers.
Carpenters, Etc.	Have Become Architects and Builders.
Miners and Mine Bosses	Have Become Foremen and Managers.
Chainmen and Surveyors	Have Become Civil Engineers.
Tinsmiths	Have Become Pattern Draftsmen.
Plumbers, Etc.	Have Become Sanitary Engineers.
Assistant Chemists	Have Become Chief Chemists.

What the International Correspondence Schools Are Doing

Second—Helping Misplaced People to Change Their Work

The man that would succeed as an electrical, mechanical, or civil engineer, or architect is filling a \$10-a-week position as salesman or assistant bookkeeper.

The woman that would be valuable in an office, drafting room, or laboratory position is earning a poor living as clerk, housekeeper, or seamstress.

These conditions exist because people do not know of any practical plan to change to the occupation of their choice.

We have solved the problem.

We have qualified hundreds for salaried positions in new lines of work, at their homes, in spare hours, and at small expense. They held their old positions until they changed to the new, with a salary better than before.

They are now earning while learning, and earn more as they learn more.

THROUGH THIS PLAN

Clerks,
Salesmen,
Bookkeepers,
Women,
Mechanics,
Farmers,
Laborers,
Teachers,
Telegraphers,
Drivers,

HAVE
BECOME

Draftsmen,
Surveyors,
Architects,
Engineers,
Chemists,
Stenographers,
Accountants,
Ad Writers,
Window
Dressers.

What the International Correspondence Schools Are Doing.

Third—Enabling Young People to Support Themselves While Learning Professions

Young men or women obliged to earn their own living are not debarred from a successful career because they have not the time or means to attend college. By our method of *education by mail* they can qualify at home, in spare time, and at small cost, for minor positions and rapidly advance.

A few months' study of Mechanical or Architectural Drawing with us will qualify young men for positions as assistant draftsmen in machine works or electrical manufacturing, or with architects. Here they can combine study with work, and advance. Through the Surveying and Mapping Course they can qualify as surveyors on engineering corps, and through further study with us advance to the highest positions.

Those that desire to enter on business life can qualify, through our instruction, for good positions as bookkeepers or stenographers, ad writers, window dressers, etc.

THROUGH THIS PLAN

Young Men	} Have Become	{	Draftsmen, Electricians, Surveyors.
Young Women			Stenographers, Bookkeepers, Designers.

The School of Mechanical Engineering

Mechanical Course.—This is intended for all those that desire a thorough knowledge of engineering calculations, mechanical drawing, and the design of steam engines, boilers, and modern machinery. It is a complete treatise on the drafting-room practice of mechanical engineering.

Mechanical Engineering Course.—This is intended for machinists, draftsmen, machine designers, mechanical engineers, apprentices, foremen, superintendents, etc. This Course comprises a thorough and exhaustive treatment of the subject of mechanics, supplemented by a complete treatise on modern shop practice, and is the most thorough Course in practical mechanical engineering ever published.

Shop Practice Course.—This is divided into five parts, as follows: Machine-Shop Practice is intended for all those that wish to get a thorough knowledge of modern machine-shop practice. Toolmaking is intended for those that desire information in regard to the latest and most approved tool-making methods. Patternmaking covers thoroughly every branch of patternmaking. Foundrywork gives thorough instruction in all phases of foundrywork. Blacksmithing and Forging is intended to give blacksmiths, helpers, etc. a thorough knowledge of the best methods employed in their trade.

Increased Salary Over 100 Per Cent.



Having been obliged to make my own way before I had a chance to get a practical education, I soon found that the man with the working education got the pay, while the others got the hard work. After studying from textbooks and at night schools with but little success, I enrolled in the I. C. S. Since I enrolled I have secured better employment, and my pay has been increased over 100 per cent. I am now first-class machinist for the U. S. Government.

HUGH J. WHITE, *Washington, D. C.*

The School of Mechanical Engineering

Refrigeration Course.—Graduates of this Course will understand the principles of refrigeration, the design and construction of refrigerating apparatus, and its installation, testing, and operation.

Gas Engines Course.—This is intended for all that desire to manufacture and install gas or oil engines, and for all that operate or repair them, or wish to qualify as gas engineers.

Farm Machinery Course.—This is intended for traction engineers, threshers, and farmers. Graduates will thoroughly understand the principles of operation of farm machinery and be able to run traction engines.

The School of Steam Engineering

Steam-Electric Course.—This is a complete Course on steam engineering and electric-lighting and railway work, and is intended for engineers, superintendents, etc. of large electric-lighting or railway plants.

Complete Steam Engineering Course.—This Course, as its name implies, is a complete treatise on stationary engineering, and is intended for engineers, superintendents, etc. in large steam plants using little or no electric apparatus.

A Chief Engineer's Opinion

I wish I could make my remarks about the I. C. S. Steam Engineering Courses strong enough to induce every engineer to enroll. The knowledge gained has been very valuable to me. I have the satisfaction of knowing not only the how but the why of my work. I now have charge of the Spring Garden Station of the Philadelphia Water Department, the largest plant owned by the city. The cost of an I. C. S. Course is insignificant.

CLARENCE D. WILLIASON,
1723 Norris St., Philadelphia, Pa.



The School of Steam Engineering

Advanced Engine Running Course.—This Course will qualify engineers, firemen, etc. to take charge of medium-sized steam plants.

Engine Running Course.—This Course is intended for engineers, firemen, helpers, etc. in small plants, and to qualify any one to get a start in steam engineering.

Engine and Dynamo Running.—This is intended for those who wish to qualify to operate and care for the apparatus in small steam-electric plants.

The School of Marine Engineering

Marine Engineers' Course.—This Course is intended for coal passers, firemen, water tenders, oilers, and other employes of steam vessels. The graduate will be qualified to pass marine license examination for any grade to which he is eligible by law.

The School of Locomotive Running

Trainmen and Carmen's Scholarships.—These three Scholarships provide trainmen with special instruction on air brakes, train rules, train heating, car lighting, etc.

What We Do for Marine Firemen



I had studied a good many versions of the so-called "Complete Treatise on the Steam Engine," but I had never found what I wanted to know until I enrolled in the Schools. Ten times what I have paid for my Scholarship would not buy my Course from me. I have risen from the position of fireman to that of chief engineer of an ocean-going steamer. Every promise you made has been faithfully kept, and you have even done more for me than I expected.

SHERMAN MALLERY, *Norfolk, Va.*

The School of Locomotive Running

Locomotive Running Scholarships.—These six Scholarships are very popular with locomotive firemen and engineers. They furnish instruction in the construction and management of the locomotive and its various appliances and fittings.

Air Brake Scholarships.—These five Scholarships are intended for enginemen and air-brake inspectors. They provide complete instruction on Westinghouse and New York air brakes.

The Roundhouse Scholarships.—These three Scholarships include either New York or Westinghouse air-brake instruction, together with instruction on locomotive boilers.

The School of Electrical Engineering

Electrical Engineering Course.—This is intended for superintendents of electrical establishments, factory artisans, draftsmen, designers, inventors, and all others that wish a thorough knowledge of the design of electrical apparatus.

Electrical Course.—This is intended for those that desire to qualify for any electrical position that does not require a knowledge of mechanical engineering.

Electric Railways Course.—This will qualify the student to construct and maintain electric railways under any system.

An Electrical Engineer's Experience

I have always thought the cost of my I. C. S. Electrical Course trifling compared with the benefits received. The last Papers of the Course on Electric Machine Design are especially valuable. Through the Schools, I was appointed electrician for Moncton, N. B. For some time I have been installing complete plants of all kinds, making the specifications myself. During the past year, I have had more work than I could handle.

GEO. M. MACDONALD,
Box 183, Moncton, N. B.



The School of Electrical Engineering

Advanced Electric Railways Course.—This Course is a complete treatise on steam engineering as applied to electric-railway plants, and is intended for superintendents, engineers, etc. in large electric-railway plants.

Advanced Electric Lighting Course.—This Course is a complete treatise on steam engineering as applied to electric-lighting plants. It is intended for superintendents, engineers, etc. in large electric-lighting plants.

Electric Lighting and Railways Course.— This is intended for dynamo tenders, stationary engineers, installation engineers, and all who work around electric-railway or lighting plants.

Electric Lighting Course.—This will qualify the student to successfully install and operate electric arc- and incandescent-lighting systems.

Dynamo Running Course.—This is a short but complete Course on the operation and care of dynamos. It is intended for dynamo tenders and for those who wish to get a start in electrical work.

Electric Car Running Course.—This Course will qualify the student to direct the equipment of a modern electric car, operate it safely under all conditions, and pass any motorman's or car inspector's examinations.

Interior Wiring.—This will qualify the student to do electric-light wiring and bellwork.

From Conductor to Head Barnman



I am a student in the Electric Car Running Course, and am pleased to say that the knowledge gained from my Course has enabled me to accept a much better position. When I enrolled, I was a motorman for the Ogdensburg Street Railway Company, but after applying myself to my Course for some time, I was promoted to head barnman for the above company, with a better salary, and have also been promised another increase.

JOHN HENRY O'NEILL,
129 New York Ave., Ogdensburg, N. Y.

The School of Electrotherapeutics

Complete Electrotherapeutic Course.—Graduates of this Course will be qualified to make all modern and approved applications of the electric current in the diagnosis and treatment of disease.

Neurological Electrotherapeutic Course.—This is intended for physicians and medical students that desire a knowledge of the treatment by electricity of the diseases of the brain, spinal cord, peripheral nerves, and muscles.

Gynecological Electrotherapeutic Course.—This is intended for physicians that wish to make a special study of the diseases peculiar to women.

Surgical Electrotherapeutic Course.—This is intended for surgeons and general practitioners.

Eye, Ear, Nose, and Throat Electrotherapeutic Course. This is intended for physicians and specialists.

Dental Electrotherapeutic Course.—This is a complete up-to-date treatise on electricity as used in dental operations.

Roentgen Rays Course.—This includes instruction in the manipulation and care of all up-to-date apparatus used by recognized Roentgen ray workers.

Nurses' Electrical Course.—The information contained in this Course will increase the efficiency and earning capacity of any nurse, and will make her more valuable to the attending physician.

Genito-Urinary Electrotherapeutic Course.—This Course is especially intended for the genito-urinary specialist.

Prominent Physicians Commend Our Course

INTERNATIONAL CORRESPONDENCE SCHOOLS,
Scranton, Pa.

Gentlemen:—After having completed the Electrotherapeutic Course I can tell you that this Course is, in my opinion, a wonderful piece of work. The physician finds everything for intelligently applying electricity, and at the same time is not bothered with superfluous things—hypothesis, opinions, names, etc. I call them superfluous in a practical Course which aims at nothing but giving the student a thorough knowledge of electricity so far as it is used as a therapeutic and diagnostic agent.

DR. A. DECKER, 425 Orchard St., Chicago, Ill.

The School of Telephone and Telegraph Engineering

Telephone Engineering Course.—This Course will qualify students to maintain and manage telephone switchboards and exchanges, make repairs anywhere in the system, and take general supervision of a plant.

Telegraph Engineering Course.—This is intended for telegraph operators in commercial or railroad business, the military or the signal service. It gives thorough instruction in all branches of telegraphy. Graduates will be qualified to fill the most responsible positions.

The School of Civil Engineering

Civil Engineering Course.—This Course is intended for practicing bridge, railroad, municipal, or hydraulic engineers, surveyors, and engineers' assistants that wish to review former studies, to acquire a thorough knowledge of other branches of the profession by studying those special branches of engineering in which they lack knowledge, or to become qualified as consulting engineers. The instruction is sufficiently advanced and comprehensive to insure to the graduate the technical training necessary to the successful prosecution of his professional work.

Responsible Position Before He Finished Study



I wish here to say that before I had finished the instruction in "Batteries," in the I. C. S. Telephone Engineering Course, the Schools helped me to obtain the position I now occupy, that of assistant electrician for the American Bell Telephone Company, at Goshen, Ind. The only recommendations I carried were the Schools' records of my progress in the Course, as far as I had gone. My technical training has been conducted by mail entirely.

W. H. Fox, *Goshen, Ind.*

The School of Civil Engineering

Bridge Engineering Course.—This is intended for surveyors, draftsmen, bridge engineers, and their assistants, and employes in bridge works. Graduates will, with some experience, be able to design and superintend the construction of modern highway and railroad bridges.

Surveying and Mapping Course.—This is intended for rodmen, chainmen, engineers' assistants, and all that wish to become surveyors. Graduates will be qualified to survey railroads, farm properties, etc.

Railroad Engineering Course.—Graduates of this Course will have the necessary education to survey and map out proposed locations for railroads, or to fill responsible positions in the work of construction or the maintenance-of-way department.

Hydraulic Engineering Course.—This is intended for those interested in irrigation, water supply, etc. Graduates will have the necessary education to design and install water-power plants, hydraulic machinery, and water-supply and irrigation systems.

Municipal Engineering Course.—This is intended for municipal engineers and contractors, city surveyors, and their assistants. It will qualify the student to survey and make maps and estimates of proposed sewerage systems, street improvements, pavements, etc., and to superintend their construction.

Earned \$30 as Teacher: Earns \$100 as Surveyor

When I enrolled in the I. C. S., I was teaching school at \$30 a month. After studying three months, I secured a position as draftsman and assistant surveyor at \$70 a month, but soon after beginning work was offered \$100 a month. In his spare time a man can gain as good an education in the International Correspondence Schools, Scranton, Pa., as at the best colleges. I heartily recommend the Schools.

LLOYD G. SMITH,
*Deputy County Surveyor,
Chinook, Choteau Co., Mont.*



The School of Mathematics and Mechanics

This is our largest School. In it are made all the preliminary examinations and corrections in mathematics and mechanics for the other Schools. The Principal is assisted by 13 Special Instructors and 130 Examiners.

While most of the mathematical subjects are taught in connection with other Courses, the following can be taken separately:

Arithmetic Course, Part 1.—This Course includes instruction from the simplest definitions of Arithmetic up to and including ratio and proportion.

Arithmetic, Parts 1 and 2.—This Course includes all the subjects in Part 1, together with instruction in the subjects: Percentage, Interest, Notes, Bank Discount, Stocks and Bonds, Average or Equation of Payments, Partnership, and Alligation.

Algebra Course.—This is a thorough Course in Algebra, including instruction in Quadratic Equations, the Remainder Theorem, the Binomial Formula, Progressions, Logarithms, etc.

Advanced Algebra Course.—This Course is intended for teachers of Algebra, for engineers, and for all others that desire thorough instruction in higher algebra.

Kellar, the Greatest Living Magician, Testifies

MONTOUR HOUSE,
Opposite Court House,
J. L. RIEHL, *Proprietor.*

DANVILLE, PA., May 2.

DEAR SIR:—In reply to your request for my opinion regarding the International Correspondence Schools, I beg to say that I consider the system of Instruction the very best that can be desired, and judging from their Course in Arithmetic, in which I am enrolled, I think any one of ordinary intelligence can acquire a thorough knowledge of mathematics by carefully reading and observing the rules of the Instruction Papers. The system is so very simple that any one who can read may learn.

Yours truly,

HARRY KELLAR (Magician).

The School of Chemistry

General Chemistry Course.—This is a Course in general and analytical chemistry, and is intended for those who desire to obtain a general knowledge of chemistry and analytical methods and thus prepare, in as short a time as possible, for general laboratory work. Since the Course includes an exhaustive treatment of organic chemistry, it enables the student to prepare for responsible positions in the manufacture of coal-tar products and various other organic materials. This Course also prepares the student for such positions as pharmaceutical chemist, toxicologist, etc.

Chemistry and Chemical Technology.—The student in this Course receives, in addition to a thorough training in general and analytical chemistry, full detailed instruction in the most important branches of applied chemistry.

Separate Courses in the following branches of applied chemistry are given: Chemistry and Manufacture of Sulphuric Acid; Chemistry and Manufacture of Alkalies and Hydrochloric Acid; Chemistry and Manufacture of Iron and Steel; Chemistry and Packing-House Industries; Chemistry and Manufacture of Cottonseed Oil and Products; Chemistry and Manufacture of Leather; Chemistry and Manufacture of Soap; Chemistry and Manufacture of Cement; Chemistry and Manufacture of Paper; Chemistry and Manufacture of Sugar; Chemistry, Petroleum, and Manufacture of Products; Chemistry and Manufacture of Gas.

Drug Clerk Becomes a Chemist

I started early in life to earn my living by clerking, and my lot was cast in a drug store. Before I enrolled in the Chemistry Course, I made several attempts to educate myself, but with little success. Thanks to the Schools, I have mastered chemistry to a degree that my ambition had never pictured. Soon after taking up my Course, I took charge of the laboratory at E. E. Bruce & Co., wholesale druggists, at a good salary, which has since been increased 45 per cent.

H. B. MOLYNEAUX, *Omaha, Neb.*



The School of Drawing

Mechanical Drawing Course.—The student that completes this Course will be able to execute neat mechanical drawings. We have met with remarkable success in teaching this subject to thousands entirely ignorant of drawing, hundreds of whom are now earning good salaries as draftsmen and designers. We guarantee to qualify ANY ONE to make neat mechanical drawings that follows our directions.

Architectural Drawing Course.—The student that completes this Course will be able to make general and detail drawings of architectural structures, and will be thoroughly qualified to take a position in an architect's office. We guarantee to qualify ANY ONE to make neat architectural drawings that follows our directions.

If you wish to learn Mechanical or Architectural Drawing, or desire to become a mechanical or electrical engineer, or architect, and support yourself while learning either of these professions, send for special circular entitled "Support Yourself."

Drawing Outfit Free

Every student in either of these two Courses is furnished with the Complete Drawing Outfit No. 1, valued at \$13.55. He is permitted to retain this Outfit for use indefinitely, provided he observes the terms of his Contract for Scholarship.

From Dentist to Draftsman



When I enrolled in the I. C. S. Mechanical Drawing Course, I was practicing dentistry. After partially finishing my Course, I commenced making drawings for the Smith Metallic Packing Co., occasionally receiving as high as \$15 for a single drawing, and having some of my work taken in preference to that done by the Master Mechanic of a Mexican railway. I have now become Chief Draftsman for the Smith Metallic Packing Company, Chicago.

M. E. HOAG,

861 Monadnock Bldg., Chicago, Ill.

The School of Architecture

Complete Architectural Course.—This is intended for architects, draftsmen, contractors and builders, carpenters, masons, bricklayers, building tradesmen, and all others desirous of qualifying themselves to design and construct buildings. Graduates will be able to design, prepare working drawings and specifications for building operations, calculate quantities, estimate costs, and will have a thorough knowledge of iron and steel construction.

Architectural Drawing and Designing Course.—This is intended for carpenters, contractors, architectural draftsmen, and all that wish to learn architectural drawing, history, and design.

Building Contractors' Course.—This will qualify carpenters, masons, bricklayers, and other building tradesmen to make and read plans, estimate accurately, and undertake general contracting and building. This is a practical Course in building construction, containing practical instruction for practical men.

Architectural Drawing Course.—This Course is described under "The School of Drawing." See previous page. Students in this Course have been so successful that we will forfeit \$100 to any student of our Architectural Drawing Course that will study as we direct, whom we cannot qualify to make neat architectural drawings.

From Telegraph Operator to Architect

When I began studying in the Complete Architectural Course of the International Correspondence Schools, of Scranton, Pa., I was a telegraph operator and knew nothing whatever of drawing, or of the profession that I am now following. I have given up my partnership at Fairmount and am now well established here, with good prospects for a very successful business. My income at present is three times what I was receiving when I enrolled.

JOHN C. TIBBETTS, *Grafton, W. Va.*



The School of Design

Ornamental Design Course.—This provides instruction in freehand drawing, sketching from nature, and conventionalizing of natural forms as a basis for decorative design, history of ornament, details of color, and construction of all typical elements, composition, pencil, pen, wash-drawing and water-color work, and the application of design to china, wall paper, lace, book covers, embroidery, furniture, stained glass, interior decoration, rugs, carpets, oilcloths, etc. The instruction is carefully graded from the simplest instrumental work to the more complicated water-color rendering of practical designs.

Drawing, Sketching, and Perspective Course.—This is intended for teachers that wish to qualify for the position of drawing inspector; also, for all that desire a thorough knowledge of freehand and perspective drawing. Even if the Course is taken only for developing artistic tastes and perceptions, it is invaluable.

The School of Sheet-Metal Work

Sheet-Metal Pattern Drafting Course.—This Course is intended for tinsmiths, cornice makers, architectural sheet-iron workers, coppersmiths, plumbers, etc. Graduates will be able to develop all kinds of sheet-metal patterns.

Doubled His Salary Since Enrolling



Before I enrolled in the I. C. S., my education consisted of but four terms in a country district school. I took up the Sanitary Plumbing, Heating, and Ventilation Course and the Sheet-Metal Pattern Drafting Course, and have received great benefits from my instruction. When I enrolled, my salary was \$9 a week. I am now foreman of a large shop doing plumbing, heating, ventilating, and cornice work.

B. M. RICHARDS,
Box 475, Everett, Wash.

The School of Plumbing, Heating, and Ventilation

Sanitary Plumbing, Heating, and Ventilation Course.

This affords an education in plumbing, heating, and ventilation that will qualify any plumber, steam fitter, or gas-fitter to fill the highest positions in his line of work. It is of particular value to such men as aspire to become master plumbers, general contractors, or heating engineers.

Sanitary Plumbing and Gas-Fitting Course.—This Course enables plumbers, gas-fitters, and apprentices to do their work easier, better, and more economically, and to pass examinations for license.

Heating and Ventilation Course.—This is intended for steam fitters, plumbers, and heating and ventilating engineers. It will qualify the student to install any system of heating or ventilation.

Sanitary Plumbing Course.—This is intended for plumbers and plumbing apprentices that do not care to study gas-fitting or heating and ventilation. It will qualify the student to pass any examination required of plumbers.

Gas-Fitting Course.—This will qualify the student to calculate illumination required, test systems, put up combination fixtures, or make drawings of proposed installations.

From Journeyman to Master Plumber and Gas-Fitter

I am under many obligations to the I. C. S. I have recently passed an examination and obtained a first-class Master Plumber's license. It has been of inestimable benefit to me. I most heartily recommend the Schools, especially to plumbers who are desirous of gaining an insight to the technical theories of the trade, which equips them to earn larger wages and the best positions as workmen.

MICHAEL L. SYLVIA,
*Plumber and Gas-Fitter,
New Bedford, Mass.*



The School of Mines

Full Mining Course.—This is intended for mine superintendents, foremen, mining engineers, and others that wish a thorough education in all branches of coal and metal mining. It fits the student to superintend either coal or metal mines, or to pass mine foreman's or State Mine Inspector's examinations.

Complete Coal Mining Course.—This is intended for mining engineers, mine officials, and miners that wish a complete education in the methods and machinery used in coal mining. It embraces a study of every detail necessary to fit a student for any position in or around either anthracite or bituminous mines, or pass the examinations for mine foreman or State Mine Inspector.

Short Coal Mining Course.—This contains only the information absolutely necessary to qualify persons to pass the mine foreman's examinations, and those that finish it will have a good knowledge of the art of mining.

Metal Mining Course.—This is intended for mine officials, metal miners, or men engaged in ore dressing and milling. It qualifies the student to take charge of modern metal mines or mining and milling machinery.

Metal Prospectors' Course.—This will qualify the student to make assays of ores and prospect for gold, silver, and other valuable minerals.

Salary Increased \$500 Per Year



Before I had finished the Complete Coal Mining Course, I obtained a first-grade Certificate of Competency as mine foreman, and also a position where my salary was increased \$500 per year. I am successfully handling the mine in which I am employed as inside manager and foreman. From the information gained in my Course, I can master the most complicated problems in mine management, and do all my own surveying and platting.

JAMES PARTON,
Monongahela City, Pa.

The School of Metallurgy

Complete Metallurgy Course.—This Course is intended for metallurgists, investors, and others who desire a knowledge of the metallurgy of gold, silver, copper, lead, and zinc. It will give the student a broad field of usefulness and will enable him to direct metallurgical operations of any kind whatever.

Milling Course.—This provides instruction for millmen, bosses, and superintendents of milling operations, and will thoroughly qualify the student for the highest positions in this particular branch of metallurgy. The Course deals with the dry methods of gold and silver extraction.

Hydrometallurgical Course.—This Course fully explains the wet methods of treating ores for gold and silver extraction, and also includes thorough instruction in electrometallurgy. It is intended for helpers, foremen, and superintendents in hydrometallurgical work, electrochemists, electricians, etc.

Smelting Course.—This Course deals with the preliminary treatment and the reduction of ores of the common metals and the refining of the crude products. Graduates of this Course will be qualified to take charge of all kinds of smelting operations.

The School of Commerce

Failed Before; Succeeded With Us

I had attended night school and had spent nearly \$100 on textbooks without success before I heard of the I. C. S. The only thing I am sorry for is that I did not hear of them sooner. I had no trouble in understanding the Complete Commercial Course that I took; my instructors taught me everything by mail easily, simply, and accurately. The value of what I learned is shown by the fact that during the year my salary has been increased 25 per cent.

ERNEST BRUNELLI,
Gardiner, N. Mex.



The School of Commerce

Complete Commercial Course.—This Course is intended for young men or women, in the city or country, that wish to equip themselves with a business education. Graduates will be able to keep books by single or double entry, or perform the work of stenographer or correspondent. They will also have a thorough knowledge of modern office methods, card systems, etc., as used by the most successful and up-to-date commercial houses.

Bookkeeping and Business Forms Course.—This Course is intended for bookkeepers, stenographers, etc. that wish a business education. Many professional bookkeepers have taken this Course to get the up-to-date labor-saving methods that it contains.

Complete Stenographic Course.—This Course will qualify the student for the position of stenographer. Faithful study and practice will enable him in a few months to acquire the necessary speed, and, as competent stenographers are always in demand, he can readily secure a good position.

The School of English Branches

English Branches Courses.—These Courses, two in number, are intended for those that lack a common-school education, or wish to try Civil-Service Examinations.

Success Won From Defeat



I made three attempts to improve myself at night school, but gave up each time, as I was so backward. So up to 1895 I could only write my name and read a little. I then took the English Branches Course and can now write and spell well. While studying, I worked 84 hours per week, as in the steel business it is necessary to work Sundays. I am now a foreman, and my salary has been increased 60 per cent. My ability to hold the position is due to the Schools.

MICHAEL SULLIVAN, *Latrobe, Pa.*

The School of Textiles

Complete Cotton Course.—This Course is intended to qualify those that complete it for the position of superintendent, or purchasing or selling agent, of a cotton mill. It will also greatly increase the efficiency of agents, superintendents, overseers, second and third hands, mechanics, spinners, loom fixers, weavers, and all other workers in a cotton mill.

Cotton Carding and Spinning Course.—This is intended for yarn-mill superintendents, carders, spinners, section hands, card grinders, and all employed in cotton-yarn mills. It is also recommended for cotton-machinery erectors, yarn agents, hosiery-mill superintendents.

Cotton Designing Course.—This Course is intended for designers and assistant designers, agents, superintendents, overseers, and second hands in weave rooms; section hands, loom fixers, weavers, in cotton mills; dry-goods merchants, salesmen in commission houses, jobbers, and all men or women working in cotton mills, interested in cotton designing, or desirous of qualifying for any of the foregoing positions.

Cotton Spinning and Warp Preparation Course.—This is intended for superintendents of thread mills, boss spinners, second and third hands in spinning rooms, overseers of beaming, slasher tenders, and all men employed in cotton-yarn mills.

A Shipping Clerk's Advancement

As a result of your training by mail I am at present bookkeeper for the Randolph Manufacturing Co., at Franklinville, N. C. When I enrolled in your Complete Cotton Course I was working as shipping clerk for the same concern, at a salary only half what I am now receiving. I heartily indorse instruction by mail in Textile Design, as conducted by the I. C. S. The lessons are plain and to the point, and contain only information that is valuable.

HUGH PARKS, *Franklinville, N. C.*



The School of Textiles

Cotton Warp Preparation and Plain Weaving Course.

This is intended for superintendents of weave mills, boss weavers, fixers, second and third hands in weave rooms, weavers, slasher tenders, etc. It is also recommended for those having a good knowledge of yarn-mill machinery that wish to qualify as general mill superintendents.

Fancy Cotton Weaving Course.—This Course is intended for designers and assistant designers, agents, superintendents, overseers, and second hands in weave rooms, section hands, loom fixers, weavers, and all others that desire a knowledge of warp preparation and weaving machinery for fancy goods.

Cotton Carding, Spinning, and Plain Weaving Course. This Course is intended for those that desire to qualify for the position of superintendent or overseer in mills making plain cotton goods.

Theory of Textile Designing Course.—This is intended for those that have a good knowledge of warp preparation and weaving machinery, but whose knowledge of the theory of designing is not complete. It is recommended to boss weavers, overseers, second hands, and others that desire instruction on the analysis and reproduction of fabrics, and the drafting of designs. Almost every textile designer in the country is hampered in his business by the lack of competent designers.

A Mill Worker's Advancement



When I enrolled in your Cotton Carding and Spinning Course I was section hand in the card room of Mill No. 3 of the Eagle & Phoenix Mill Co., of Columbus, Ga. I am now second hand in their Mill No. 1. Your training has increased my salary and bettered my prospects. Correspondence instruction is the only means by which millmen may qualify themselves for promotion. My advice to ambitious young men is that they enroll in the I. C. S. and study in their spare time.

A. W. PITTS, *Phoenix City, Ala.*

The School of Textiles

Complete Textile Designing Course.—This is intended for designers, assistant designers, agents, superintendents, overseers, and second hands in weave rooms; and for section hands, loom fixers, and weavers in cotton, woolen, worsted, or silk mills.

Woolen and Worsted Designing Course.—This Course is intended for designers and assistant designers, agents, superintendents, overseers, and second hands in weave rooms; section hands, loom fixers, and weavers in woolen and worsted mills; dry-goods merchants, salesmen in commission houses, jobbers.

Complete Woolen Course.—This Course is intended for agents, superintendents, overseers, and section hands in woolen mills, master mechanics, weavers, loom fixers; in fact, for any that hold a position in a woolen mill.

Woolen Carding and Spinning Course.—This is especially recommended to superintendents, boss carders, boss spinners, section hands, card and mule erectors, woolen-machinery salesmen, woolen-yarn agents and merchants, and also for the superintendents and overseers of knitting mills using woolen yarns.

Woolen Warp Preparation and Weaving Course.—This is intended for all that desire a knowledge of the preparing of woolen warps, and the weaving and elementary designing of woolen goods.

Promoted to Pattern Weaver

When I enrolled for your Textile Designing Course I was a weaver. Since then, the instruction I have gained from it, together with my practical experience, has enabled me to advance to the position of pattern weaver, with the Springville Manufacturing Company. As a result of my experience, I am free to state that I consider your correspondence instruction in the manufacture of textiles to be all that is claimed for it.

C. W. SEIFERT,
128 West Main St., Rockville, Conn.



The School of Pedagogy

Teachers' Course.—This Course is intended especially for men and women that wish to qualify for remunerative and responsible positions in the teaching profession.

Methods of Teaching Course.—This Course provides teachers with thorough instruction in the best modern methods of teaching, and enables them to qualify for the highest salaried positions.

Drawing, Sketching, and Perspective Course.—This is described under the School of Design, page 22.

The School of Lettering and Sign Painting

Lettering and Sign Painting Course.—This is intended for sign painters, window dressers, designers of lithographs, book and magazine covers, advertising matter, etc., wood and metal engravers, jewelers, stone cutters, wood carvers, draftsmen, stencil makers, retail merchants, clerks, and all others that wish a knowledge of correct styles of lettering.

Show-Card Writing Course.—This is intended for business men, clerks, window trimmers, letterers, and all others that desire a knowledge of effective and artistic show-card writing. It includes the only complete and up-to-date instruction on the subject that has ever been prepared.

Lettering Plates Are Unexcelled



I think your Course is the most satisfactory that can be given. The Bound Volumes constitute a reference library that is the best to be had, and even if I did not go through the Course, I have it printed so plainly that I could teach myself without assistance, and your lettering Plates are unexcelled; they are the correct thing. My position now is Sign and Ticket Writer for the Robert Simpson Co., Limited, Department Store, of Toronto.

SIDNEY SMITH,
414 Ossington Ave., Toronto, Ont., Can.

The School of French

French Course.—This is intended for people of culture, civil engineers, business men, and all that desire a conversational knowledge of the French Language. The Course is taught by a system of instruction with the phonograph, designed and used only by the Schools.

The School of German

German Course.—This is taught with the aid of the phonograph, and is intended for professional and business men and all that come in contact with German-speaking people.

The School of Spanish

Spanish Course.—This Course is intended for lawyers, physicians, professional and business men, and all those whose duties bring them in contact with the Spanish-speaking people of South America, Cuba, Philippines, or Spain. This Course is taught with the aid of the phonograph.

The School of Navigation

Ocean Navigation Course.—This Course will qualify seamen, etc. to pass examinations for master or mate of ocean-going vessels. It is the most complete Course on this subject that has ever been published, and graduates of this Course will be qualified to pass any nautical examination in the United States or foreign countries.

Lake Navigation Course.—This Course is intended particularly for those that wish to pass examinations for masters or pilots of lake and coast vessels, and also for those that wish to increase their grade of license. It is the first and only Course on this subject that has yet appeared. Graduates of this Course will be qualified to pass examinations for master's or pilot's license for vessels engaged in lake and coast navigation.

The School of Advertising

Advertising Course.—This Course is designed to teach the student how to analyze any article of merchandise, find its selling points, and write and lay out a first-class ad that will so present these selling points as to impel the reader to become a purchaser of the article advertised.

The Course also includes instruction in the preparation of miscellaneous retail advertising matter; retail advertising management; department-store advertising; and the establishment of an ad-writing business to be conducted at home or in an office. The Course is designed to qualify the student to write good advertising for any or all retail lines of business, including department stores, and to train him for later advancement to the position of advertising manager.

The School of Window Dressing

The Window Dressing Course.—This Course is an elaborate treatise on window dressing, store decoration, etc. in all its branches. It embraces practically every form of commercial decorative treatment, for nearly all classes of business. The Course is an epitome of the ideas and writings of the most prominent decorators in this country. This Course will not only advance the window dresser to the front ranks of his profession, but will also qualify any one to become a competent window dresser.

The School of Law

Commercial Law Course.—This Course is designed to aid persons to become better equipped to conduct their business; through it an opportunity is offered them to attain a knowledge of the well-established legal rules and principles that apply to and govern business transactions.

The Course will be found especially useful by the following: Law students, mercantile persons, manufacturers, superintendents, bankers, insurance officials, bookkeepers, administrators, justices, conveyancers, etc.

